

APPLYING VOLATILITY AND EVT MODELS TO U.S., CHINESE AND RUSSIAN
STOCK MARKETS

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Master of Science

by
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ABSTRACT

Purpose: The purpose of this study is to explore the ability of EWMA, GARCH (1,1) and EGARCH (1,1) to forecast volatilities of S&P500, SSEC and MICEX, reference to two time periods in the timeframe of the Syrian war. VaR is derived using the HS approach which incorporates in its calculation the volatility of the best chosen model. The added value is the application of EVT in order to determine VaR results, which are compared and analyzed to the results of the HS approach, to define the most accurate approach.

Methodology of Work: Returns of the in-sample period prices are used in estimating the parameters of the three applied models. The calculated in-sample parameters are used to estimate the in-sample and out-of-sample volatilities. RMSE, MAE and MAPE are the error statistics applied in comparing volatility results, to obtain the most accurate volatility model, for both sample periods. VaR derived from the Historical Simulation volatility is calculated using the chosen model's parameters. On the other hand, VaR results are also obtained when applying the Extreme Value Theory using Matlab. EVT VaR and HS VaR are compared to the realized VaR to analyze the accuracy of the models.

Findings: Analysis and comparison of results show that the EVT VaR approach outperformed the HS VaR approach through providing more accurate results as compared to the realized VaR. On the other hand, the GARCH (1, 1) model outperforms EGARCH (1, 1) model for S&P 500, for both the in-sample and out-of-sample period. Moreover, our results show that EGARCH (1, 1)

model outperforms GARCH (1, 1) model for the out-of-sample period; which prove that EGARCH (1, 1) among the asymmetric models outperformed symmetric models used.

Limitations and implications of the research: One limitation relates to the limited number of countries chosen in the portfolio tested; which comprises of only three indices. This is mainly due to the strong influence of the chosen counties on the world's military production. Moreover, the chosen in-sample period extending from January 2015 till December 2016 might not be the ideal period, since it did not witness the burst of the Syrian war. However, if an earlier time period was to be chosen to entail former years of war, then the studied results would be obsolete. Finally, it would be interesting to derive a panel of daily EVT VaR results over a specific period to assess the trend of variations in these results, instead of deriving the results on a one-day basis.

Practical implications: Results concluded in this study are helpful for decision makers (investors, firms, governments, etc.) willing to invest in any of U.S., China and Russia. Results depict the degree of influence of the Syrian war on the studied countries' economies based on their high degree of intervention. Accordingly, investors can forecast and manage their risk exposure to limit any possible future losses.

Originality/value: This study tackles a combination of three stock market indices to form a portfolio of the three most powerful military countries of the world; U.S., China and Russia. Specifically, this is done to study the impact of the intervention of the chosen countries in the Syrian war.

Keywords: S&P500, SSEC, MICEX, volatility, realized volatility, implied volatility, EWMA, GARCH, EGARCH, in-sample, out-of-sample, VaR, EVT, Historical Simulation approach.

Chapter 1

Introduction

1.1. General Background

Political uncertainty occurs due to many factors like elections and changes in the government or parliament, changes in policies, strikes, minority disdain, foreign intervention in national affairs and others. In many cases, these uncertainties lead to further complications affecting the economy and financial market of the concerned country. Accordingly, the currency could devalue, prices of assets, commodities and stocks could fluctuate and the growth of the economy would be hindered. From this perspective, countries strive to keep political risks controlled to be able to endure the cost or consequence of any sudden political unrest. This is the main reason behind the intervention of powerful countries in the political and military affairs of less powerful countries. However, this is done at a high cost for both the powerful country and the unstable countries. This is mainly due to high budget costs, loss of resources, defocus on profitable forgone investment opportunities and other. This thesis studies the impact of the intervention of the three most powerful military countries in the world in the Syrian war on their market volatility to provide further guidance for investors and decision makers.

In March 2011, large peaceful protests broke in Syria to call for economic and political reforms with few armed protesters, leading to man arrests. Events evolved into violent acts from the side of the government using artillery and aircrafts, antigovernment rebels, terrorists and extremists' attacks, suicide attacks, explosive operations, intervention of foreign countries,

chemical weapons and others leading to a humanitarian crisis. In 2015, Russia started supporting the Syrian president through financial aids and military support (Humud et al., 2017). In the meantime, the United States was providing support for the local Syrians. Sooner, the United States and Russia increased their intervention in the war through arms, aircrafts and planned troop attacks; each supporting their own political interests and allies. In the meantime, China's involvement was shifting from humanitarian assistance and weapon exports (Swaine, 2012) to armed forces and increased weapon exports to support its allies' objectives from this war (O'Connor, T. 2018).

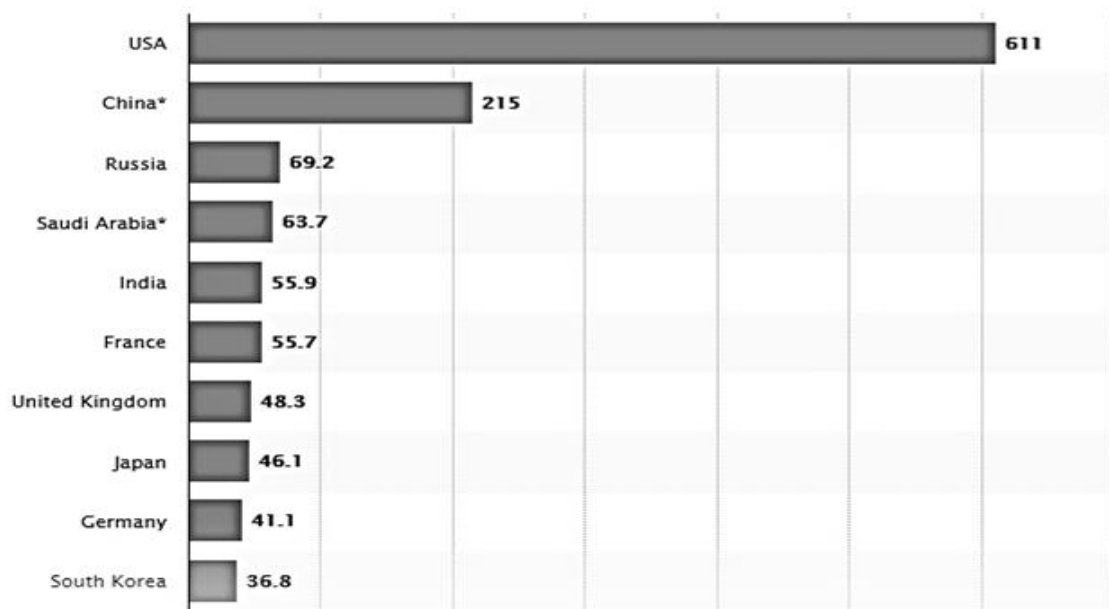


Figure 1: The 10 countries with the highest military spending worldwide in 2016 (in billion U.S. dollars) (Statista, 2018)

Figure 1 represents countries with the highest military spending in the world for 2016 (in billion U.S. dollars). The U.S spends the highest budget in the world on defense forces. This budget

rounded up to \$590 billion during 2015 and \$ 593 billion during 2016 for U.S. based on the International Institute for Strategic Studies (IISS, 2017). Based on the Department of Defense (DoD, 2016) statistics, the U.S. budget for 2015 was \$ 583 billion. However, this budget reaches \$ 611 billion during 2015 based on Stockholm International Peace Research Institute (SIPRI, 2015). In addition, the defense spending of the United States alone is higher than the next eight highest defense spending countries combined. These countries include China, Russia, Saudi Arabia, India, France, UK, Japan and Germany (SIPRI, 2015).

The country with the second highest defense budget is China. Its budget reached \$ 142 billion during 2015 and \$144 billion during 2016 based on the International Institute for Strategic Studies (IISS, 2017). Based on the Department of Defense statistics, the Chinese budget for 2015 was \$ 180 billion (China Power, 2015). However, this budget reaches \$ 215 billion during 2015 based on Stockholm International Peace Research Institute (SIPRI, 2015).

As for Russia, its budget reached \$ 52 billion during 2015 and \$44 billion during 2016 based on the International Institute for Strategic Studies (IISS, 2017). However, this budget reaches \$ 69 billion during 2015 based on Stockholm International Peace Research Institute (SIPRI, 2015).

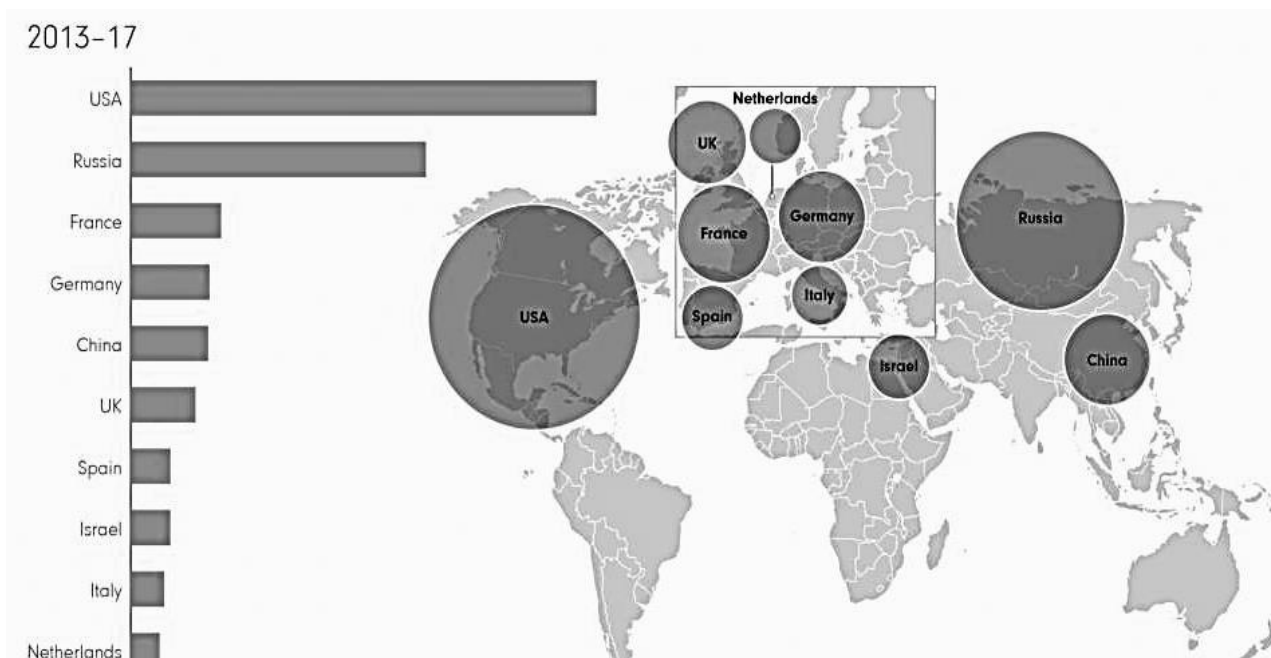


Figure 2: The 10 Largest Arms Exporters (SIPRI, 2017)

Figure 2 represents the 10 largest arms exporters in the world between 2013 and 2017. Besides having the highest budgets on defense, U.S. and Russia are also the top exporters of arms and China is among the top 5 worldwide countries.

Based on the above information, the importance of U.S., China and Russia between military countries is highly reinforced. For this reason, it is crucial to study their financial markets to comprehend the risks and opportunities they might face, which would affect their worldwide exposure. Nevertheless, forecasting volatility for asset returns is necessary to make effective risk management decisions. Financial institutions are increasingly using Value at Risk (VaR) as a tool for risk management. A critical input in calculating VaR is the future volatility of returns. As a

result, overestimating volatility leads to lost opportunities due to locked capital and underestimating volatility leads to unprotected risks (Ding and Meade, 2010).

Value at Risk (VaR) became of great importance to managers due to the huge losses incurred on financial institutions after several financial crises in the 1990's (Piroozfar, 2009). VaR was founded by JP Morgan in an attempt to measure the total risk exposure of a certain portfolio in one number that managers can comprehend and accordingly supervise and regulate possible losses. In general, risk consists of different types including: credit, liquidity, operational and market. The main approaches used to calculate market risks using VaR are the historical Simulation approach, Model Building approach, variance-covariance, Monte Carlo simulation and Extreme Value Theory (Hull, 2012).

Diverse models to monitor volatility are used in an attempt to manage portfolio risks. Among these models is the EWMA model that provides lower weight to older, less recent data to monitor changes in volatility. EWMA verifies the assumption that most recent data have more influence on volatility results than older data. EWMA model assumes that volatility is not constant through time and responds to changes when they occur (Korkmaz and Aydın, 2002).

The GARCH (1, 1) model is used to calculate a "long-run average variance rate" based on the latest data observations and latest variance estimations. This model is assumed to be a particular calculation of the GARCH (p, q), which is the general model (Hull, 2012). The GARCH (1, 1) is the most popularly used model of all GARCH models. The sum of the parameters in GARCH (1, 1) should be equal to one and the maximum likelihood function is applied to estimate them.

Limitations of the GARCH (1,1) model are tackled in second generation GARCH models. Among these models is the Exponential GARCH (EGARCH). The main difference between the EGARCH model and the GARCH model is the log of the variance (Ali, 2013). EGARCH model is able to capture the “leverage effect” in the return from stocks in order to model asymmetric variance effects. As a result, the sign of the parameters can be negative or positive (Schmitt, 1996).

When traditional VaR models are used, researchers would be 95% confident about the outcome; which is not enough since a 5% margin of loss could cost investors millions of dollars. For this reason, the Extreme Value Theory (EVT) was introduced to calculate VaR with a confidence level higher than traditional VaR methods. This is done through estimating the distribution of tails in a given portfolio to study possible volatilities. EVT is best used when the distribution of returns is fat tailed, since other VaR models like variance-covariance model and the historical simulation approach lead to inaccurate results, especially when the long term VaR is being forecasted. Moreover, EVT is used for the examination of univariate distributions; whereas other VaR models consider many risky factors in the market under study. Another difference is the requirement of a bootstrap procedure, which increases the computational process to reach the desired outcome (Odening and Hinrichs, 2003).

1.2. Purpose of the Study

This research attempts to explore and assess several types of volatility models applied on the stock market indexes of United States, China and Russia in the period of their intervention in the

war in Syria. United States, China and Russia are chosen since they are the most dominant military powers of the world and are intervening in the Syrian war.

The studied stock market indexes are S&P 500 for U.S., SSEC for China and MICEX for Russia. Data for 3 years are used to conduct in-sample and out-of-sample forecasts. Data for the in-sample forecast include years 2015 and 2016, taking it as a reference for a “stressed market” state since the beginning of the intervention of the chosen countries in the war in Syria (Humud et al., 2017). Data for the out-of-sample forecast include years 2017 and 2018. Both data samples are tested using the previously mentioned models in order to reach the objectives discussed below.

The main objective of this thesis can be achieved by answering the following questions:

- Which type of volatility model is best for the stock market indexes of the world’s most authoritative military forces: EWMA, GARCH (1, 1) or EGARCH (1, 1) models?
- Which type of volatility model is the best model for each studied period?
- Will the VaR results of Extreme Value Theory be more accurate than the actual VaR?
- What would be the recommendation to investors wishing to invest in U.S., Chinese and Russian economies, based on the results of volatility and VaR models?

1.3. Need for the Study

In general, this thesis highlights the effect of a world crisis/war on the economy of the most powerful military countries of the world. Moreover, researchers could refer to this thesis in their studies on the Russian financial market, since few articles and publications are implemented on

the Russian market. This is evident in the upcoming section (Literature Review). Moreover, this thesis covers the Russian financial market from a risk perspective through combining and comparing the latter's analysis to those of U.S. and China. Thus, the financial markets of the three most powerful military countries are studied in relation to their intervention in the Syrian crisis.

Results concluded in this thesis are helpful for decision makers (investors, firms, governments, etc.) willing to invest in any of U.S., China and Russia. Results depict the degree of influence of the Syrian war on the studied countries' economies based on their high degree of intervention. Accordingly, investors can forecast and manage their risk exposure to limit any possible future losses.

From a theoretical perspective, this thesis compares and evaluates the results of different volatility models applied on different countries in reference to two chosen time periods (in-sample and out-of-sample). Based on the outcomes of EWMA, GARCH (1, 1) and EGARCH (1, 1), results will be examined and analyzed. The best model will be identified and the requirement for asymmetry and leverage effect will be discussed in relevance to the studied stock market indexes. Another added value is the application of the Extreme Value Theory to calculate VaR. EVT allows researchers to explore extreme deviations in the available data in order to calculate the probability that such events might occur. EVT is applied using a specialized platform, known as the EVIM software package from MATLAB.

1.4. Overview of Upcoming Chapters

This thesis consists of 5 chapters; starting with the current chapter being the Introduction. The following chapter cover a concise literature review on the implemented volatility and VaR models. The findings of several research papers are included and compared with a detailed focus on the papers that cover U.S., China and Russia and more specifically cover the indexes chosen in the thesis.

Chapter 3 details the methodologies used and specific features and conditions of each model. The Jarque-Bera normality test is implemented on the S&P 500 for U.S., the SSEC for China and MICEX for Russia. Moreover, descriptive statistics are presented for both sample periods. The EWMA model, GARCH (1, 1) model, EGARCH (1, 1) model and EVT are discussed theoretically.

Chapter 4 covers the results obtained from applying the discussed methods. Analysis and comparison of results are clearly identified at the end of each section. Returns of the in-sample period prices are used in estimating the parameters of the three applied models. Moreover, parameters are derived from NumXI upon implementing the following distributions: normal distribution, student t-distribution, and GED. Volatility estimates are derived for the in-sample and out-of-sample periods using the chosen parameters. RMSE, MAE and MAPE are the error statistics applied in comparing volatility results, to obtain the most accurate volatility model, for both sample periods. Parameters of the chosen model are used for to calculate the Historical Simulation volatility update VaR. On the other hand, VaR results are also obtained when applying

the Extreme Value Theory using Matlab. EVT VaR and volatility update VaR are compared to the realized VaR to analyze the accuracy of the models.

Chapter 5 concludes the explanation of the results obtained. Moreover, limitations and suggestions for future analysis are provided.

Chapter 2

Literature Review

Financial institutions' main concern is to observe market variables and monitor their volatilities. These market variables include interest rates, exchange rates, equity and commodity prices and other financial instruments that form a financial portfolio. Volatility is not recognized as a constant value, since at certain periods it is low and other periods it is high. To keep track of such variations, several models are designed to calculate volatility throughout specified time periods (Hull, 2012). These models include EWMA and GARCH; which are further expanded to include other models like GJR GARCH, EGARCH and Quadratic GARCH. Several researchers applied these models on U.S., Chinese and Russian stock markets, where results and comparisons are discussed in the literature.

In his paper, Wei (2002) was interested in forecasting stock market volatility with non-linear GARCH models on the Chinese stock market returns. He specifically applied the Quadratic GARCH (QGARCH) and the Glosten, Jagannathan and Runkle model GARCH (GJR GRACH) as nonlinear modifications to the original GARCH model. Weekly data for seven years are observed for the Shanghai Stock Exchange Composite (HSEC) and Shenzhen Stock Exchange Component (ZSEC) in China. Wei concluded that for China's stock market indices the QGARCH outperformed the linear GARCH model and did not recommend the GJR GARCH model.

Similar conclusions were established by Romero and Kasibhatla (2013), who examined price equity indices and returns of the emerging markets of the BRICs countries (Brazil, Russia,

India and China). Out-of-sample forecasts were performed on returns using GARCH models. The student-t distribution for errors was concluded to be most accurate for Russia's RTS equity index for both symmetric and non-symmetric GARCH models applied; whereas the generalized error distribution (GED) was most accurate for China's SSE equity index. When forecasting returns' conditional volatility, nonlinear models outperform linear models of volatility.

Lin and Fei (2012) concluded that the APGARCH model outperformed other GARCH models on different time scales in estimation of "long memory property of Chinese stock market". The forecast included the Shanghai and Shenzhen stock markets. In short, nonlinear volatility models are recommended to describe long memory of stock more than linear models, due to the accuracy of their results.

Hou and Li (2015) investigated the transmission of information between U.S. and China's index futures markets using an asymmetric DCC GARCH approach. Their work focused on daily return and volatility spillover between well-known U.S. stock market represented through the S&P 500 stock index futures market, and the newly founded Chinese stock market, represented through the CSI 300 stock index futures market. A bivariate GARCH structure is used to study the relation between the mentioned markets. The authors reached a conclusion that using the ADCC GARCH model past information is the base for the correlation between U.S. and Chinese index futures market. This correlation increases with the rise of negative shocks in these markets. Moreover, it is affirmed that the U.S. index futures market is more efficient in its price adjustment compared to the Chinese market, since it is older and more mature.

