VOLATILITY AND VALUE AT RISK: CRYPTO VERSUS FIAT CURRENCIES

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ABSTRACT

Purpose: The purpose of this thesis is to investigate the ability of EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1) to forecast volatilities of Bitcoin, Ripple, EURUSD, GBPUSD and CNYUSD. The optimal volatility model for each fiat and virtual currency is used to measure the accuracy of VaR by incorporating the volatility update into the Historical Simulation approach.

Design/Methodology/Approach: In-sample returns are calculated from daily closing prices and are used in estimating the parameters of the selected models. The calculated in-sample parameters are applied to estimate and forecast the volatilities during the in-sample and out-of-sample periods. The studied period extended from March 01, 2016 to February 28, 2019. The in-sample period extended from March 01, 2016 through February 28, 2018 while the out-of-sample period covered March 01, 2018 to February 28, 2019. Three error metrics (RMSE, MAE and MAPE) are used to determine the optimal model for each currency and cryptocurrency in both sample periods. Scenarios of future returns are generated for each day for each of the selected market variables to measure VaR. These scenarios are calculated by incorporating volatility updating to the historical simulation. VaR values for the last 250 days of the data sample are calculated on four confidence levels: 90%, 95%, 97.5% and 99%. The Kupiec test is applied to determine the accuracy of the model.

Findings: By comparing the calculated volatilities to the realized volatility, the EWMA model outperformed the rest of the models for all of the selected currencies and cryptocurrencies during the in-sample period. In the out-of-sample period, the GARCH (p, q) was the optimal model for the CNYUSD and Ripple, and the EWMA proved to be the best model for the EURUSD, GBPUSD and Bitcoin. The calculated volatilities were compared to the implied volatility for the selected fiat currencies. During the in-sample period, the GARCH (1, 1), GARCH (6, 6) and EWMA were the optimal models for the EURUSD, CNYUSD and GBPUSD, respectively. However, in the out-of-sample period, the EWMA model was optimal for the EURUSD and CNYUSD. As for the GBPUSD, the EGARCH (1, 1) was selected as the best model. Finally, VaR results are back-tested using Kupiec test. The results were accepted for the EURUSD, GBPUSD and Bitcoin at all confidence levels. As for the CNYUSD, the results were rejected at 90% and 95% confidence levels. Ripple's results were only accepted at 90% and 99% confidence levels.

Research Limitations/Implications: In this study, we only considered the EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1) models, whereby there are other models that could be used. Also, when calculating VaR, we solely examined incorporating volatility update into the historical simulation model, while disregarding other models. Furthermore, when calculating the volatility and VaR, altering the number of generated scenarios and the sample size might have led to different results. Finally, we only back-tested VaR results using the Kupiec test. Other tests might have been applied such as the independence test suggested by Christoffersen, where consecutive and frequent exceptions are taken into consideration.

Practical Implications: Our results are helpful for decision makers (investors, firms, governments, etc.) willing to invest in cryptocurrencies. Bitcoin and generally cryptocurrencies cannot act as alternatives to fiat currencies at the moment. This is due to their volatile behavior significantly different from the fiat currencies behavior. Second, investors need to be prudent when considering an investment in cryptocurrencies, given their high risk and extremely volatile behavior. Finally, market participants aiming at diversifying their portfolios or seeking a risky position could consider cryptocurrencies, given their unique behavior compared to other instruments.

Originality/Value: This study is original since it tackles two cryptocurrencies and three fiat currencies by comparing their volatility and VaR behavior. To our knowledge, this is the first time the GARCH (p, q) and incorporating volatility into historical simulation are used in cryptocurrencies assessment for the period March 01, 2016 through February 28, 2019.

Keywords: Bitcoin, Ripple, EURUSD, GBPUSD, CNYUSD, volatility, realized volatility, implied volatility, EWMA, GARCH (1, 1), GARCH (p, q), EGARCH (1, 1), in-sample, out-of-sample, log-likelihood ratio, incorporating volatility into historical simulation, Value at Risk (VaR), Kupiec test.

Chapter 1

Introduction

1.1 General background about the topic

From the beginning of times, almost everything has been used as money to facilitate transactions between human beings. Starting from beans, pearls, animals, silver, gold and even slaves, all of which led to modern times shifting away from barter to using fiat money (Jenks, 1964). Nowadays there has been an introduction of electronic money or cryptocurrency such as Bitcoin, Litecoin, Ripple and many others. Cryptocurrencies work through the Blockchain technology allowing fast transfer or payments with little to no transaction cost.