To focus on the U.S. stock market, Awartani and Corradi (2005) predicted the volatility of the S&P 500 Composite Price Index using GARCH models to show the function of asymmetries found using out-of-sample time horizons. Squared returns of the data is used, since it measures volatility and locks accurate ranking of losses. As a conclusion, asymmetric GARCH models outperform GARCH (1, 1) model, which highlight the predictive ability of the asymmetric models. Nevertheless, the lowest predictive model is the Risk Metrics variance model.

Zhe (2018) selected the SSE Composite Index to conduct empirical analysis using GARCH models on a period ranging from 2013 till 2017. The uncertainties in the Chinese stock market lead to constant changes in prices which affects the market return. The SSEC index series distribution shows that returns are leptokurtic. Zhe used GARCH models due to its adaptability to price fluctuations. Specifically, the forecasting performance of the SSEC index is tested knowing that the Chinese stock market is highly volatile. Multiple models are applied which comprise of the symmetric GARCH (1, 1) model and the asymmetric Threshold GARCH (TGARCH) (1, 1) and EGARCH (1, 1) models. The asymmetric models outperformed symmetric models in the forecasting results. This is due to the influence of negative and positive return shocks. Results show that the EGARCH (1, 1) among the asymmetric models outperformed the other models used in his research.

To further expand the research on risk limits, Furio and Climent (2013) argue that fluctuations in stock prices are of a high frequency yet have a small impact on returns. For this

reason, these variations are considered to be normally distributed when studied in a larger scale. On the other hand, when stock prices highly fluctuate this leads to massive losses and in certain cases to a market crash. However, such severe events occur a lot more than expected under the normal assumption of returns. For this reason, Furio and Climent (2013) found that it is important to highlight in their work on the distribution of tails, to study extreme movements in the return of several stock prices. They worked on three index returns that represent “important financial areas in the world”: S&P500, FTSE 100 and NIKKEI 225. They focused on analyzing and comparing the estimates of GARCH-type models to EVT estimates. Results point out that more accurate estimates are derived from EVT calculations in both in-sample and out-of-sample, as compared to less accurate estimates using the GARCH model. This helps investors and decision makers to predict such fluctuations in prices and take advantage of this information to take a strategic position in the market.

From this perspective, the Extreme value theory (EVT) is applied to estimate the tails of a distribution. In other words, its aim is to model and measure extreme risks in a given distribution. EVT is used to estimate VaR with high confidence levels and to calculate the expected shortfall (ES) (Hull, 2012). EVT application could be achieved through 5 different distributions including the generalized Pareto distribution (GPD) and the generalized Extreme value (GEV) and the generalized Logistic (GL). Moreover, for the above mentioned distributions, different models are used like the Block Maxima Minima method (BMM) and the Peak over Threshold (POT) (Hussain and Li, 2014). From the above discussed methods, each one has certain limitations. When

comparing traditional VaR measures to EVT, EVT avoids big error problems since it is reinforced by concrete mathematical models and principals. VaR and ES based on EVT lead to strong “analytical expression”. EVT is most appropriately used when aiming for high confidence intervals. However, when integrated with other models like Historical Simulation or variance-covariance model; this leads to more accurate VaR estimates with lower confidence intervals (Wang et al., 2010). Wang et al. also propose the extension of EVT from a univariate analysis to a multivariate analysis that can accurately measure VaR of a portfolio. We present below the work of various authors on the application of EVT with other models and different comparisons to reach the best VaR outcome for different countries.

Wang et al. (2010) implemented an EVT based VaR and ES to estimate the exchange rate risk of the Chinese currency CNY. They compared the results of the Historical Simulation approach and the variance-covariance method; where they found that the EVT based VaR estimation produces accurate results for the currency exchange rate risks of EUR/CNY and JPY/CNY. This is validated by passing the back testing process. However, EVT underestimated this risk for USD/CNY and HKD/CNY; which was evidenced by back testing the results. This could be due to the continuous appreciation of the Chinese currency against the U.S. dollar and Hong Kong dollar. However, when VaR results from the Historical Simulation approach and variance-covariance method are compared to EVT value, the latter provides more accurate risk measures for EUR/CNY and JPY/CNY exchange rates.

To continue the assessment on the Chinese market, Chen et al. (2012) estimated VaR and ES by applying EVT on 13 worldwide stock indices. They concluded that China ranks first for VaR and ES with negative returns and ranks third for positive returns with high levels of risk. Moreover, a positive correlation proved to exist between the fluctuations of China's stock market and other stock markets of the countries under study. However, when comparing large fluctuations in values of the stock market of China to all other countries, it is important to note that such movements are "asymptotically independent" from the movements in the stock market of all other countries. Moreover, when creating an international portfolio of equity funds, investors in Chinese stocks tend to diversify their portfolio through choosing other international stocks with lower dependency to the Chinese market.

"Multi-fractal variations" in prices are studied for developed markets as well as in emerging markets. In their work, Wei et al. (2013) focus their study on the developing market of China. They apply their study on the SSE index of China and introduce a new method of studying price variability by combining a multifractal volatility (MFV) model and EVT. GARCH models like GARCH (1, 1), IGARCH and EGARCH (1, 1) are implemented to compare VaR results of the previous method and then back tested to examine the performance of the models. As a conclusion, VaR results obtained from the MFV-EVT method are more accurate and outperform the other GARCH models applied.

Hussain and Li (2014) focus on the effect of extreme returns in stock markets on risk management, by studying the case of China's emerging market and the SSE index. SSE

extreme daily returns are used in applying EVT with its three familiar distributions: Generalized Extreme Value (GEV), Generalized Logistic (GL) and Generalized Pareto distributions. Various time intervals of extreme daily returns are studied using a Block Maxima Minima method (BMM). Results show that the Generalized Extreme Value (GEV) distribution is the best fit model for extreme upward maxima series of stock movements; which contradicts the findings of other stock markets like the United States. The Generalized Logistic (GL) distribution is the best fit model for extreme downward minima series of stock movements. These results improve the calculation of VaR for China's stock market, which in turn influences the market's risk management.

Based on the work of Peng et al. (2006) on fat tailed distributions, they try to verify whether the EVT General Pareto Distribution (GPD) is superior to certain GARCH models, implemented on the Shanghai Stock Exchange Index. The sample data used is the index closing prices from 1 July 1999 to 10 May 2005, with 1408 observations. Peng et al. state that EVT and GARCH models both have the statistical characteristics needed to study the fat tail performance in their specified time period given the high frequency data available. Peng et al. test and estimate the tail index in addition to forecasting the VaR. Empirical results of the extreme value theory and modeling methods are analyzed and compared. The concluding outcome demonstrated that the EVT- GPD method outperforms the GARCH models used to estimate VaR. The main advantage of the GPD is that the latter was able to expand and reach out of the specified data sample to estimate more accurate VaR results. In specific, the GPD model outperformed the GARCH (normal), GARCH (GED) and GARCH (t-student). In turn, when comparing the performance of the GARCH models

only, the GARCH (t-student) model showed superiority to the other models. The last in rank is the GARCH (normal) model.

The work of most authors reviewed in this thesis recommended the use of EVT over other GARCH models to calculate VaR. On the other hand, others argued that when EVT is integrated into the Historical simulation approach lower confidence intervals are attained. From this perspective, we aim at applying the above discussed models on U.S., China and Russia to calculate VaR and reach conclusions about the most effective combination of models. Moreover, the lack of studies applied based on the Russian stock market is evident in the literature, with little work implemented on the BRICS countries. This is considered as an added value for this paper since it not only covers Russia in its estimation, but also includes it in the comparison of the VaR of the three most powerful military countries of the world.

Chapter 3

Procedures and Methodology

3.1. Introduction

Financial risk managers' primary concern is to forecast future possible variations in returns. VaR is estimated to disclose the worst expected loss given a specific time frame and confidence level not to be exceeded. VaR uses market and historical prices or data to compare investments in different markets. VaR estimation methods include variance-covariance, Monte Carlo Simulation and Historical Simulation approach (HS). Variance of returns of a given financial asset could be forecasted using a time series financial model (Wei, 2002). In the previous chapter, the review of literature gave a brief overview about the different variables that researchers used to control the focus of their study. The variables presented in this chapter include the volatility models, data and time frame specified and VaR models selected. Volatility models selected in this thesis are applied in details. These models include the EWMA, GARCH (1,1) and EGARCH (1,1). The Extreme Value Theory (EVT) is "a branch of statistics dealing with the extreme deviations from the median of probability distributions" (Cao et al., 2015). Moreover, the main objective of using EVT is to forecast extreme events that will take place in the future. From this perspective, EVT is implemented to derive VaR estimates with high confidence interval.

3.2. Data Selection

Data for 3 years are extracted from the Bloomberg platform for the three selected stock market indexes: S&P 500 for U.S., SSEC for China and MICEX for Russia. In-sample forecasts include

data ranging from January 2015 till December 2016, representing two years of stressed market conditions in relevance to the deteriorated conditions of the war in Syria. Whereas out-of-sample forecasts include data extending from January 2017 till May 2018. The above mentioned data is manipulated to derive the return from the closing prices of the three stock market indexes. For the in-sample period, 458 daily observations are studied. While for the out-of-sample forecast, 315 daily observations are extracted.

The studied stock market indexes S&P 500, SSEC, MICEX are chosen since they are considered as a proxy for the performance of the market of their representative country. The S&P 500 (Standard & Poor's) is considered to be the leading instrument in the equities market in U.S. S&P 500 index consists of the leading 500 companies from major industries employed in the economy of U.S. (S&P 500, 2009). The SSEC (Shanghai Stock Exchange Composite) Index constituted of the stocks listed on the Shanghai Stock Exchange; A and B shares. The Shanghai stock market's performance could be concluded from the performance of the SSEC index (CSIndex, 2018). As for the MICEX (Moscow Interbank Currency Exchange) index, it is composed of Russian stocks of the top 50 largest issues in the Moscow Exchange. The performance of the MICEX index is considered as a scale for the performance of the Russian stock market. In November 2017, the name of the MICEX index was officially changed to MOEX Russia Index, representing the "Russian stock market benchmark" (Moscow Exchange, 2017).

Table 1 represents the descriptive statistics of S&P 500, SSEC, and MICEX for the in-sample period, while Table 2 represents descriptive statistics for the out-of-sample period.

Stock Markets	S&P 500	SSEC	MICEX
Mean	0.000224932	-0.000167852	0.000968412
Standard Deviation	0.009523965	0.020954624	0.011858006
Skewness	-0.158919226	-1.136622175	-0.163607662
Kurtosis	3.326696814	4.617609276	0.971300062
Median	0.000145283	0.001028966	0.000398498
Minimum	-0.040211416	-0.108323598	-0.044036995
Maximum	0.047498924	0.060292716	0.044034431
1st Quartile	-0.41%	-0.66%	-0.65%
3rd Quartile	0.49%	0.87%	0.85%

Table 1: Descriptive Statistics of S&P 500, SSEC, and MICEX: Jan. 05, 2015 – Dec. 30, 2016

Stock Markets	S&P 500	SSEC	MICEX
Mean	0.000601	-0.000008	0.000098
Standard Deviation	0.007113	0.007737	0.010253
Skewness	-1.16	-1.05	-1.10
Kurtosis	7.93	4.83	12.45
Median	0.000602	0.000757	-0.000101
Minimum	-0.041843	-0.04137	-0.080255

Maximum	0.027590	0.021461	0.038873
1st Quartile	-0.001880	-0.00357	-0.005098
3rd Quartile	0.003406	0.004059	0.005469

Table 2: Descriptive Statistics of S&P 500, SSEC, and MICEX: Jan. 03, 2017 - May 31, 2018

The skewness of the returns of S&P 500, SSEC, and MICEX (January 05, 2015 to December 30, 2016), and S&P 500, SSEC, and MICEX (January 03, 2017 to May 31, 2018) is approximately equal to 0. The returns of S&P 500, SSEC, and MICEX in reference to the two chosen time periods display excess in kurtosis. This implies that the distributions of returns are not normal.

The results of Jarque-Bera tests for the in sample and out of sample time periods are presented in table 3 and table 4 below. NumXL is used to conduct the above mentioned test to examine the normality of returns of the three studied markets.

Stock Markets	Score	C.V.	P-Value
S&P 500	198.66	5.99	0.0%
SSEC	483.75	5.99	0.0%
MICEX	18.12	9.21	0.0%

Table 3: Jarque-Bera Normality Test of S&P 500, SSEC, and MICEX: Jan. 05, 2015- Dec. 30, 2016

Stock Markets	Score	C.V.	P-Value
S&P 500	865.78	5.99	0.0%
SSEC	352.63	5.99	0.0%
MICEX	2027.03	5.99	0.0%

Table 4: Jarque-Bera Normality Test of S&P 500, SSEC, and MICEX: Jan. 03, 2017 - May 31, 2018

The p-values of the distribution of returns of S&P 500, SSEC, and MICEX for the in-sample and out-of-sample periods are equal to 0. Accordingly, their distributions of errors are not normal.

Test (5%)	STAT	P-Value	C.V.	Stationary
No Constant	-28.4	0.1%	-1.9	True
Const.-only	-28.4	0.1%	-2.9	True
Const. + Trend	-28.4	0.0%	-1.6	True
Const. + Trend + Trend²	-28.4	0.0%	-1.6	True

Table 5: ADF Stationarity Test of S&P 500

Test (5%)	STAT	P-Value	C.V.	Stationary
No Constant	-12.8	0.1%	-1.9	True
Const.-only	-12.8	0.1%	-2.9	True
Const. + Trend	-12.7	0.0%	-1.6	True
Const. + Trend + Trend²	-12.7	0.0%	-1.6	True

Table 6: ADF Stationarity Test of SSEC

Test (5%)	STAT	P-Value	C.V.	Stationary
No Constant	-26.9	0.1%	-1.9	True
Const.-only	-11.6	0.1%	-2.9	True
Const. + Trend	-11.6	0.0%	-1.6	True
Const. + Trend + Trend^2	-11.6	0.0%	-1.6	True

Table 7: ADF Stationarity Test of MICEX

The above tables 5, 6 and 7 present results of the Augmented Dicky Fuller (ADF) test for stationarity conducted on NumXL. The results for the three indexes S&P 500, SSEC and MICEX for the period 2015-2018 show that no transformation of data is needed and that the distributions of returns are stationary. As observed in tables 5, 6 and 7, the statistics calculated is lower than the critical value (C.V.) for all the indexes. Moreover, the P-values for S&P 500, SSEC and MICEX are less than one, thus the null hypothesis for the presence of a Unit-root is rejected.

3.3. Exponentially Weighted Moving Average (EWMA) Model

The EWMA model is defined through the following equation:

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1 - \lambda)u_{n-1}^2 \quad (1)$$

Where, σ_n^2 is the variance of today

The exponential factor λ is a number between 0 and 1.

n is the number of days under observation.

σ_{n-1}^2 is the variance of the previous day.

u_{n-1} is the daily most recent percentage change

EWMA model assumes that volatility is not constant through time and responds to changes when they occur (Korkmaz and Aydın, 2002). As a result, as the observed data move backward through a given time span, u_i declines at a rate λ . Consequently, any weight today is λ times the previous weight. Moreover, when applying equation (1), the volatility estimate increases when the realized value of u_{n-1}^2 exceeds its expected value and vice versa.

A unique characteristic the EWMA approach holds is that most recent data observations are subject to heavier weight. The most recent value of the market variables and current variances are needed at any given time. With every new observation, updates of the daily percentage change and variance rate are calculated and data from the previous observations are not used any more.

λ is the value that defines the extent to which daily percentage changes respond to daily volatility updates. From this perspective, when λ is low, a high weight is specified to u_{n-1}^2 as volatility is calculated. Accordingly, successive days of such volatility estimates will lead to high volatility. On the other hand, when λ is high (close to 1), daily volatility estimates respond in a slower manner to the changes in daily data. As a result, JPMorgan's RiskMetrics database uses an estimate of λ equal to 0.94 for daily data in their EWMA model. Based on different variables and forecasts, the specified value for lambda provides the closest forecast of the variance rate as compared to the realized rate (Hull, 2012).

3.4. Generalized Autoregressive Conditional Heteroskedastic (GARCH) Model

The GARCH (p, q) model is the broad form of the model, where multiple lags exist. This model is depicted in equation (2):

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^p \beta \sigma_{n-i}^2 + \sum_{i=1}^q \alpha u_{n-i}^2 \quad (2)$$

The GARCH (1,1) model is the most popularly used model of all GARCH models. When applying the GARCH (1,1) model, data from the most recent daily observations and variance rate estimate are used. For this reason, “(1,1)” defines that one observation is under study; as opposed to “(p, q)” where p estimates of the variance rate and q observations of u^2 are studied. The volatility estimate in this model is derived from a “long run average variance rate” (Hull, 2012, pp. 218). The GARCH (1,1) model is depicted in equation (3):

$$\sigma_n^2 = \gamma V_L + \beta \sigma_{n-1}^2 + \alpha u_{n-1}^2 \quad (3)$$

Where, σ_n^2 is the variance calculated today.

V_L is the variance reached in the long-run.

σ_{n-1}^2 is the previous day's variance.

u_{n-1}^2 is the previous day's square of return.

V_t has a weight γ , u_{n-1}^2 has a weight α and σ_{n-1}^2 has a weight β as assigned in equation (3). The Maximum Likelihood function is applied to estimate the value of each parameter. It is interesting to note that EWMA model is a specific case of GARCH (1, 1) model with different values for the parameters ($\gamma=0$, $\alpha=1-\lambda$ and $\beta=\lambda$).

Another format for the equation for the GARCH (1, 1) model is as follows:

$$\sigma_n^2 = \omega + \beta\sigma_{n-1}^2 + \alpha u_{n-1}^2 \quad (4)$$

Where $\omega = \gamma v_t$

This format makes it easier to calculate the model's parameters ω , β and α . Therefore, $\gamma = 1 - \beta - \alpha$ and the long term variance is ω/γ . To ensure the stability of the model, the sum of the parameters should be equal to one and $\alpha + \beta < 1$. When $\alpha + \beta > 1$, then the previous constraint is not achieved and the weight of V_L is negative. This is considered to be “mean fleeing” instead of “mean reverting”. When the variance rate is “mean reverting”, the reversion rate is equal to the weight assigned to the long run variance rate. For this reason, if V_L is negative the GARCH (1, 1) model is not stable. Moreover, if $\omega = 0$, hence V_L is zero and the GARCH (1, 1) model switches to the EWMA model.

In the application of the GARCH (1,1) model, it is important to examine the trueness of the variance process applied. To prove that the model is built right, an error series is conducted and should result in a constant variance and mean. Moreover, autocorrelation is tested using a Ljung box test with 15 lags (Engle, 2007). GARCH (1,1) is non-stationary, leptokurtic, includes extreme

values, accounts for the clustering of volatility in its calculation and has positive conditional variance (Galdi and Pereira, 2007). One limitation of the GARCH (1,1) model is that it fails to capture asymmetric performance, since it is symmetric by nature. GARCH (1,1) doesn't account for the leverage effect which is tackled by the EGARCH (1,1), as discussed in the following section.

3.5 Exponentially Generalized Autoregressive Conditional Heteroskedastic (EGARCH)

Model

The GARCH (1,1) model was further extended by researchers to realize the Exponential GARCH (1,1) model. The EGARCH model's main difference compared to the GARCH model is that it is asymmetric by nature. EGARCH accounts for leptokurtosis, skewness and leverage effect (Hull, 2012). Equation (5) represents the equation of EGARCH (1,1):

$$\log \sigma_n^2 = \gamma v_l + \beta g(z_{n-1}) + \alpha \log \sigma_{n-1}^2 \quad (5)$$

Where, σ_n^2 is the variance calculated today.

V_L is the variance reached in the long-run.

σ_{n-1}^2 is the previous day's variance.

$g(z_{n-1})$ is the explanatory variable for the "leverage effect".

V_L has a weight γ , σ_{n-1}^2 has a weight α and $g(z_{n-1})$ has a weight of β as assigned in equation (5); which includes in its structure the Maximum Likelihood function to calculate the parameters. The

GARCH and EGARCH models share some common characteristics including mean reversion and skewness. This part of the model $\beta g(z_{n-1})$ is able to capture the “leverage effect” in the return from stocks in order to model asymmetric variance effects. It is important to note that negative shocks tend to have a greater impact on volatility than positive shocks. However, the EGARCH model needs a positivity constraint in its formation (probability of conditional variance function positive and equal to one for all parameters). Thus, this model is stationary (Anyfantaki and Demos, 2012).

3.6 Extreme Value Theory

Extreme Value Theory (EVT) studies the distribution of the tail of an estimated sample (Gencay et al., 2001). EVT is used to calculate VaR estimates with high confidence interval, leading to more accurate results (Hull, 2012). For this reason, EVT is considered to be a risk management tool. The approach behind EVT is gaining more interest from experts in different fields including environmental science, insurance and finance in order to model and measure extreme events based on historical data and occurrences.

In general, there are two types of models used to calculate EVT: the “block maxima” model and the “peaks-over-threshold” (POT) model. The first model collects data from large samples of identical observations (daily or hourly records). The second model collects data from large samples surpassing a certain threshold or limit. The POT model is in turn divided into the “semi-parametric” models and the “parametric” model defined by the generalized Pareto distribution (GPD) which is the model used in this paper and detailed below (McNeil, 1999).

The results of VaR helps investors to quantify a certain loss based on a specific confidence level and time horizon. This loss denotes the market risk of a portfolio presented in one number that is representative. The main emphasis of EVT is the tails of the distribution sample. Accordingly, extreme fluctuations are detected to alert for extreme losses. (Marimoutou et al., 2009). This is done through “extrapolating” the tails of the distribution under study. To measure extreme losses of a distribution, consider $F(v)$ with variable v as the cumulative loss distribution over a given time period. Like any distribution, $F(v)$ has a right tail and a left tail and u is a value in the right hand distribution.

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} \quad (7)$$

Where, $F(u+y) - F(u)$ is the probability that v lies between u and $u+y$, with y greater than zero and v greater than u . As u increases, $F_u(y)$ converts to a generalized Pareto distribution (GPD) given in the following equation:

$$G_{\epsilon, \beta}(y) = 1 - \left[1 + \epsilon \frac{y}{\beta} \right]^{-1/\epsilon} \quad (8)$$

Where, ϵ determines the heaviness of the tail and β is a scale parameter. ϵ and β are calculated using the maximum Likelihood method. When v has a normal distribution, ϵ is equal to zero. And when the heaviness of the tails of the distribution increases, ϵ increases. In general, ϵ has a positive value between 0.1 and 0.4. In order to calculate these parameters, a probability density function of

the GPD is derived from calculating the GPD as a function of y . The probability density function $g_{\epsilon, \beta}(y)$ is as follows:

$$g_{\epsilon, \beta}(y) = \frac{1}{\beta} \left[1 + \frac{\epsilon y}{\beta} \right]^{-\frac{1}{\epsilon-1}} \quad (9)$$

Consequently, maximizing the following logarithmic equation which is used to find the values of the parameters ϵ and β .

$$\sum_{i=1}^{n_q} \ln \left[\frac{1}{\beta} \left(1 + \frac{\epsilon (v_i - u)}{\beta} \right)^{-\frac{1}{\epsilon-1}} \right] \quad (10)$$

A specific confidence interval is denoted with q in order to calculate VaR with equation (11) and the expected shortfall with equation (12).

$$VaR = u + \frac{\beta}{\epsilon} \left\{ \left(\frac{n}{n_q} (1 - q) \right)^{-\epsilon} - 1 \right\} \quad (11)$$

$$\text{Expected Shortfall} = \frac{VaR + \beta - \epsilon u}{1 - \epsilon} \quad (12)$$

(Hull, 2012)

Excel is usually used for the computation of EVT VaR for one day. However, to calculate a series of VaR results for the portfolio of indexes chosen in this thesis, more complex software is

needed. Consequently, analysis and calculation of the Extreme Value Theory is applied using EVIM software package from MATLAB (Gencay et al., 2001).

3.7 Conclusion

The discussed models are applied using three indices distributed into in-sample and out-of-sample periods. The in-sample period extends from January 05, 2015 till December 30, 2016 and the out-of-sample period extends from January 03, 2017 till May 31, 2018. The three indices S&P500, SSEC and MICEX represent U.S., China and Russia respectively. These indices are used in the application of volatility models. The indices chosen are a proxy of the performance of their countries. The combination of countries and time periods are selected due to their direct interdependence. In this thesis, we try to evaluate the performance of the most powerful military powers of the world during their period of military intervention in the War in Syria.

To conclude, the various characteristics of each model used in this thesis and the difference between each model are discussed. Each of the studied volatility models EWMA, GARCH (1,1) and EGARCH (1,1) have specific limitations. The EWMA model doesn't account for mean reversion and calculates overvalued volatility outcomes after severe price fluctuations. As opposed to second generation GARCH models, GARCH (1,1) doesn't incorporate leverage effect in its structure and by nature is symmetric. Whereas EGARCH (1,1) is asymmetric and incorporates the leverage effect. However, for the EGARCH (1,1) model to be stable, its parameters should be positive or less than 1. Moreover, the studied EVT model's main limitation is the need for a large number of collected data, since EVT disregards any data that is not an extreme value and renders

the observed records into a smaller dataset. Aside this, EVT is an important model used in risk management analysis that complements VaR models and extrapolates results to reach a confidence interval on tail estimates (Embrechts et al., 1999).

Chapter 4

Findings

4.1. Introduction

In the previous chapter, the process of data selection was presented, in addition to the descriptive statistics of each index. Furthermore, the methodology of processing each of the EWMA model, GARCH (1, 1) model, EGARCH (1, 1) model and Extreme Value Theory were detailed from the theoretical perspective, in addition to deriving VaR using each of the volatility estimates in the Historical Simulation approach and Extreme Value Theory.

On the other hand, this chapter covers the data findings collected from performing the selected models. First, in-sample returns are demonstrated through the descriptive statistics, the Jarque-Bera normality test and ADF stationarity test. The Jarque-Bera test defines the distribution of errors to check whether the data are normally distributed and the ADF test shows the stationarity of returns and whether they should be transformed. The next section presents the results of the estimated parameters with respect to each of the volatility models incorporated in this thesis. In order to identify the most appropriate parameters out of the estimated ones, the goodness of fit and residual analysis are compared for each of the studied distributions. Excel and NumXL are used to calculate parameters of the volatility models. The derived results are compared and the most favorable parameters in terms of their suitability are used in the implementation of EWMA, GARCH (1, 1) and EGARCH (1, 1) in order to calculate their respective volatilities.

Realized and implied volatilities are compared to the previously calculated in-sample volatilities for each stock index. The calculated volatilities of the previously discussed models are compared to the realized volatility in order to derive the most accurate model in computing and forecasting volatility estimates. This is accomplished through applying RMSE, MAE, and MAPE error statistics. Results are compared and the model with the smallest error difference is concluded to be the most accurate model. The in-sample parameters of the chosen model are applied using the out-of-sample data to calculate the volatility estimates of the out-of-sample period. The above error statistics are used to apply the same process to determine whether the in-sample and the out-of-sample models share the same accurate volatility model. Then, the best model is used in the incorporate volatility update in the Historical Simulation approach to reach VaR estimates of the portfolio of the selected stock indices. The calculated VaR estimates are compared to their respective VaR estimates from EVT, in order to determine the accuracy of the results.

EVT and copulas are implemented to calculate the market risk through VaR. This is modeled using a portfolio of the three stock indices. The process is applied on MATLAB with the assistance of a user-guide from The MathWorks, Inc. Visual illustrations of several outcomes are presented, including the price fluctuations of the stock indices, logarithmic returns, Auto-Correlation Function (ACF) of returns and squared returns, filtered residuals, Upper Tail of Standardized Residuals and others. The Gaussian kernel estimate and the Generalized Pareto Distribution (GPD) are used for the interior marginal Cumulative Distribution function (CDF) and to estimate the upper and lower tails respectively. Thus, VaR from the EVT process is derived for the portfolio of the three indexes over a one-month period. Consequently, the calculated VaR estimates from HS

are compared to their respective VaR estimates from EVT, in order to determine the accuracy of the results.

4.2. In-Sample Descriptive Statistics

In-sample forecasts include data ranging from January 2015 till December 2016 for the closing prices of the three chosen stock market indexes: S&P 500 for U.S., SSEC for China and MICEX for Russia. The in-sample period includes 458 daily observations. The chosen observations are used to calculate the index's daily in-sample returns, which in turn will be used to estimate the index's volatility. The descriptive statistics of the calculated daily returns for each of the three indices are presented in Table 8.

Stock Markets	S&P 500	SSEC	MICEX
Mean	0.000224932	-0.000167852	0.000968412
Standard Deviation	0.009523965	0.020954624	0.011858006
Skewness	-0.158919226	-1.136622175	-0.163607662
Kurtosis	3.326696814	4.617609276	0.971300062
Median	0.000145283	0.001028966	0.000398498
Minimum	-0.040211416	-0.108323598	-0.044036995
Maximum	0.047498924	0.060292716	0.044034431
1st Quartile	-0.41%	-0.66%	-0.65%
3rd Quartile	0.49%	0.87%	0.85%

Table 8: Descriptive Statistics of S&P 500, SSEC, and MICEX: Jan. 05, 2015-Dec. 30, 2016

The average returns of S&P 500, SSEC, and MICEX 35 are 0.000224932, -0.000167852 and 0.000968412 respectively with a standard deviation of 0.009523965, 0.020954624, and 0.011858006 respectively. The distributions of the three stock market indices are not normal since their kurtosis is different than 3.

Stock Markets	Score	C.V.	P-Value
S&P 500	198.66	5.99	0.0%
SSEC	483.75	5.99	0.0%
MICEX	18.12	9.21	0.0%

Table 9: Jarque-Bera Normality Test of S&P 500, SSEC, and MICEX: Jan. 05, 2015- Dec. 30, 2016

The results of p-values relative to the Jarque-Bera test for the S&P500, SSEC, and MICEX are 0.0%. For any distribution to be normal, the calculated p-values should be greater than 1%, 5%, and 10%. Based on the finding in Table 9, we conclude that the in-sample returns for S&P500, SSEC, and MICEX don't follow a normal distribution.

Test	STAT	P-Value	C.V.	Stationary
No Constant	-12.8	0.1%	-1.9	True
Const. - only	-12.8	0.1%	-2.9	True
Const. + Trend	-12.8	0.0%	-1.6	True
Const.+Trend+Trend ^ 2	-12.8	0.0%	-1.6	True

Table 10: In-Sample ADF Stationarity Test for S&P 500 Daily Returns

Test	STAT	P-Value	C.V.	Stationary
No Constant	-20.0	0.1%	-1.9	True
Const.-only	-20.0	0.1%	-2.9	True
Const. + Trend	-20.0	0.0%	-1.6	True
Const. + Trend + Trend ²	-20.0	0.0%	-1.6	True

Table 11: In-Sample ADF Stationarity Test for SSEC Daily Returns

Test	STAT	P-Value	C.V.	Stationary
No Constant	-20.6	0.1%	-1.9	True
Const.-only	-20.7	0.1%	-2.9	True
Const. + Trend	-20.7	0.0%	-1.6	True
Const. + Trend + Trend ²	-20.8	0.0%	-1.6	True

Table 12: In-Sample ADF Stationarity Test for MICEX Daily Returns

Tables 10, 11 and 12 present the results of the Augmented Dicky Fuller (stationarity) test for S&P500, SSEC and MICEX respectively, derived using NumXL. Results for the three indices show that the statistics calculated is lower than C.V. (critical value). Moreover, results show the non-existence of a unit root since the P-values of all the indices are less than 1%. Thus, the null hypothesis is rejected and the indices' in-sample distributions are stationary.

4.3. Parameters' Estimation

The first model to estimate its parameter is EWMA; where lambda is calculated using excel for each of the 3 indices. This is done by computing the daily returns of the closing prices of the

indices, then calculating the variance using the EWMA model, as described in the previous chapter. The next step is to maximize the Log Likelihood function using the Solver add-in from excel options, in order to acquire the best lambda parameters. The Ljung-box test is applied to determine the stability and efficacy of the applied EWMA model. If results of the Ljung-box test are greater than 25, then the studied model is perceived to be unstable.

GARCH (1,1) and EGARCH (1,1) parameters are calculated using the closing prices' daily returns for each index on NumXI. The order of the GARCH volatility model is set to 1. Through NumXI, results of the normal distribution, the student's t-distribution and GED (generalized error distribution) are derived. As previously discussed, the Goodness of Fit test and Residual Analysis are applied in order ensure that the assumptions of the applied models are met. The derived parameters are calibrated on NumXI. The last step is to maximize the Log Likelihood function in order to acquire the best parameters. The parameters derived from NumXI are compared to the calculated models' parameters in excel, in order to determine the best parameters to be used for every volatility model.

4.3.1. EWMA

The daily returns of the closing prices for S&P500, SSEC and MICEX are used to apply the EWMA model. In order to obtain the best Lambda parameters, solver function in excel is used to maximize the log likelihood function. Accordingly, Lambda and the Likelihood function results are as presented in Table 13 below.

EWMA	S&P500	SSEC	MICEX
Lambda	0.95	0.95	0.93825
Likelihood	1921.43	1630.387	1819.635

Table 13: EWMA Test Results

4.3.2. GARCH (1,1)

GARCH (1,1) model is implemented on NumXL using in-sample data related to S&P500, SSEC and MICEX, as presented in the below sections. Results of the parameters, goodness of fit and residual analysis relevant to the three distributions; normal distributions, student t-distributions and GED; are also presented.

4.3.2.1. S&P500 GARCH (1,1)

	Normal Dist.	Student's t-dist.	GED
Long-run mean (μ)	0.000388	0.000188	0.000249
Omega (ω)	0.000010	0.000060	0.000007
ARCH component (α)	0.221322	0.160057	0.205113
GARCH component (β)	0.670779	0.124098	0.729621

Table 14: S&P500 GARCH (1,1) Estimated Parameters

Reference to Table 14, GARCH (1,1) estimated parameters relevant to S&P500 are presented. However, when comparing the calculated GARCH components under the three distributions, it is

evident that the results for the normal distribution and GED are somewhat close and higher than the results under student's t-distribution. In short, the higher the GARCH components, the higher the persistence of shocks in the studied market.

	LLF	AIC	Check
Normal Dist.	1525.055	-3044.11	1
Student's t-dist.	1522.677	-3037.35	1
GED	1542.351	-3076.7	1

Table 15: S&P500 GARCH (1,1) Goodness of Fit

Reference to Table 15, "LLF" stands for "Log Likelihood Function" and "AIC" stands for the "Akaike Info. Criterion", both of which represent the goodness of fit of the S&P 500 GARCH (1,1) model. The outcome under the "Check" section is one for the three distributions, which imply the stability of the model and the application of the assumptions of the studied model.

	AVG.	STDEV.	SKEW.	KURT.	Noise	Normal	ARCH
Normal Dist.	-0.04129	1.000581684	-0.47768	1.897508	TRUE	FALSE	FALSE
Target (normal)	0	1	0	0			
Student's t-dist.	-0.0014	1.005105939	-0.20206	1.897947	TRUE	FALSE	TRUE
Target (t-dist.)	0	1	0	47658790			
GED	-0.02824	1.000894863	-0.53216	2.142191	TRUE	FALSE	FALSE
Target(GED)	0	1	0	1.7772			

Table 16: GARCH (1,1) Residual Analysis for S&P500

Reference to Table 16, GARCH (1,1) residual analysis relevant to S&P500 are presented. When comparing the results of the three distributions to each other, we notice that the results of the normal distribution and GED are somewhat close compared to the results of the student's t-distribution. In addition, the calculated average, standard deviation, skewness, and kurtosis for the normal distribution and GED are rather closer to their target when compared to the results of the student's t-distribution. Results of the Noise test indicate that the three distributions are not auto-correlated. On the other hand, the Normality test results is "False" for the three distributions, which shows that errors are not normally distributed. Finally, the ARCH tests' result is positive only in the student's t-distribution. In other words, the ARCH effect reflects a time series that is not auto-correlated but has conditional variance in its squared series.

In short, based on the above analysis of GARCH (1,1) Estimated Parameters, Goodness of Fit and Residual Analysis for S&P500, the student's t-distribution can be overlooked. Based on the results of the two remaining distributions, the GED distribution is applied for GARCH (1,1).

4.3.2.2. SSEC GARCH (1,1)

	Normal Dist.	Student's t-dist.	GED
Long-run mean (μ)	-0.00009	-0.00009	-0.00009
Omega (ω)	0.00000	0.00028	0.00034
ARCH component (α)	0.05790	0.08350	0.08353
GARCH component (β)	0.94120	0.08042	0.08158

Table 17: SSEC GARCH (1,1) Estimated Parameters

The GARCH (1,1) estimated parameters relevant to SSEC are presented in Table 17. When comparing the calculated GARCH components under the three distributions, it is evident that the results for the student's t-distribution and GED are somewhat close and higher than the results under normal distribution; except for the GARCH component. However, the higher the GARCH components, the higher the persistence of shocks.

	LLF	AIC	Check
Normal Dist.	1211.987	-2417.97	1
Student's t-distribution	1187.039	-2366.08	1
GED	1196.923	-2385.85	1

Table 18: SSEC GARCH (1,1) Goodness of Fit

The LLF and AIC represent the goodness of fit of the SSEC GARCH (1,1) model, as shown in Table 18. The outcome under the "Check" section is one for the three distributions, which imply the stability of the model and the application of the assumptions of the studied model.

	AVG.	STDEV.	SKEW.	KURT.	Noise	Normal	ARCH
Normal Dist.	0.026901	1.002647318	-0.84585	3.398488	TRUE	FALSE	TRUE
Target (normal)	0	1	0	0			
Student's t-dist.	0.002782	1.10964378	-1.13497	4.477101	TRUE	FALSE	TRUE
Target (t-dist.)	0	1	0	3.77E+08			
GED	0.002323	1.008684573	-1.1315	4.467316	TRUE	FALSE	TRUE
Target(GED)	0	1	0	2.999			

Table 19: GARCH (1,1) Residual Analysis for SSEC

Reference to Table 19, GARCH (1,1) residual analysis relevant to SSEC are presented. When comparing the results of the three distributions to each other, we notice that the results of the student's t-distribution and GED are somewhat close compared to the results of the normal distribution. In addition, the calculated average, standard deviation, skewness, and kurtosis for the student's t-distribution and GED are rather closer to their target when compared to the results of the normal distribution. Results of the Noise test indicate that the three distributions are not auto-correlated. Whereas results of the Normality tests indicates that errors are not normally distributed among the three distributions. Finally, the ARCH tests' results are positive for the three distributions.

In short, based on the above analysis of GARCH (1,1) Estimated Parameters, Goodness of Fit and Residual Analysis for SSEC, the normal distribution can be overlooked. Comparing results of the student's t-distribution and GED, GARCH (1,1) under GED distribution will be adopted.

4.3.2.3. MICEX GARCH (1,1)

	Normal Dist.	Student's t-dist.	GED
Long-run mean (μ)	0.00081	0.00087	0.00080
Omega (ω)	0.00000	0.00000	0.00000
ARCH component (α)	0.08312	0.07960	0.08187
GARCH component (β)	0.91305	0.91781	0.91484

Table 20: MICEX GARCH (1,1) Estimated Parameters

Reference to Table 20, GARCH (1,1) estimated parameters relevant to MICEX are presented. When comparing the calculated GARCH components under the three distributions, it is evident that the results for the normal distribution and GED are somewhat close compared to the results of the student's t-distribution.

	LLF	AIC	Check
Normal Dist.	1405.381	-2804.76	1
Student's t-dist.	1406.81	-2805.62	1
GED	1405.87	-2803.74	1

Table 21: MICEX GARCH (1,1) Goodness of Fit

The LLF and AIC represent the goodness of fit of the MICEX GARCH (1,1) model, as shown in Table 21. The outcome under the “Check” section is one for the three distributions, which imply the stability of the model and the application of the assumptions of the studied model.

	AVG.	STDEV.	SKEW.	KURT.	Noise	Normal	ARCH
Normal Dist.	0.007479	0.966951	-0.18812	0.442888	TRUE	FALSE	FALSE
Target (normal)	0	1	0	0			
Student’s t-dist.	0.000974	0.964661	-0.1939	0.462949	TRUE	FALSE	FALSE
Target (t-dist.)	0	1	0	0.5803			
GED	0.007892	0.967011	-0.18942	0.450605	TRUE	FALSE	FALSE
Target(GED)	0	1	0	0.211			

Table 22: GARCH (1,1) Residual Analysis for MICEX

Reference to Table 22, GARCH (1,1) residual analysis relevant to MICEX are presented. When comparing the results of the three distributions to each other, we notice that the results of the normal distribution and GED are somewhat close compared to the results of the student’s t-distribution. In addition, the calculated average, standard deviation, skewness, and kurtosis for the normal distribution and GED are rather closer to their target when compared to the results of the student’s t-distribution. Results of the Noise test indicate that the three distributions are not auto-correlated. On the other hand, the Normality test results is “False” for the three distributions, which shows that errors are not normally distributed. Finally, the ARCH tests’ results are false for all of the three distributions.

In short, based on the above analysis of GARCH (1,1) Estimated Parameters, Goodness of Fit and Residual Analysis for MICEX, the student's t- distribution can be overlooked. Comparing results of the normal distribution and GED, GARCH (1,1) under GED distribution will be adopted.

To summarize, GARCH (1,1) under GED distribution is chosen to be adopted for the 3 indices based on the above analysis. Table 23 presents a summary of the parameters of the chosen distribution.

	S&P500	SSEC	MICEX
Long-run mean (μ)	0.000249	-0.00009	0.00080
Omega (ω)	0.000007	0.00034	0.00000
ARCH component (α)	0.205113	0.08353	0.08187
GARCH component (β)	0.729621	0.08158	0.91484

Table 23: GARCH (1,1) GED distribution Estimated Parameters

4.3.3. EGARCH (1,1)

EGARCH (1,1) parameters are calculated on NumXL using in-sample data. EGARCH (1,1) results of the three stock indices, S&P500, SSEC and MICEX, are divided in the below sections; which include the estimated parameters, goodness of fit, and residual analysis related to three distributions: the normal distributions, student t-distributions, and GED. In reference to the GARCH (1,1) estimated parameters, EGARCH (1,1) parameters include an additional parameter

known as the leverage coefficient. The role of the latter is to measure the effect of shocks, whether positive or negative, on the volatility of the studied stock markets.

4.3.3.1. S&P 500 EGARCH (1,1)

	Normal Dist.	Student's t-dist.	GED
Long-run mean (μ)	-0.00018	0.00018	0.00030
Omega (ω)	-0.82926	-10.51345	-0.89049
ARCH component (α)	0.06071	0.40337	0.13123
Leverage coefficient (γ)	-4.76951	-0.16786	-2.28336
GARCH component (β)	0.91790	-0.09338	0.91897

Table 24: S&P500 EGARCH (1,1) Estimated Parameters

Reference to Table 24, the EGARCH (1,1) estimated parameters for S&P500 are presented relative to the normal distribution, student t-distribution, and GED. The high value of the GARCH component for GED indicates more persistence of shocks, as opposed to lower values of the GARCH component for the normal distribution and t-distribution.

	LLF	AIC	Check
Normal Dist.	1544.377	-3078.75	1
Student's t-dist.	1517.692	-3023.38	1
GED	1556.722	-3101.44	1

Table 25: S&P500 EGARCH (1,1) Goodness of Fit

The AIC and LLF represent the EGARCH (1,1) goodness of fit results for S&P500, as shown in Table 25. The outcome under the “Check” section is one for the three distributions, which imply the stability of the model and the application of the assumptions of the studied model.

	AVG.	STDEV.	SKEW.	KURT.	Noise	Normal	ARCH
Normal Dist.	0.01539	1.005027	-0.56433	2.392028	TRUE	FALSE	FALSE
Target (normal)	0	1	0	0			
Student’s t-dist.	-0.00032	0.987698	-0.11991	1.921805	TRUE	FALSE	TRUE
Target (t-dist.)	0	1	0	50958.36			
GED	-0.04953	1.00852	-0.63669	2.678038	TRUE	FALSE	FALSE
Target(GED)	0	1	0	1.3679			

Table 26: EGARCH (1,1) Residual Analysis for S&P500

Reference to Table 26, EGARCH (1,1) residual analysis relevant to S&P500 are presented. When comparing the results of the three distributions to each other, we notice that the results of the normal distribution and GED are somewhat close compared to the results of the student’s t-distribution. In addition, the calculated average, standard deviation, skewness, and kurtosis for the normal distribution and GED are rather closer to their target when compared to the results of the student’s t-distribution. Results of the Noise test indicate that the three distributions are not auto-correlated. On the other hand, the Normality test results is “False” for the three distributions, which shows that errors are not normally distributed. Finally, the ARCH tests’ results are false for all of the normal distribution and GED

In short, based on the above analysis of EGARCH (1,1) Estimated Parameters, Goodness of Fit and Residual Analysis for S&P500, the student's t- distribution can be overlooked. Comparing results of the normal distribution and GED, EGARCH (1,1) under GED distribution is chosen.

4.3.3. 2. SSEC EGARCH (1,1)

	Normal Dist.	Student's t-dist.	GED
Long-run mean (μ)	-0.00010	-0.00009	-0.00009
Omega (ω)	-0.16835	-7.24614	-7.50935
ARCH component (α)	0.13715	0.20748	0.20038
Leverage coefficient (γ)	-0.09513	-0.11253	-0.11235
GARCH component (β)	0.99196	0.12456	0.12712

Table 27: SSEC EGARCH (1,1) Estimated Parameters

Reference to Table 27, the EGARCH (1,1) estimated parameters for SSEC are presented relative to the normal distribution, student t-distribution, and GED. The high value of the GARCH component for the normal distribution indicates more persistence of shocks, as opposed to lower values of the GARCH component for the GED and t-distribution.

	LLF	AIC	Check
Normal Dist.	1210.192	-2410.38	1
Student's t-dist.	1186.815	-2361.63	1
GED	1187.6	-2363.2	1

Table 28: SSEC EGARCH (1,1) Goodness of Fit

EGARCH (1,1) goodness of fit results for S&P500, as shown in Table 28, are presented by AIC and LL. The outcome under the “Check” section is one for the three distributions, which imply the stability of the model and the application of the assumptions of the studied model.

	AVG.	STDEV.	SKEW.	KURT.	Noise	Normal	ARCH
Normal Dist.	0.02101	1.0214673	-1.01598	4.254564	TRUE	FALSE	TRUE
Target (normal)	0	1	0	0			
Student’s t-dist.	0.002387	1.1586819	-1.10585	4.352801	TRUE	FALSE	TRUE
Target (t-dist.)	0	1	0	3995796			
GED	0.00297	1.3451584	-1.10483	4.358168	TRUE	FALSE	TRUE
Target(GED)	0	1	0	2.999			

Table 29: EGARCH (1,1) Residual Analysis for SSEC

Reference to Table 29, EGARCH (1,1) residual analysis relevant to SSEC are presented. When comparing the results of the three distributions to each other, we notice that the results of the normal distribution and GED are somewhat close compared to the results of the student’s t-distribution. In addition, the calculated average, standard deviation, skewness, and kurtosis for the normal distribution and GED are rather closer to their target when compared to the results of the student’s t-distribution. Results of the Noise test indicate that the three distributions are not auto-correlated. On the other hand, the Normality test results is “False” for the three distributions, which shows that errors are not normally distributed. Finally, the ARCH test’s results are positive for the three distributions.

In short, based on the above analysis of EGARCH (1,1) Estimated Parameters, Goodness of Fit and Residual Analysis for SSEC, the student's t- distribution can be overlooked. Comparing results of the normal distribution and GED, EGARCH (1,1) under GED distribution is chosen.

4.3.3.3. MICEX EGARCH (1,1)

	Normal Dist.	Student's t-dist.	GED
Long-run mean (μ)	0.00109	0.00154	0.00142
Omega (ω)	-0.36194	-0.18087	-0.20518
ARCH component (α)	0.16239	0.07314	0.08007
Leverage coefficient (γ)	0.15843	0.90173	0.83883
GARCH component (β)	0.97378	0.98594	0.98390

Table 30: MICEX EGARCH (1,1) Estimated Parameters

Reference to Table 30, the EGARCH (1,1) estimated parameters for MICEX are presented relative to the normal distribution, student t-distribution, and GED. The high values of the GARCH component for all the distributions indicate more persistence of shocks.

	LLF	AIC	Check
Normal Dist.	1403.344	-2796.69	1
Student's t-dist.	1402.665	-2793.33	1
GED	1401.304	-2790.61	1

Table 31: MICEX EGARCH (1,1) Goodness of Fit

EGARCH (1,1) goodness of fit results for S&P500, as shown in Table 31, are presented by AIC and LL. The outcome under the “Check” section is one for the three distributions, which imply the stability of the model and the application of its assumptions.

	AVG.	STDEV.	SKEW.	KURT.	Noise	Normal	ARCH
Normal Dist.	-0.0097	0.987514	-0.17554	0.518726	TRUE	FALSE	FALSE
Target (normal)	0	1	0	0			
Student’s t-dist.	- 0.04369	0.988681	-0.20274	0.754138	TRUE	FALSE	TRUE
Target (t-dist.)	0	1	0	0.685891			
GED	- 0.03335	0.991258	-0.20002	0.726122	TRUE	FALSE	TRUE
Target(GED)	0	1	0	0.29454			

Table 32: EGARCH (1,1) Residual Analysis for MICEX

Reference to Table 32, EGARCH (1,1) residual analysis relevant to MICEX are presented. When comparing the results of the three distributions to each other, we notice that the results of the student’s t-distribution and GED are somewhat close compared to the results of the normal distribution. In addition, the calculated average, standard deviation, skewness, and kurtosis for the student’s t-distribution and GED are rather closer to their target when compared to the results of the normal distribution. Results of the Noise test indicate that the three distributions are not auto-correlated. On the other hand, the Normality test results is “False” for the three distributions, which shows that errors are not normally distributed. Finally, the ARCH test’s results are positive for the three distributions.

To sum up the above analysis on MICEX using the EGARCH (1,1) model and given the presented findings, the GED distribution is chosen.

To summarize, EGARCH (1,1) under GED distribution is chosen to be adopted for the 3 indices based on the above analysis. Table 33 presents a summary of the parameters of the chosen distribution.

	S&P500	SSEC	MICEX
Long-run mean (μ)	0.00030	-0.00009	0.00142
Omega (ω)	-0.89049	-7.50935	-0.20518
ARCH component (α)	0.13123	0.20038	0.08007
Leverage coefficient (γ)	-2.28336	-0.11235	0.83883
GARCH component (β)	0.91897	0.12712	0.98390

Table 33: EGARCH (1,1) GED Estimated Parameters

4.4. In-Sample Findings

The chosen volatility models applied in this paper are detailed below.

4.4.1. Realized and Implied Volatility

The daily realized and implied volatilities for S&P 500 are used to calculate the error statistics as previously discussed. However, the implied volatility for SSEC and MICEX are not available and will be excluded from the comparison. Volatility data are extracted from Investing.com.

4.4.2. Volatility of GARCH (1, 1) model

In order to calculate the volatility of the GARCH (1, 1) model for the three stock indices, the following steps are implemented on excel. The log of the daily return of the closing prices is calculated. The difference between the calculated daily returns and the average of returns is squared. Equation (4) is implemented and the GARCH (1, 1) volatility is the square root of the variance multiplied by 250, in order to deduce the daily volatility.

	S&P500	SSEC	MICEX
Omega (ω)	8.5218E-06	1.1035E-06	2.9655E-06
ARCH component (α)	0.1972	0.0569	0.0653
GARCH component (β)	0.7105	0.9331	0.9065

Table 34: GARCH (1,1) In-Sample Estimated Parameters

Parameters in Table 34 are chosen to be implemented in the calculation of VaR, since the Long-Term volatility is more accurate using the above described process; as opposed to parameters from NumXl.

4. 4. 3. Volatility of EGARCH (1, 1) model

In order to calculate the volatility of the EGARCH (1, 1) model for the three stock indices, the following steps are implemented. The logarithm of the returns and average returns of the daily closing prices are derived. The daily difference between the return and average of returns is calculated. The average of the last difference is also calculated. $Z(t)$ is calculated by dividing the daily difference between returns and average returns over square root of the variance. The next

step is to derive $G[Z(t)]$ using the following equation: $\gamma * Z(t) + \alpha * [ABS(Z(T)) - SQRT(2/PI())]$. To proceed with the estimation, calculate the Log conditional variance through setting its first value to -10 and estimating the following equation: $\omega + g[Z(t)] + \beta * [Log \sigma^2(n-1)]$. The conditional variance is the exponential of its derived log. The EGARCH (1, 1) volatility is the square root of the variance multiplied by 250.

	S&P500	SSEC	MICEX
Omega (ω)	-0.72879	-0.08598	-0.87318
ARCH component (α)	0.04766	0.20530	0.30477
Leverage coefficient (γ)	-0.29149	-0.01362	0.05345
GARCH component (β)	0.92471	0.98689	0.90237

Table 35: EGARCH (1,1) In-Sample Estimated Parameters

Parameters in Table 35 are chosen to be implemented in the calculation of VaR, since the Long-Term volatility is more accurate using the above described process; as opposed to parameters from NumXI.

4.4.4. Final Results

The three error statistics: Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) are implemented using NumXI, to determine the best applied volatility model. The best model is chosen based on the error statistics that ranks first when subtracting its volatility from the realized volatility, for every index. Theoretically, to calculate the previously mentioned errors statistics, the following formulas are used:

- i. $RMSE = \sqrt{(\sum_{t=1}^n (f - Y)^2)/n}$
- ii. $MAE = \frac{1}{n} \sum_{t=1}^n |f - Y|$
- iii. $MAPE = \frac{100}{n} \sum_{t=1}^n \left(\frac{|Y-f|}{Y}\right)$

Where n is the number of periods, Y is the true value and f is the prediction value.

The NumXI results of the three error statistics for the in-sample period of the chosen indices are presented in Tables 36, 37 and 38.

S&P500	RMSE	Rating	MAE	Rating	MAPE	Rating
Implied Vol.	0.047694	2	0.039028	2	0.00377	3
GARCH(1,1)	0.040995	1	0.028954	1	0.002376	1
EGARCH(1,1)	0.049947	3	0.039066	3	0.003261	2

Table 36: In-Sample Period Error Statistics for S&P 500

SSEC	RMSE	Rating	MAE	Rating	MAPE	Rating
GARCH(1,1)	0.09567	1	0.080424	1	0.002879	1
EGARCH(1,1)	0.11588	2	0.090075	2	0.003134	2

Table 37: In-Sample Period Error Statistics for SSEC

MICEX	RMSE	Rating	MAE	Rating	MAPE	Rating
GARCH(1,1)	0.188566	2	0.179723	2	0.004997	2
EGARCH(1,1)	0.184631	1	0.173261	1	0.004775	1

Table 38: In-Sample Period Error Statistics for MICEX

The GARCH (1, 1) ranks 1st for S&P 500 and SSEC. However, the EGARCH (1,1) ranks 1st for MICEX. Consequently, the best model to be used for the in-sample period is the GARCH (1, 1) model. The graphical fluctuation of GARCH (1, 1) volatility in comparison to the realized volatility of S&P 500 and SSEC, and the EGARCH (1, 1) volatility in comparison to the realized volatility of MICEX are presented in the below figures. Reference to Figure 3, it is visually evident that the volatility estimates of GARCH (1,1) are moving together with the estimates of the realized volatility.

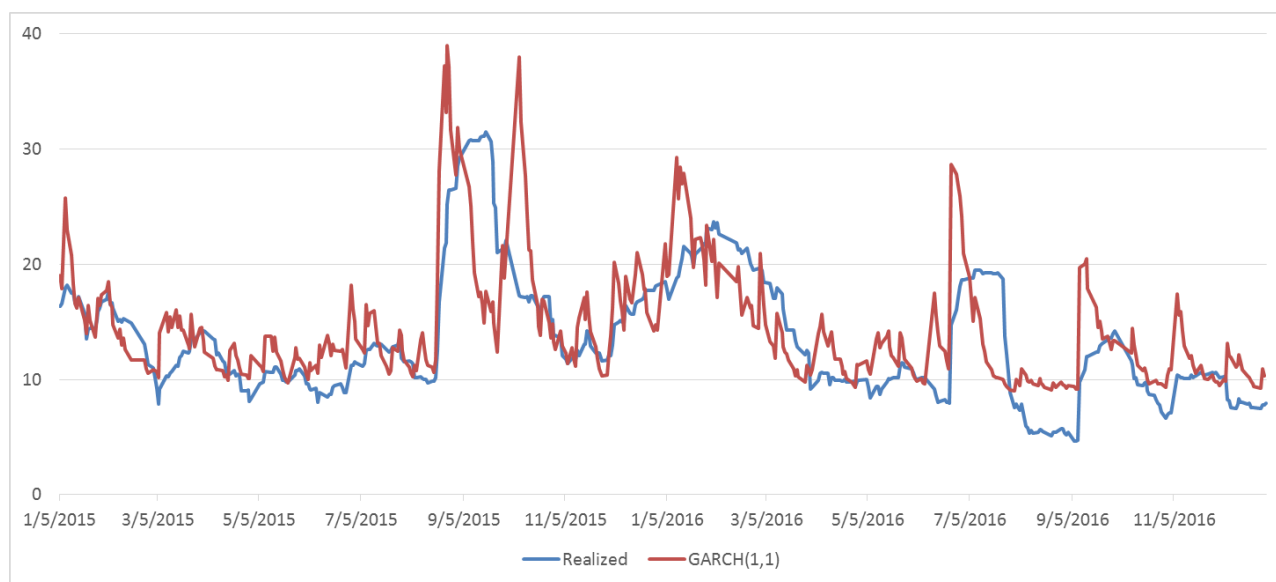


Figure 3: In-Sample Realized and GARCH (1,1) Volatilities for S&P500

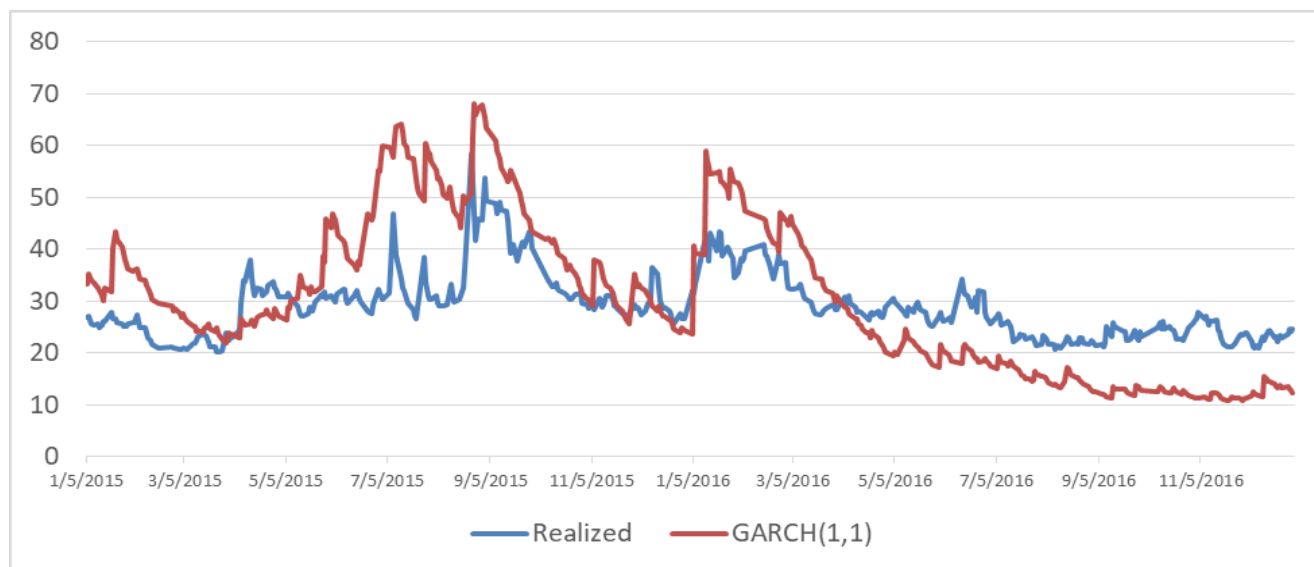


Figure 4: In-Sample Realized and GARCH (1,1) Volatilities for SSEC

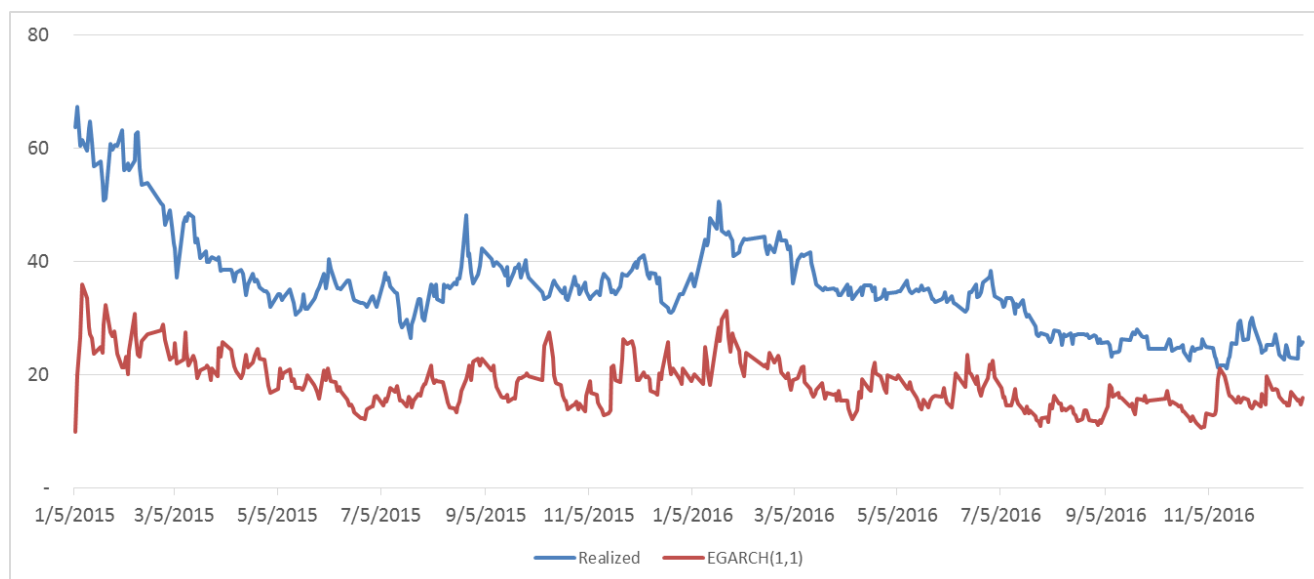


Figure 5: In-Sample Realized and EGARCH (1,1) Volatilities for MICEX

4.4.5. Out-of-Sample Findings

In order to assess the ability of the calculated in-sample parameters in forecasting future fluctuations in the studied market, these parameters are chosen to calculate the out-of-sample volatilities using the returns calculated from out-of-sample data. The derived out-of-sample volatilities are evaluated based on the RMSE, MAE, and MAPE error statistics. The best model in the out-of-sample period is compared to the best model previously concluded to be the best for the in-sample period.

The NumXI results of the three error statistics for the out-of-sample period of the chosen indices are presented in Tables 39, 40 and 41.

S&P500	RMSE	Rating	MAE	Rating	MAPE	Rating
Implied Vol.	0.04809	3	0.04325	3	0.006092	3
GARCH (1,1)	0.040024	1	0.035135	1	0.004896	1
EGARCH (1,1)	0.047078	2	0.041034	2	0.005689	2

Table 39: Out-of-Sample Error Statistics for S&P 500

SSEC	RMSE	Rating	MAE	Rating	MAPE	Rating
GARCH (1,1)	0.08478	2	0.080437	2	0.004033	2
EGARCH (1,1)	0.07113	1	0.063535	1	0.00322	1

Table 40: Out-of-Sample Error Statistics for SSEC

MICEX	RMSE	Rating	MAE	Rating	MAPE	Rating
GARCH (1,1)	0.071894	2	0.064616	2	0.002909	2
EGARCH (1,1)	0.069381	1	0.061135	1	0.00271	1

Table 41: Out-of-Sample Error Statistics for MICEX

The GARCH (1,1) ranks the 1st for S&P 500. The EGARCH (1, 1) ranks 1st for SSEC and MICEX; with a difference of around 0.03 and 0.006 units respectively from the results of the GARCH (1, 1) model. Therefore, GARCH (1, 1) is chosen to be superior to EGARCH (1, 1) for S&P500 in both the in-sample and out-of-sample period. EGARCH (1, 1) is chosen to be superior to GARCH (1, 1) for MICEX in both the in-sample and out-of-sample period. However, for SSEC, GARCH (1,1) was superior in the in-sample period and EGARCH (1,1) was superior in the out-of-sample period. The graphical fluctuation of GARCH (1, 1) volatility in comparison to the realized volatility of S&P 500, and the EGARCH (1 ,1) volatility in comparison to the realized volatility of SSEC and MICEX are presented in the below figures.

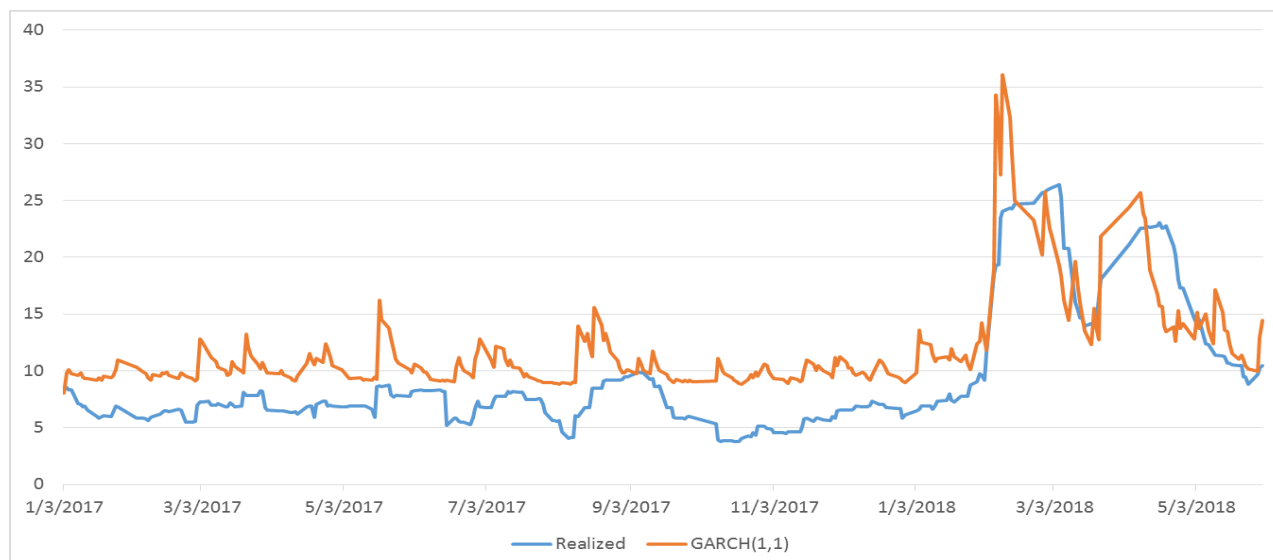


Figure 6: Out-of-Sample Realized and GARCH (1,1) Volatilities for S&P 500

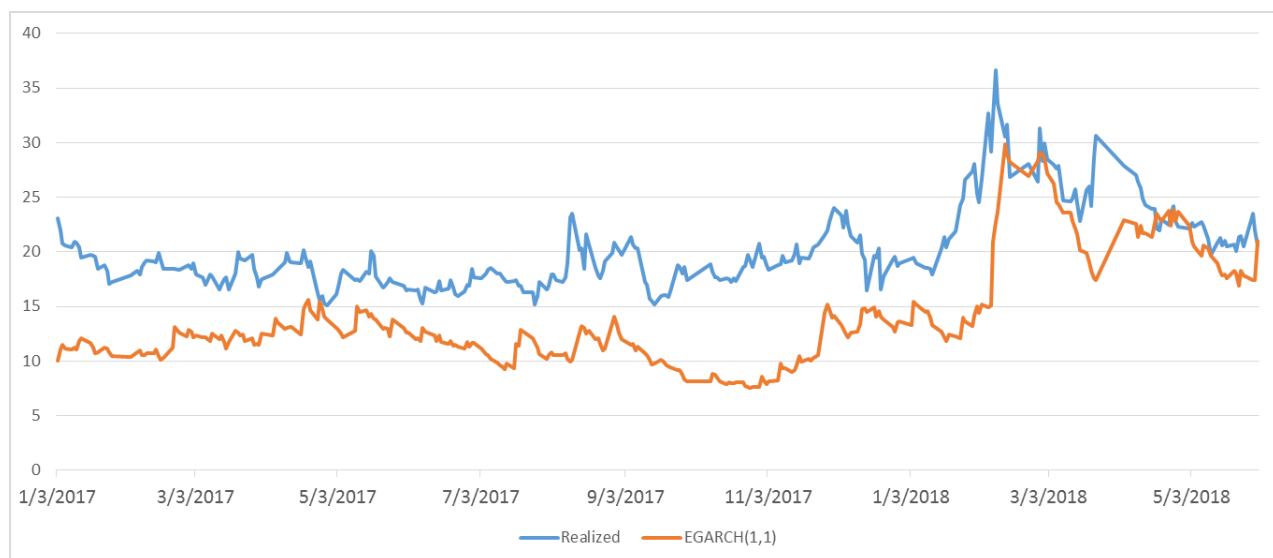


Figure 7: Out-of-Sample Realized and EGARCH (1,1) Volatilities for SSEC

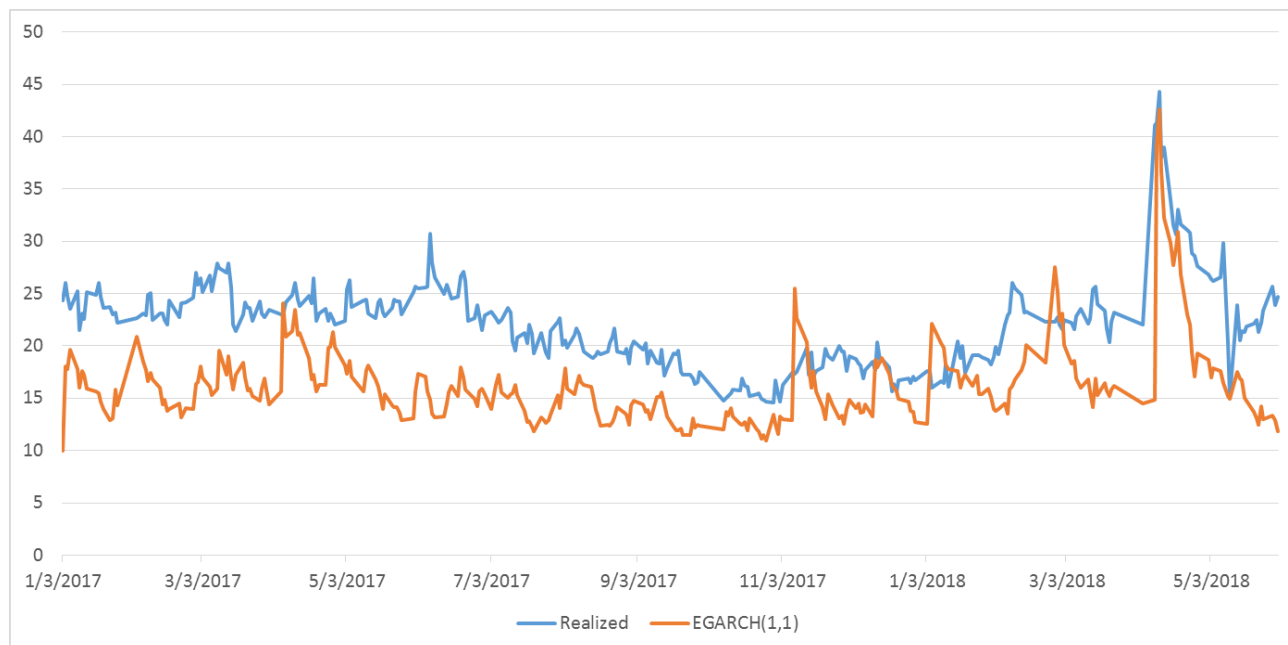


Figure 8: Out-of-Sample Realized and EGARCH (1,1) Volatilities for MICEX

4. 4. 6. VaR Results

The parameters based on the models that ranked 1st (reference to each index and period) are implemented in the Historical Simulation model to incorporate the volatility update using the daily return of S&P500, SSEC and MICEX, in order to reach the desired portfolio VaR. The latter represents the Value at Risk of the portfolio of the three indices having the same weight. The variance and volatility are calculated using the chosen model's parameters for every index. Scenarios are derived and ranked using the previously calculated volatility updates. The estimated losses for the in-sample portfolio and out-of-sample portfolio are ranked, starting with the highest losses. Accordingly, the 99% VaR is the loss that corresponds to 1% of all the studied observations.

Below is a summary table of the in-sample and out-of-sample portfolio 90%, 95% and 99% VaR results.

Outcome	HS In-Sample	HS Out-of-Sample
Simulated 90% VaR	0.68%	0.48%
Simulated 95% VaR	1.03 %	0.71%
Simulated 99% VaR	2.31%	1.65%

Table 42: HS VaR Summary Results

4.5. Extreme Value Theory (EVT)

The market risk is modeled through a portfolio of the three studied stock indices with a Monte Carlo simulation method including student t-copula and EVT. The filtered residuals of each return series are extracted using an asymmetric GARCH model. The Gaussian kernel estimate is used for the interior marginal Cumulative Distribution function (CDF) and the Generalized Pareto Distribution (GPD) is applied to estimate the upper and lower tails. The Student t-copula is implemented to the portfolio's data in order to reveal the correlation among the residuals of every index. Consequently, the Value at Risk (VaR) is computed for the portfolio of the three indexes over a one-month period.

4.5.1 In-Sample Findings

In-sample estimates include data ranging from January 2015 till December 2016 for the studied portfolio, which constitutes the closing prices of the three chosen stock market indexes: S&P 500

for U.S., SSEC for China and MICEX for Russia. The in-sample period includes 458 daily observations. The below figures and analysis disclose the different steps applied on MATLAB to reach the one-month VaR of the portfolio. First, price movements of the portfolio's indices are presented in Figure 9. The closing prices of all indices were normalized to unity, in order to enable easy comparison relative to the performance of each index.

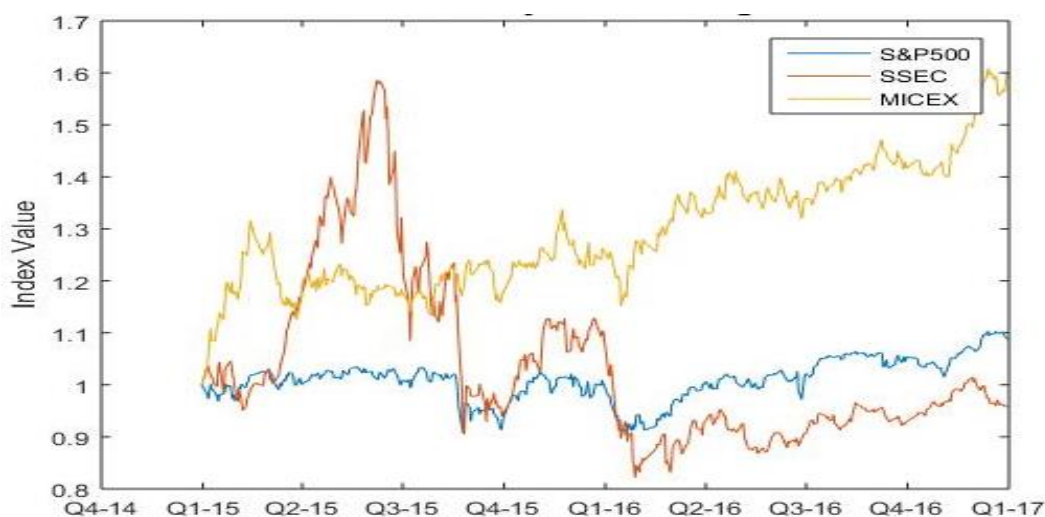


Figure 9: Relative Daily Index Closings of the In-Sample Portfolio

Daily logarithmic return series are derived using the closing prices of the chosen indices. These returns are used in the applied modeling approach. The return series of every index are illustrated in figures 10, 11 and 12.

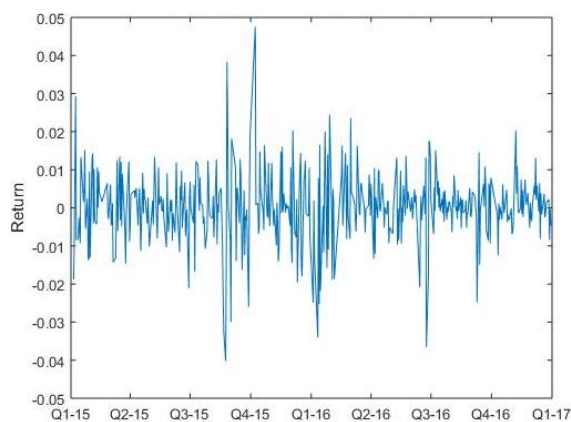


Figure 10: Daily Logarithmic Returns of S&P500

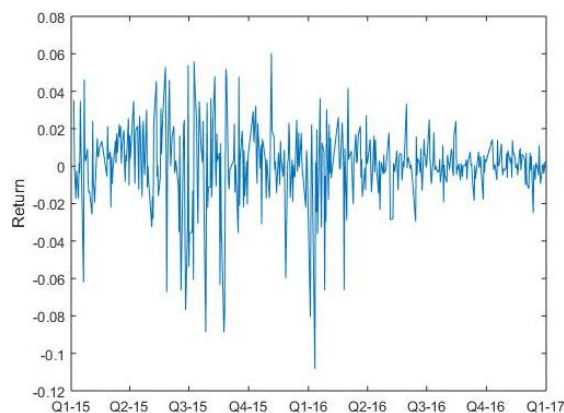


Figure 11: Daily Logarithmic Returns of SSEC

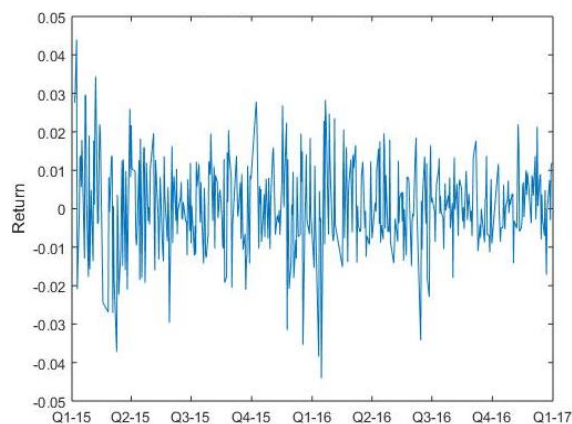


Figure 12: Daily Logarithmic Returns of MICEX

The return series of every index are filtered to adjust observations to be “independent and identically distributed (i.i.d.)”; which is required to apply the GPD in modeling the distribution’s tails. The autocorrelation functions (ACF) of S&P500, SSEC and MICEX are presented below. Usually, return series demonstrate a certain level of autocorrelation and heteroskedasticity. However, “the autocorrelation functions (ACF) of squared returns reflects the persistence of the variance and indicates that the GARCH model could fit the data into the estimation of the tails process” (The MathWorks, Inc., 2014). In other words, negative and null observations would be disregarded when squared. The ACF returns and ACF of squared returns of S&P500, SSEC and MICEX are compared below.

Reference to figures 13, 15 and 17, the ACF of return series of S&P 500, SSEC and MICEX demonstrate some degree of sequential correlation. Figures 14, 16 and 18 reveal that the variance of squared returns is persistent in the three indices.

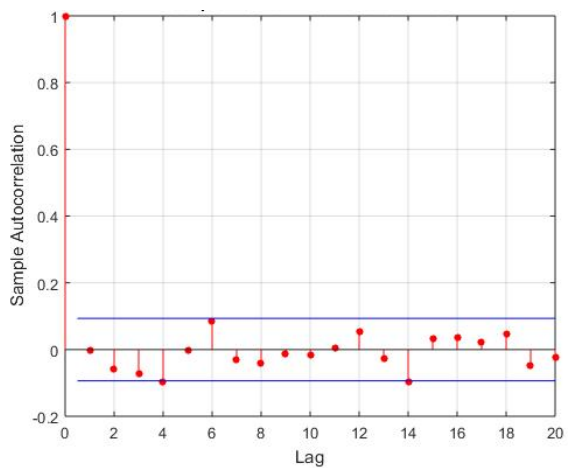


Figure 13: ACF of Returns of S&P500

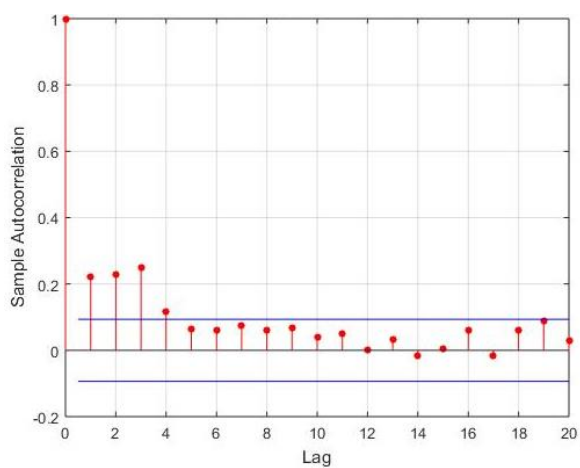


Figure 14: ACF of Squared Returns of S&P500

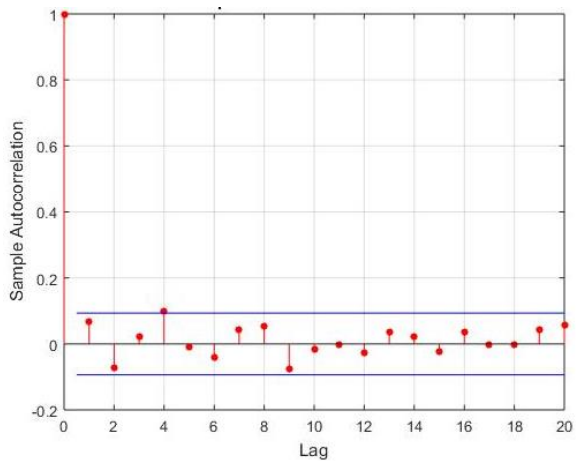


Figure 15: ACF of Returns of SSEC

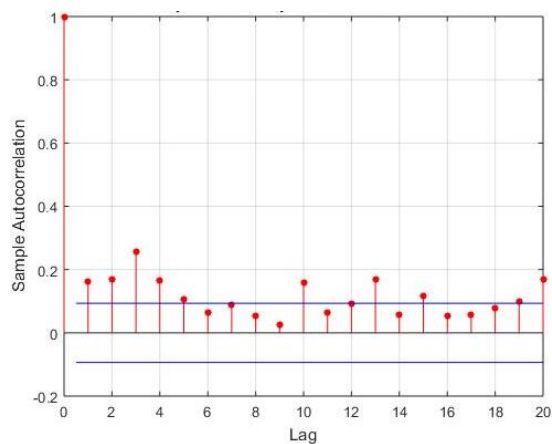


Figure 16: ACF of Squared Returns of SSEC

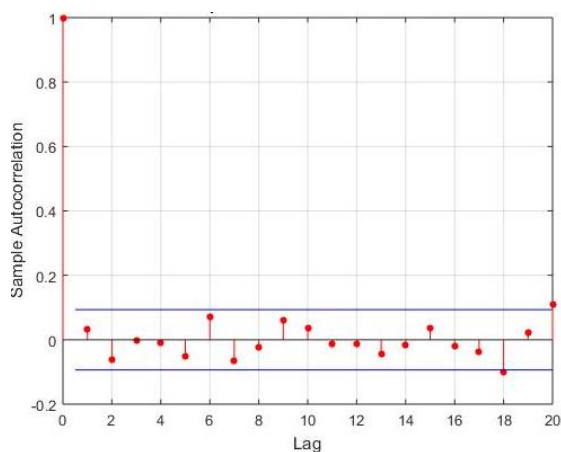


Figure 17: ACF of Returns of MICEX

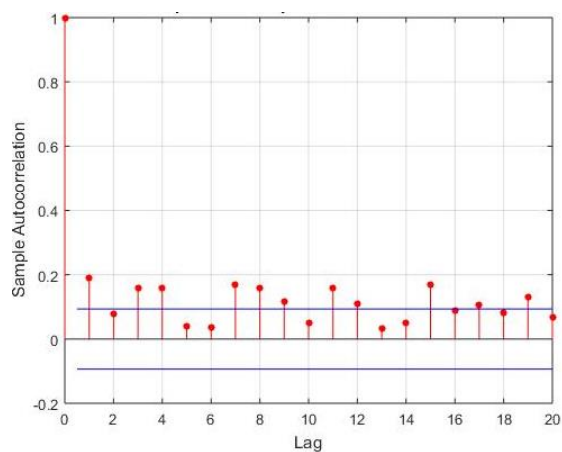


Figure 18: ACF of Squared Returns of MICEX

In order to generate i.i.d observations needed for the calculation of EVT, the first order autoregressive model is used to fit the conditional mean of the return series of the three chosen indices; which accounts for autocorrelation. The asymmetric GARCH model is fit to the conditional variances of the indices; which accounts for heteroskedasticity. Moreover, the student t-distribution is used to model the standardized residuals of every index; which accounts for the fat tails of the distribution. The return series of every index are used to derive the filtered residuals and conditional variances. The filtered residuals, which demonstrate heteroskedasticity, and the filtered conditional standard deviation of the three indices are compared below.

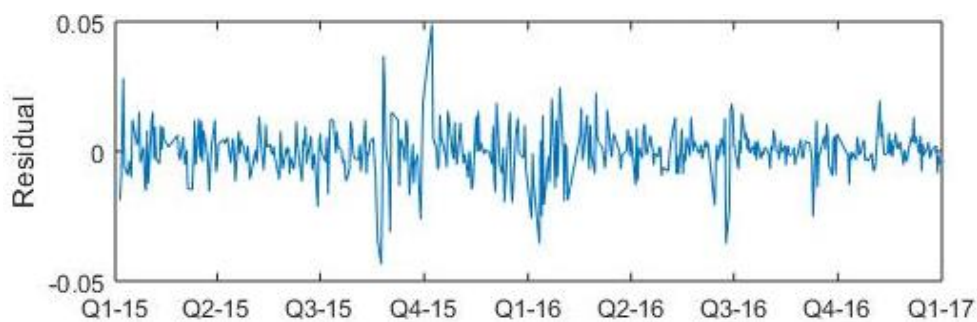


Figure 19: Filtered Residuals of S&P 500

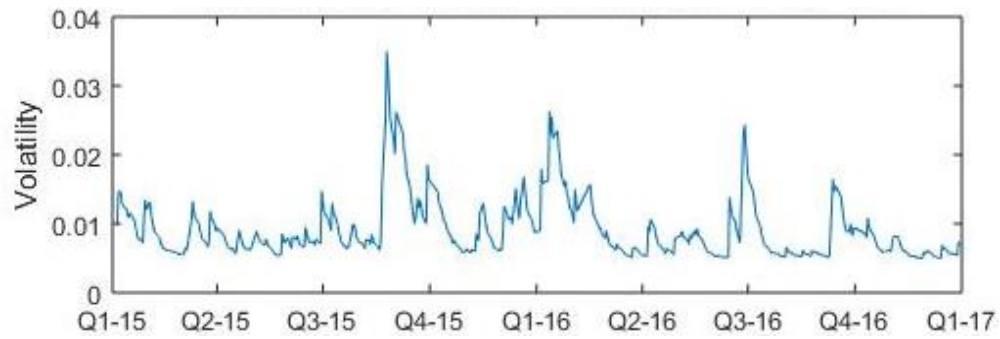


Figure 20: Filtered Conditional Standard Deviation of S&P 500

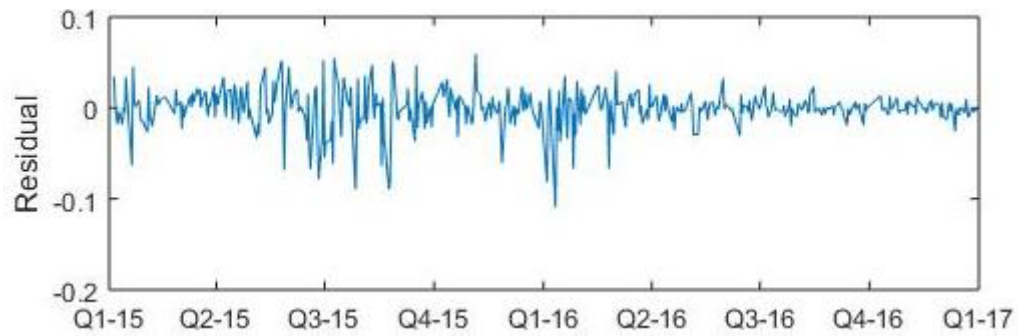


Figure 21: Filtered Residuals of SSEC

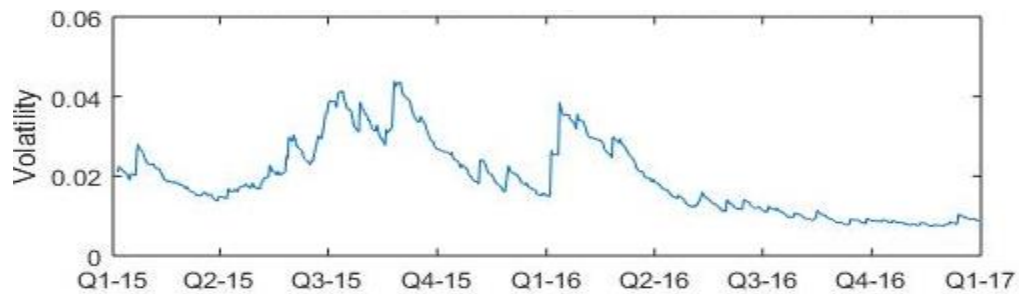


Figure 22: Filtered Conditional Standard Deviation of SSEC

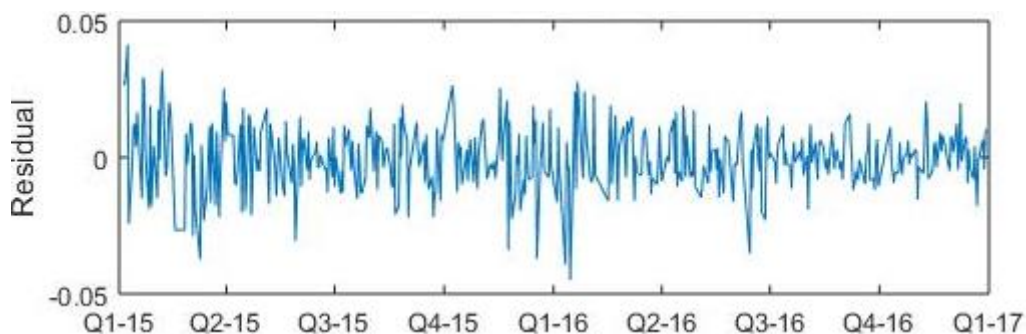


Figure 23: Filtered Residuals of MICEX

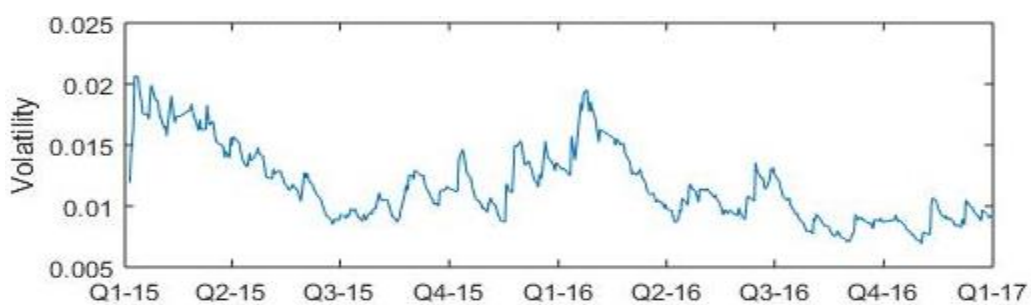


Figure 24: Filtered Conditional Standard Deviation of MICEX

After filtering the residuals of every return series, these residuals are standardized by their conditional standard deviation to exhibit the zero mean, unit variance and i.i.d series to be used in the EVT calculation of the portfolio's CDF tails. The below figures demonstrate a comparison of the results of the ACF of standardized residuals and ACF of squared standardized residuals of the three indices, which reveal that the transformed standardized residuals are almost i.i.d.

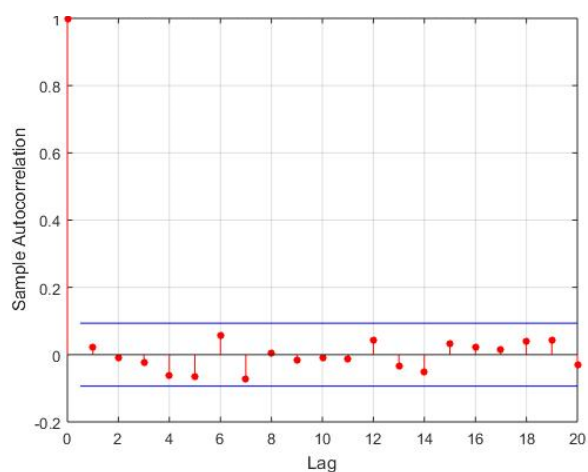


Figure 25: ACF of Standardized Residuals of S&P500

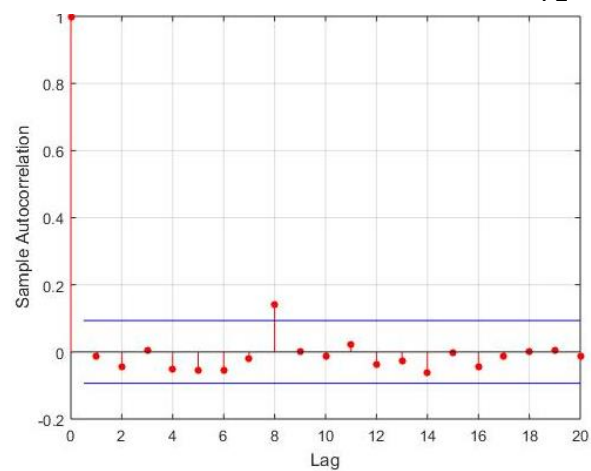


Figure 26: ACF of Squared Standardized Residuals of S&P500

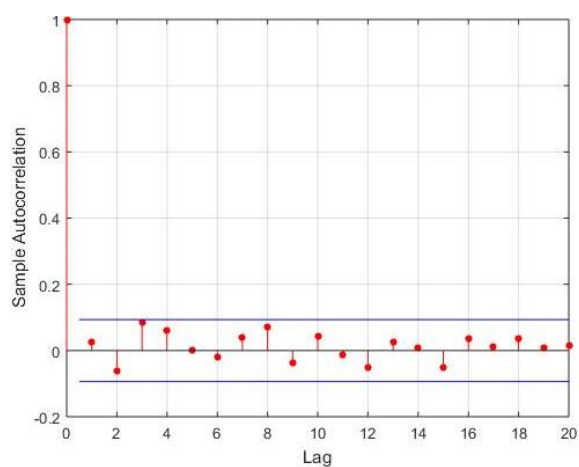


Figure 27: ACF of Standardized Residuals of SSEC

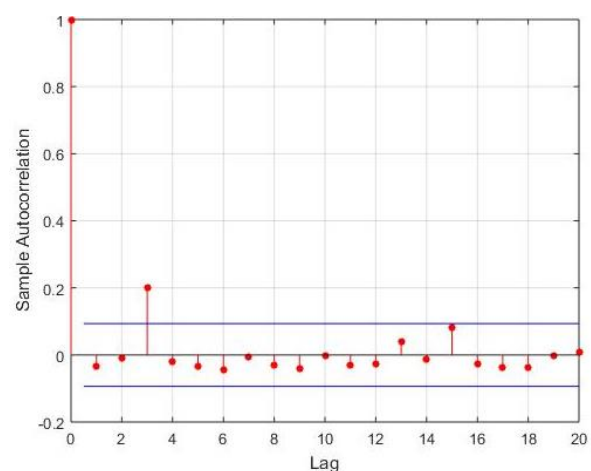


Figure 28: ACF of Squared Standardized Residuals of SSEC

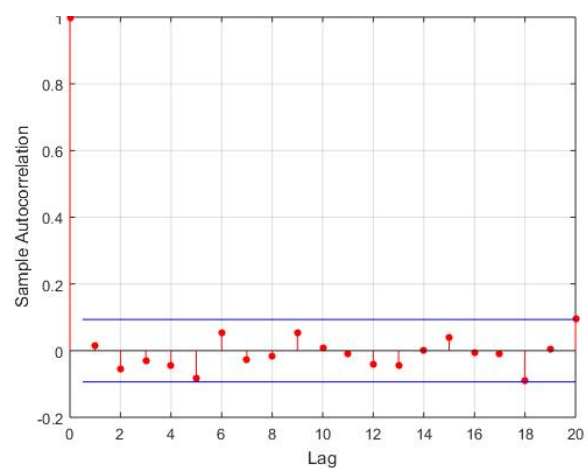


Figure 29: ACF of Standardized Residuals of MICEX

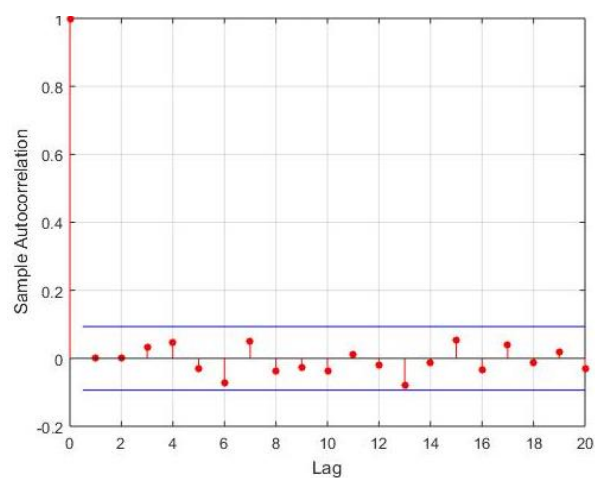


Figure 30: ACF of Squared Standardized Residuals of MICEX

After deriving the standardized residuals, the empirical CDF is calculated using the Gaussian Kernel for every index. However, the CDF estimates are more suitable for the interior distribution rather than the tails. For this reason, EVT is applied to the residuals of the upper and lower tails. As a result, the calculated thresholds for the upper and lower tails account for 10% of the reserved residuals of every tails. The maximum likelihood function is applied to fit the extreme tails' residuals (above the defined threshold) to the parametric GPD, which is reflected in the following figures for the upper tail exceedances. For the three indices, the distribution of exceedances is visually close to the fitted data, which implies that the GPD model is appropriate in our study.

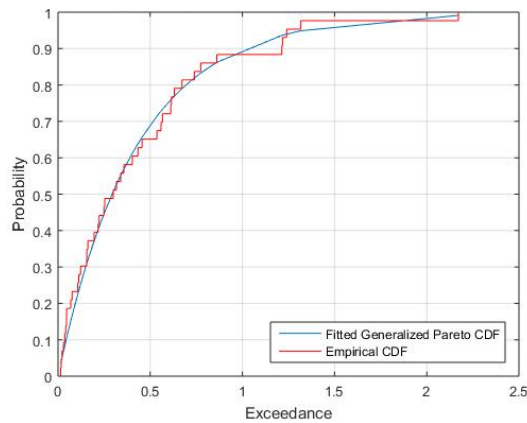


Figure 31: S&P500 Upper Tail of Standardized Residuals

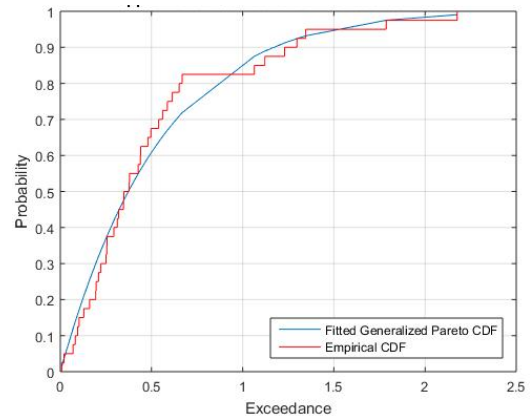


Figure 32: SSEC Upper Tail of Standardized Residuals

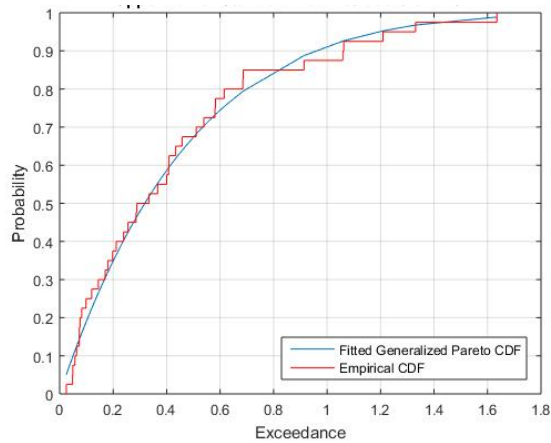


Figure 33: MICEX Upper Tail of Standardized Residuals

After deriving the standardized residuals, the next step is to standardize the t Copula through estimating the parameter of the degrees of freedom and building the linear correlation matrix of the t copula, using the Statistics and Machine Learning Toolbox in MATLAB. This is done through approximating the log-likelihood function for the degrees of freedom parameter and comparing them to the linear correlation matrix. The residuals are standardized to uniform variates using the previously calculated semi-parametric empirical CDF and then fitted to the t copula to the data, known as the Canonical Maximum Likelihood (CML). The t copula parameters are used to simulate dependent returns of the indices by simulating their respective standardized residuals. This is done by transforming the uniform variates into dependent standardized residuals using the semi-parametric marginal CDF relative to every index. As a result, these dependent standardized residuals are used to simulate 2,000 independent trials, calculated over one month/ 22 trading days' horizon. This outcome, which is in the form of a uni-variate series of returns of several trials, is reshaped into a multi-variate series of returns of one particular trial. Consequently, an equally weighted portfolio of returns for the three indices is formed, noting that the equal weights remain constant through both the in-sample and out-of-sample periods. Then, the cumulative returns are

calculated by adding the results of the returns for the specified period. Finally, the required results over a one-month period are simulated using the cumulative returns and are summarized in the below table. (The MathWorks, Inc., 2014)

Outcome	EVT In- Sample
Estimated 90% VaR	2.93%
Estimated 95% VaR	4.83%
Estimated 99% VaR	8.39%

Table 43: EVT- VaR Summary Results of In-sample Period

4.5.2. Out-of-Sample Findings

Out-of-sample estimates include data ranging from January 2017 till May 2018 for the studied portfolio, which constitutes the closing prices of the three chosen stock market indexes: S&P 500 for U.S., SSEC for China and MICEX for Russia. The out-of-sample period includes 315 daily observations. The below figures and analysis disclose the different steps applied on MATLAB to reach the one-month VaR of the portfolio. The same methodology of interpretation used in the In-sample findings has been used for the Out-of-sample findings. First, price movements of the portfolio's indices are presented in Figure 34.

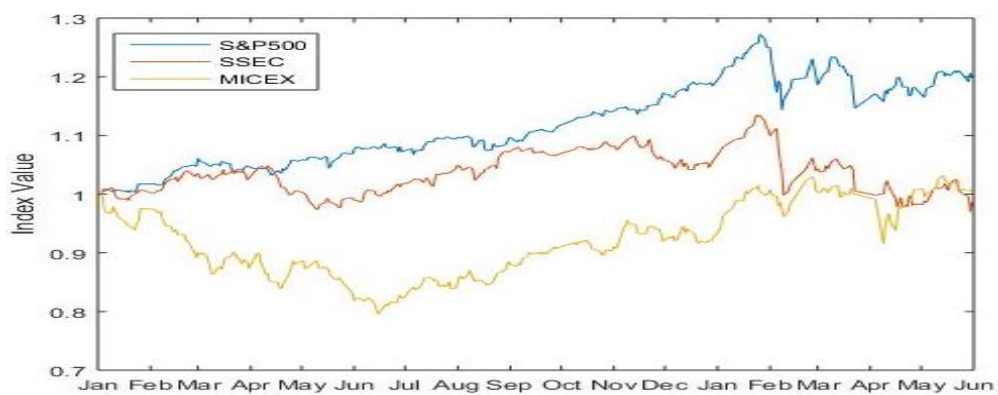


Figure 34: Relative Daily Index Closings of the Out-of-Sample Portfolio

The return series of every index are illustrated in figures 35, 36 and 37.

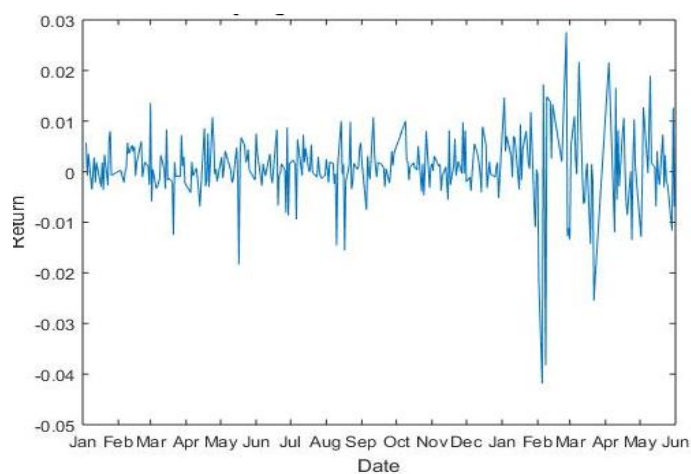


Figure 35: Daily Logarithmic Returns of S&P500

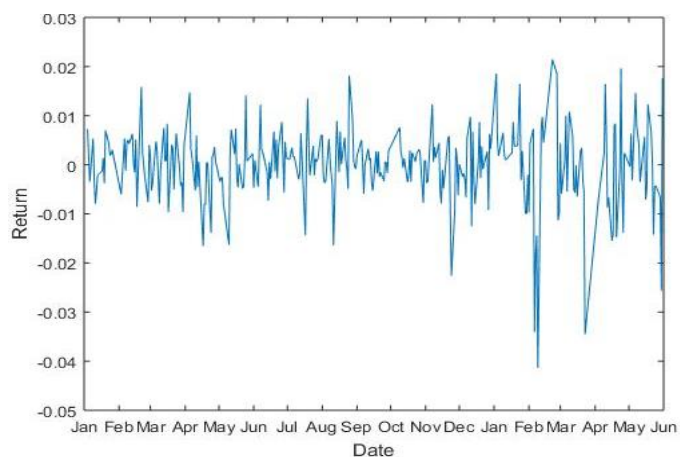


Figure 36: Daily Logarithmic Returns of SSEC

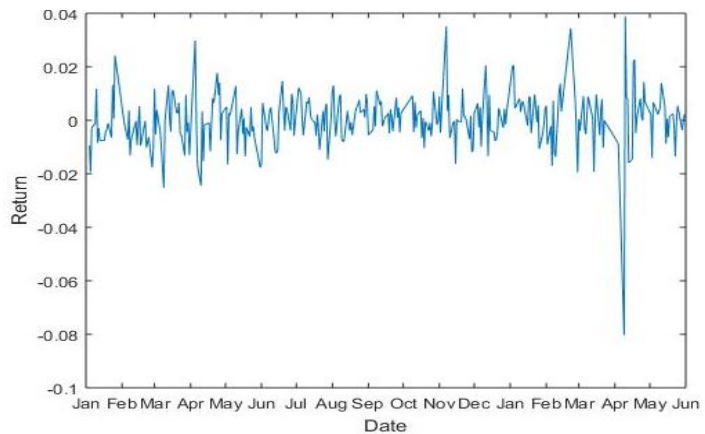


Figure 37: Daily Logarithmic Returns of MICEX

The ACF returns and ACF of squared returns of S&P500, SSEC and MICEX are presented in figures 38 through 43.

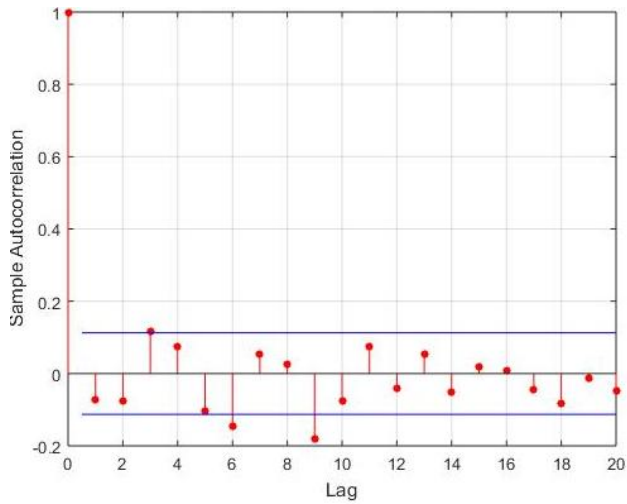


Figure 38: ACF of Returns of S&P500

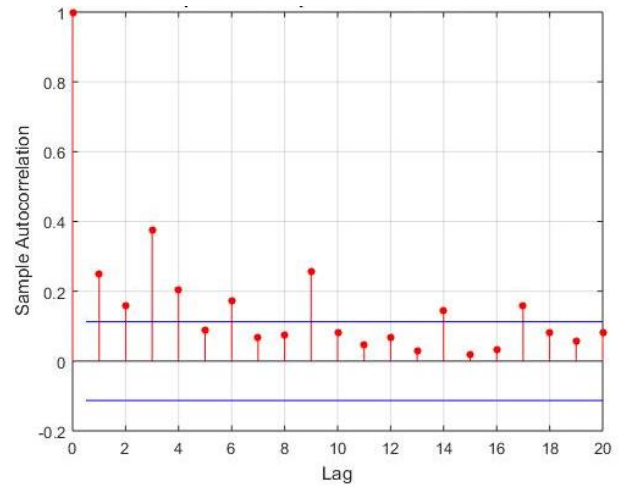


Figure 39: ACF of Squared Returns of S&P500

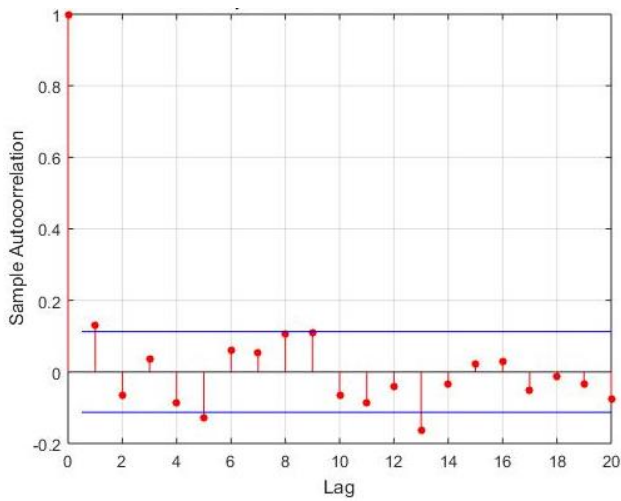


Figure 40: ACF of Returns of SSEC

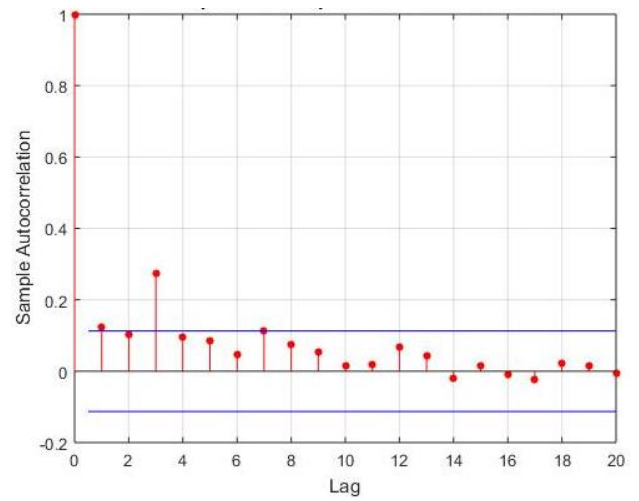


Figure 41: ACF of Squared Returns of SSEC

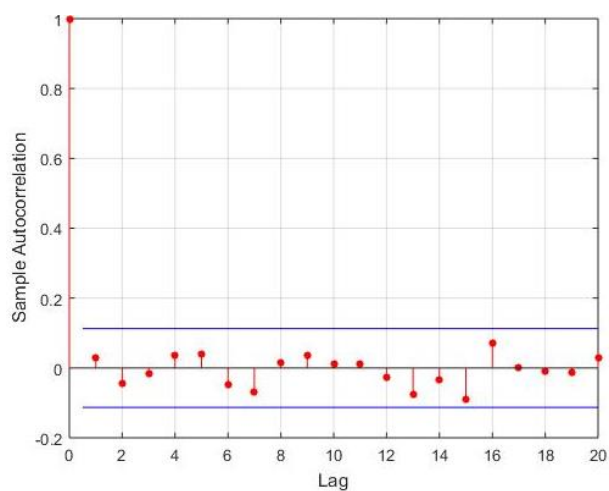


Figure 42: ACF of Returns of MICEX

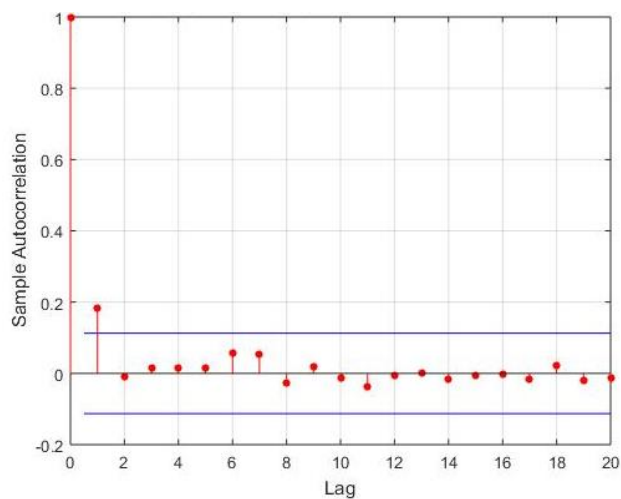


Figure 43: ACF of Squared Returns of MICEX

Reference to figures 44, 46 and 48, the ACF of return series of S&P 500, SSEC and MICEX demonstrate some degree of sequential correlation. Figures 45, 47 and 49 reveal that the variance of squared returns is persistent in the three indices.

The filtered residuals, which demonstrate heteroskedasticity, and the filtered conditional standard deviation of the three indices are compared below.

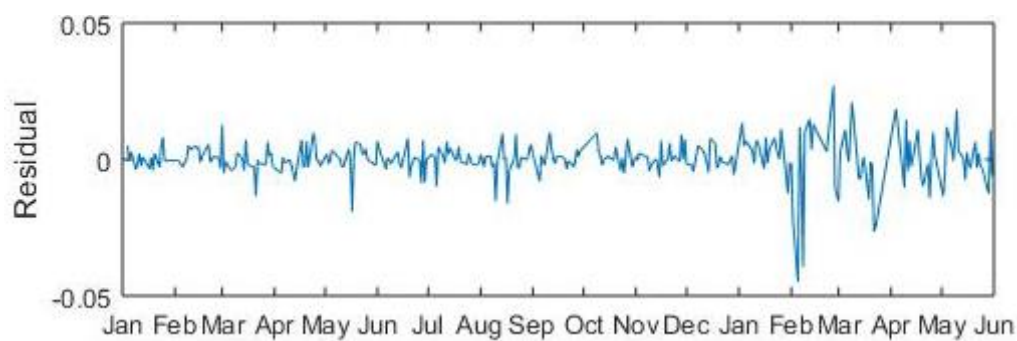


Figure 44: Filtered Residuals of S&P 500

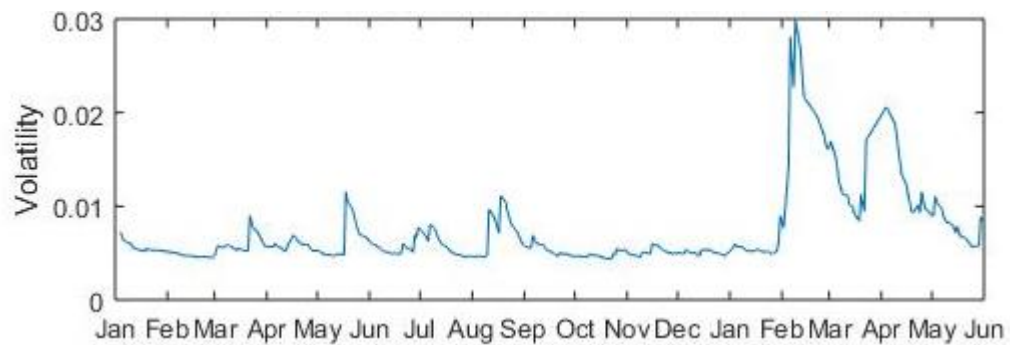


Figure 45: Filtered Conditional Standard Deviation of S&P 500

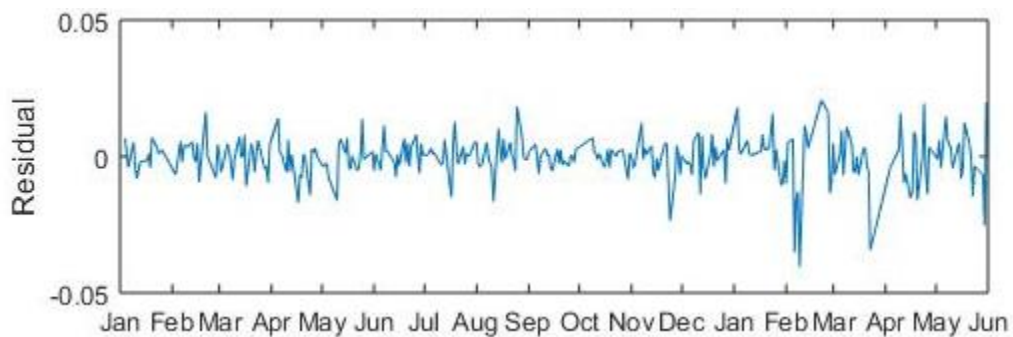


Figure 46: Filtered Residuals of SSEC

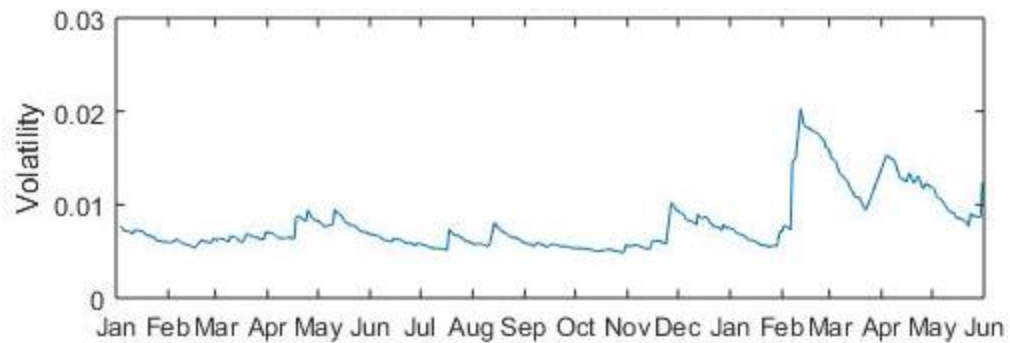


Figure 47: Filtered Conditional Standard Deviation of SSEC

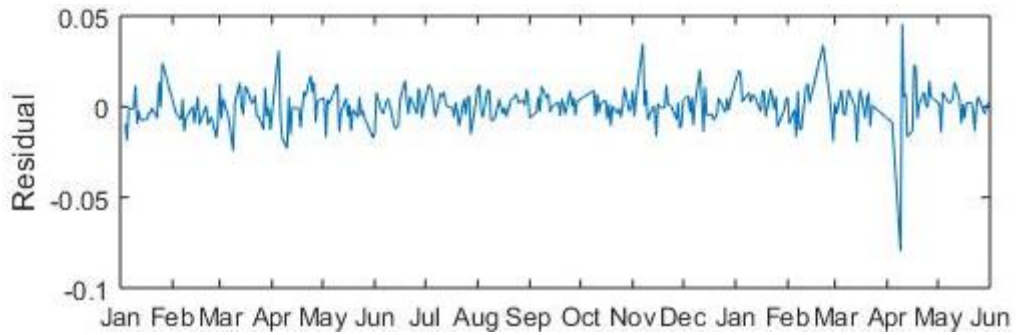


Figure 48: Filtered Residuals of MICEX

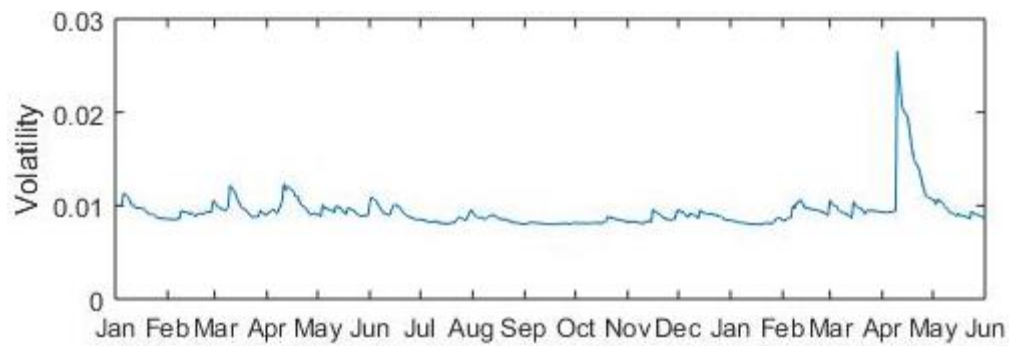


Figure 49: Filtered Conditional Standard Deviation of MICEX

The below figures demonstrate a comparison of the results of the ACF of standardized residuals and ACF of squared standardized residuals of the three indices, which reveal that the transformed standardized residuals are almost i.i.d.

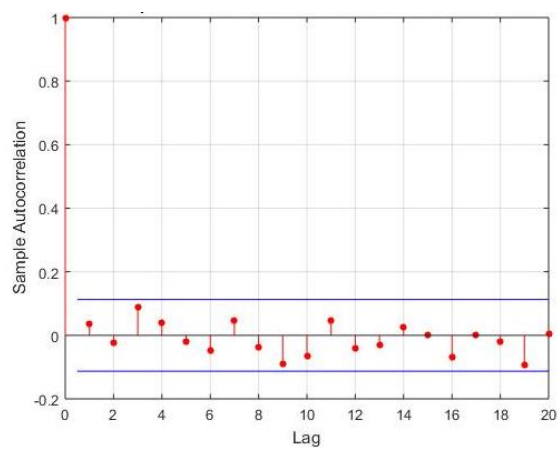


Figure 50: ACF of Standardized Residuals of S&P500

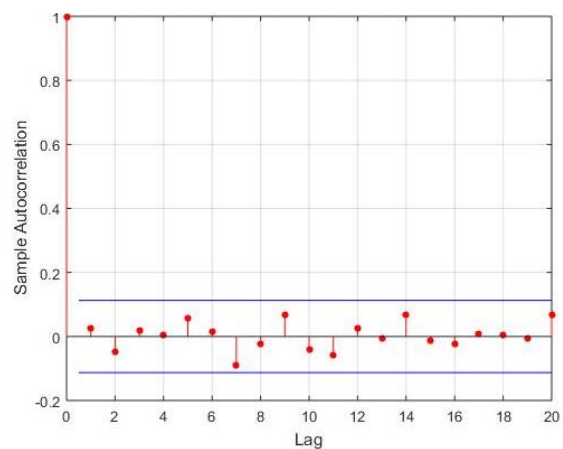


Figure 51: ACF of Squared Standardized Residuals of S&P500

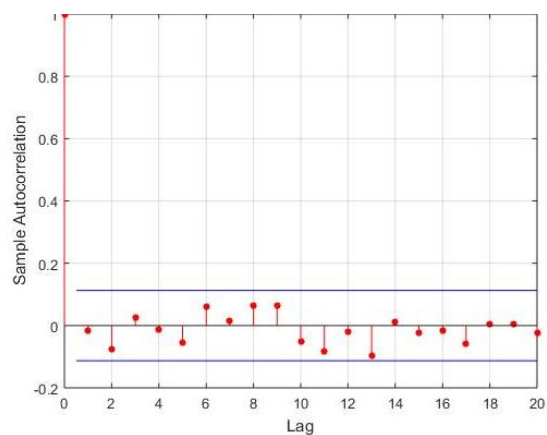


Figure 52: ACF of Standardized Residuals of SSEC

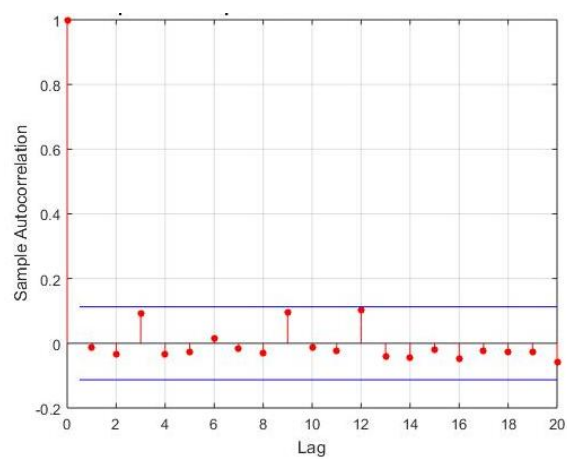


Figure 53: ACF of Squared Standardized Residuals of SSEC

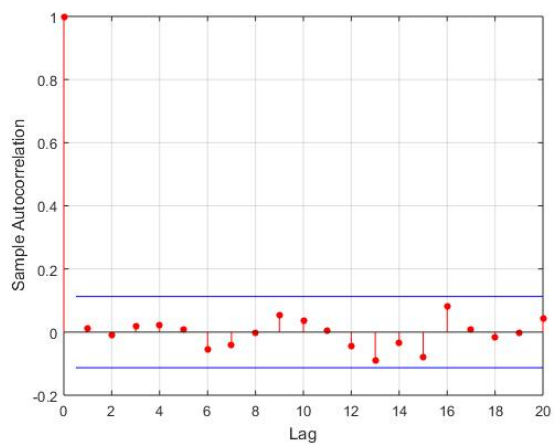


Figure 54: ACF of Standardized Residuals of MICEX

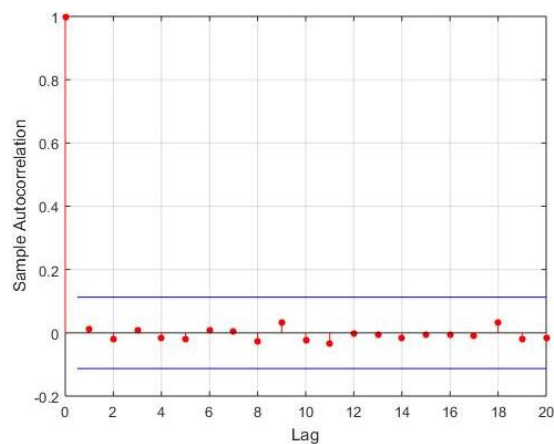


Figure 55: ACF of Squared Standardized Residuals of MICEX

After deriving the standardized residuals, the empirical CDF is calculated using the Gaussian Kernel for every index. However, the CDF estimates are more suitable for the interior distribution rather than the tails. For this reason, EVT is applied to the residuals of the upper and lower tails. As a result, the calculated thresholds for the upper and lower tails account for 10% of the reserved residuals of every tails. The maximum likelihood function is applied to fit the extreme tails' residuals (above the defined threshold) to the parametric GPD, which is reflected in the following figures for the upper tail exceedances. For the three indices, the distribution of exceedances is visually close to the fitted data, which implies that the GPD model is appropriate in our study.

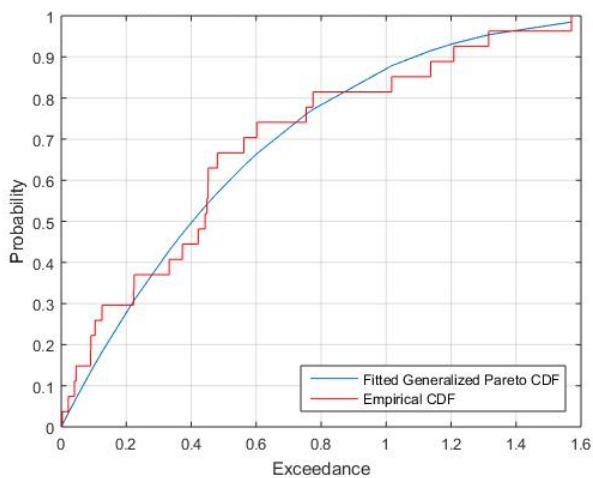


Figure 56: S&P500 Upper Tail of Standardized Residuals

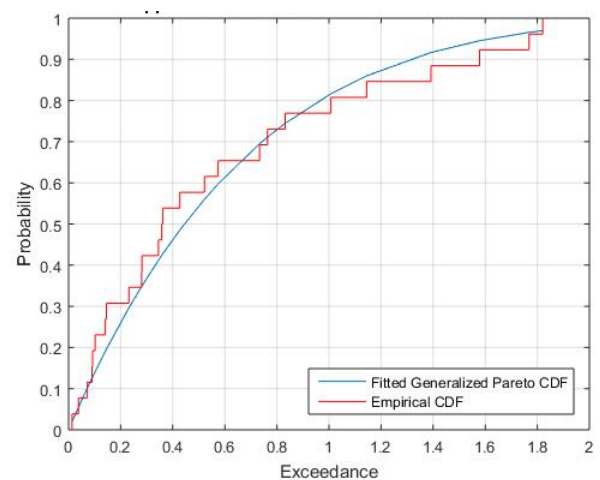


Figure 57: SSEC Upper Tail of Standardized Residuals

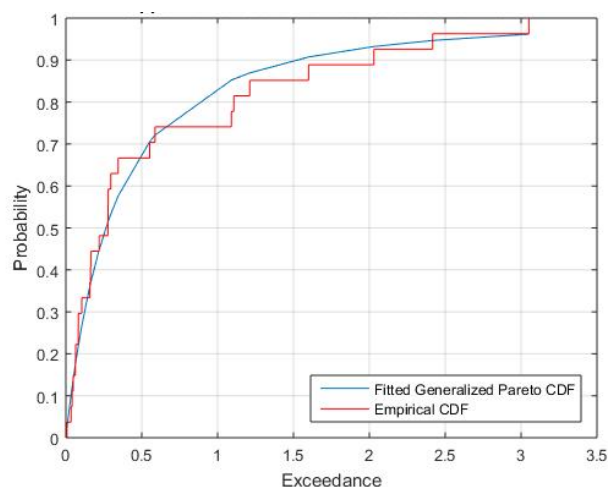


Figure 58: MICEX Upper Tail of Standardized Residuals

Following the above illustrations and reference to the interpretation of the In-sample findings, an equally weighted portfolio of returns for the three indices is formed and the EVT-VaR results over a one-month period are simulated and summarized in the below table.

Outcome	EVT In- Sample	EVT Out-of-Sample
Estimated 90% VaR	2.93%	3.76%
Estimated 95% VaR	4.83%	5.47%
Estimated 99% VaR	8.39%	9.60%

Table 44: EVT- VaR Summary Results

4.6. Final VaR Estimates

The Real VaR is calculated using a portfolio of daily returns of the three chosen stock market indices using excel. In order to calculate the standard deviation of the portfolio's daily returns, the following excel formula is used: STDEV.S. The 90% VaR is also estimated on excel using the following formula: N-1 (0.90) multiplied by STDEV.S. (which is the previously calculated standard deviation). The same formulas are applied to calculate the 95% and 99% Real VaR.

Tables 45 and 46 summarize all the VaR estimates calculated for the in-sample and out-of-sample periods respectively, in reference to various confidence intervals and volatility models and in comparison to the Real VaR estimates.

When comparing results in Tables 40 and 41, it is evident that VaR estimates increase as the confidence level increases from 90% to 99%. EVT VaR results are closer to the Real VaR results in the in-sample period as compared to the out-of-sample period. However, HS VaR results are closer to the Real VaR results in the out-of-sample period as compared to the in-sample period.

Results show that EVT-VaR is higher than HS-VaR for all observations. As such, it could be concluded that EVT measures and forecasts VaR more accurately as compared to the HS incorporated volatility approach. Moreover, results of the in-sample HS-VaR are greater than the results of the out-of-sample HS-VaR as opposed to results of the in-sample EVT-VaR which are lower than the out-of-sample EVT-VaR. In addition, VaR increases as we move to a higher confidence level. On the other hand, the difference between VaR outcomes of EVT and HS increases when comparing the results of the in-sample period to those of the out-of-sample period.

Outcome	HS	EVT	Real VaR
90% VaR	0.68%	2.93%	1.31%
95% VaR	1.03 %	4.83%	1.68%
99% VaR	2.31%	8.39%	2.38%

Table 45: In-Sample VaR Summary Results

Outcome	HS	EVT	Real VaR
90% VaR	0.48%	3.76%	0.73%
95% VaR	0.71%	5.47%	0.94%
99% VaR	1.65%	9.60%	1.33%

Table 46: Out-of-Sample VaR Summary Results

4.7. Conclusion

As a general conclusion to the derived results, GARCH (1,1) is the best volatility model for the in-sample period for S&P500 and SSEC and EGARCH (1,1) for MICEX, in reference to the outcome of the error statistics RMSE, MAE and MAPE. This outcome is somehow validated for the out-of-sample period where the EGARCH (1,1) model proves to be the superior model for SSEC and MICEX; whereas GARCH (1,1) superior for the S&P500 index.

Furthermore, when comparing the RMSE and MAE error statistics, results show that out-of-sample findings are slightly smaller than the in-sample findings. In other words, the out-of-sample

volatility is lower than the in-sample volatility. In short, low volatility periods provide better outcomes when volatility models are used, as opposed to high volatility periods, when applying GARCH (1,1) and EGARCH (1,1) models.

Most importantly, when comparing VaR estimates of the Extreme Value Theory and the Historical Simulation incorporated volatility approach to the Real VaR estimates, we can conclude that EVT performs better in high volatility periods as compared to lower periods (out-of-sample period).

Chapter 5

Final Conclusion and Recommendations

5.1. Introduction

This thesis tackles a combination of three stock market indices to form a portfolio of the three most powerful military countries of the world; U.S., China and Russia. Specifically, the thesis studies the impact of the intervention of the chosen countries in the Syrian war. This is done through studying the market volatility and VaR estimates of the portfolio in reference to the in-sample period extending from January 2015 till December 2016, and the out-of-sample period extending from January 2017 till May 2018.

In order to forecast the volatility of the portfolio, the following volatility models were applied: EWMA, GARCH (1, 1) and EGARCH (1, 1). For the in-sample and out-of-sample periods, parameters were estimated on Excel using the methodologies discussed in Chapter 3. Moreover, parameters were also estimated using NumXL following three distributions: normal distribution, t-distribution and GED. Results revealed that for the three indices, in-sample parameters estimated using the GED distribution are the most accurate, for both GARCH (1, 1) and EGARCH (1, 1). On the other hand, for the out-of-sample period, results using NumXL were different. The most accurate distribution; for both GARCH (1, 1) and EGARCH (1, 1); is presented as follows: the normal distribution is chosen for S&P, GED is chosen for SSEC and t-distribution is chosen for MICEX. In addition, the Jarque-Bera test was implemented to study the normality of the distribution and the ADF (Augmented Dicky Fuller) to test for stationarity. However,

parameters derived from Excel are used for further volatility calculations, since the Long Term volatility is more accurate using these parameters.

To further assess and choose which volatility model is to be implemented in the Historical Simulation approach; RMSE, MAE, and MAPE error statistics for the in-sample period are calculated and compared. Results show the supremacy of the GARCH (1, 1) model for S&P 500 and SSEC; and EGARCH (1, 1) for MICEX.

In specific, the chosen in-sample parameters are used in the out-of-sample volatility calculation in order to assess the ability of the chosen volatility model in forecasting future volatilities. Results of the error statistics for the out-of-sample period show the supremacy of the EGARCH (1, 1) model for SSEC and MICEX; and GARCH (1, 1) model for S&P 500.

A portfolio of volatility updates is established based on the superior model for each index and relative to each sample period. The Historical Simulation approach is applied to the created portfolio and VaR results with confidence levels of 90%, 95% and 99% are concluded.

On the other hand, the portfolio of the three indices is used to calculate VaR estimates using Extreme Value Theory along with a Student t-copula technique applied on Matlab. This is done using the filtered residuals of the indices of the portfolio applied in a GARCH model. Moreover, the ACF (Sample Autocorrelation Function) of returns and those of squared returns is plotted and compared in reference to the degree of variance persistence for each index. Additionally, filtered residuals and filtered conditional standard deviations are plotted from the

initial returns in order to demonstrate the level of variations of volatility and illustrate the heteroskedasticity of filtered residuals.

The Gaussian kernel estimations for the inner function and the generalized Pareto distribution (GPD) estimations for the lower and upper tails are calculated and plotted using the marginal CDF (cumulative distribution function). Student t-copula is fitted to the derived data to establish correlation between the residuals simulated for each index of the portfolio. The applied process paves the way to the estimation of portfolio VaR results of the chosen period, over a horizon of 1 month and confidence levels of 90%, 95% and 99%.

5.2. Analysis of the Findings

The added value of this thesis is supported by the derived results, which pointed out that more accurate estimates are derived from EVT calculations in both in-sample and out-of-sample, as compared to less accurate estimates using the GARCH model. These findings accurately reflect the results concluded by Furio and Climent (2013), who found that it is important to highlight on the distribution of tails, to study extreme movements in the return of several stock prices. This is also validated by the work of Wang et al. (2010), who compared the results of the Historical Simulation approach and the variance-covariance method; where they found that the EVT based VaR estimation produces accurate results for the currency exchange rate risks. Furthermore, the work of Peng et al. (2006) on fat tailed distributions verifies that EVT General Pareto Distribution (GPD) is superior to certain GARCH models, implemented on the Shanghai Stock Exchange

Index. In specific, the GPD model outperformed the GARCH (normal), GARCH (GED) and GARCH (t-student).

We took a further step in order to assess the accuracy of our results by deriving EVT VaR for a portfolio of stock indices of low volatility countries, as compared to high volatility countries studied in this thesis. The chosen low volatility countries include Finland, Sweden and Ecuador with their relative stock market indices: OMX Helsinki, OMX 30 Sweden and ECU Ecuador General index. Data extending from January 2017 till May 2018 was extracted, relevant to the out-of-sample period of the thesis. VaR estimates using Extreme Value Theory along with a Student t-copula technique was applied on Matlab using the newly extracted data. VaR results for confidence intervals of 90%, 95% and 99% were 2.23%, 3.49 % and 6.45% respectively. These results affirm that the chosen countries have low volatility, especially when compared to the out-of-sample VaR estimates being 3.76%, 5.47% and 9.60% for confidence levels of 90%, 95% and 99% respectively. Accordingly, we can conclude that the most powerful military countries of the world being U.S., China and Russia are highly volatile and their portfolio holds a relatively high VaR estimate; when compared to a portfolio of less volatile countries.

Results from our thesis show that GARCH (1, 1) model outperforms EGARCH (1, 1) model for S&P 500, for both the in-sample and out-of-sample period; which is contrary to the results of Awartani and Corradi (2005). They predicted the volatility of S&P 500 using GARCH models to show the function of asymmetries found in an out-of-sample time horizon. In their

results, they proved that asymmetric GARCH models outperform GARCH (1, 1) model, which highlight the predictive ability of the asymmetric models.

Moreover, our results show that EGARCH (1, 1) model outperforms GARCH (1, 1) model for the out-of-sample period, which is similar to the results of Zhe (2018) who proved that EGARCH (1, 1) among the asymmetric models outperformed symmetric models used in his research in forecasting volatility.

5.3. Research Limitations

One limitation relates to the limited number of countries chosen in the portfolio tested; which comprises of only three indices. This is mainly due to the strong influence of the chosen counties on the world's military production.

Moreover, the chosen in-sample period extending from January 2015 till December 2016 might not be the ideal period, since it did not witness the burst of the Syrian war. However, if an earlier time period was to be chosen to entail former years of war, then the studied results would be obsolete.

Finally, the calculated EVT VaR is derived on a one-day basis; reference to the chosen periods. However, it would be interesting to derive a panel of daily EVT VaR results over a specific period to assess the trend of variations in these results.

5.4. Recommendations for Future Studies

The first recommendation is introducing additional volatility models to increase the opportunity to compare between a wider selection of implemented models and generalize results. The role of additional models is to better understand the volatility of the studied countries. Moreover, the selected in-sample period could be extended to cover the earliest days of the Syrian war. This would enrich the study with a wider spectrum of observations to assess and evaluate volatility fluctuations reference to the circumstances that took place.

It is also recommended to compare the VaR of a portfolio comprising of the primary producers and exporters of military equipment to a portfolio of countries which are the primary importers of military equipment. In this comparison, the balance of payments and military budgets of the chosen countries could be tackled to highlight any relation between the budget spent on military equipment and volume of exports or imports to the forecasted VaR. From a similar perspective, this analysis could also be studied to assess the relationship between the volume of military production and level of political influence of the exporting countries.

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