Cryptocurrencies suggest an alternative or a switch from the traditional financial system while operating under algorithms and peer to peer mechanisms. This new type of assets enables transparency and security which oppose the current less transparent monetary system (Samuelson, 1968). After recent economic crises, the public’s trust in the financial system had tumbled, which increased the popularity of alternative and new concepts such as cryptocurrency (Glaser et al., 2014). Gupta (2017) pointed at the inefficiencies and transaction costs associated with regular banking, all of which led to the creation of Bitcoin.

The first decentralized cryptocurrency (Bitcoin) was created in 2009 and as a result of its huge success, it has now more than 1600 competitors (Corbet et al., 2017). The founder of Bitcoin, Satoshi Nakamoto, defines cryptocurrency as a digital asset aimed to work as a mean of exchange using cryptography. The blockchain technology allows rapid transactions where their history is saved in a chain (Nakamoto, 2008). Miners solve cryptographic puzzles to validate a transaction, where a reward namely a fraction of a Bitcoin is awarded afterwards (Brière et al., 2015).

Blockchain is described as a ledger technology, which acts as a data base and aims to keep a copy of the performed transactions synced and verified. The blockchain is still in its beginning stage, but its main advantage is that it aims to eliminate the need for third parties and acts as a level of trust in exchanging data which are known as transactions. The blockchain technology was able to affect many business models across the industries (Seppala, 2016).

Furthermore, Allen (2017) focused on the importance of blockchain technology and that it was implemented in Bitcoin to store information over currency. Furthermore, any type of information needs a third-party intermediary to verify the transaction done, but the blockchain technology aims to eliminate this third party and become independent. However, there is still riskiness in the cryptocurrency market since it is a possible source of uncertainty and financial instability (Omari et al., 2019).

Cap (2018) described the growth in the digital currencies market as exponential with a total market capitalization of more than 128 billion dollars. Moreover, as of January 2019, Bitcoin dominated the cryptocurrency market with a 53% market share. On the other hand, Ripple has 9% of the cryptocurrency market capitalization and works through a centralized form of the Blockchain technology where the control of its activity is under a single authority away from the presence of miners.

However, and regardless of its huge capitalization, cryptocurrencies have been subject to many accusations such as price bubbles and money laundering (Corbet el al., 2018). Even after ten years of its initiation, there still exists debates on the classification of cryptocurrencies, while some describe them as currencies, while others argue that they should be classified as commodities (Naimy and Hayek, 2018).

Recent studies by Katsiampa (2017) and Osterrieder and Lorenz (2017) pointed at the high volatility and return behavior observed in cryptocurrencies. This is pushing governments and central banks to regulate this new type of asset, since they are concerned about its trading mechanisms, to protect financial institutions from undesirable risks.

Since risk assessment is at the core attention of any firm or government, it is important to estimate the volatility of any security as it assists risk managers in pointing at investment risks and possible unwanted outcomes, therefore facilitating risk decisions. In addition, volatility is a major component of pricing an option, as it represents the rate and magnitude of price changes (Jorion, 2001).

Following this standpoint, many complex models for estimating volatility have been created and are usually used such as GARCH and EWMA. GARCH models capture the volatility of the studied data, however they fail to capture errors distribution since they are not symmetric or normally distributed (Kosapattarapim, 2013). The EWMA model assumes that volatility is not constant, and the model adapts to new modifications upon occurrence. New data have a more significant effect on the future or estimated volatility, which is the reason for most recent data having a higher weight than older data (Korkmaz and Aydin, 2002). On the other hand, EGARCH or the Exponential GARCH model points that positive shocks have less influence than negative shocks. This is usually witnessed due to the leverage effect, thus showing asymmetry in variances. This leads to different signs in the model's parameters (Schmitt, 1996).

1.2 Need for the study

It is essential to note that digital currencies are still an important topic in financial debates as their classification as asset or currency is still ambiguous (Glaser et al, 2014). However, there hasn't been enough studies on this topic since it is relatively new.

This research attempts to explore and assess several types of volatility models applied on cryptocurrencies (Bitcoin, Ripple) and fiat currencies (EURUSD, GBPUSD, CNYUSD) to choose the best model suitable for each instrument and to check whether cryptocurrencies have the properties of foreign exchange. This highlights the importance of this thesis as it evaluates the volatility of traditional currencies versus new digital currencies.

Many models have been used to determine volatility, but this research will tackle the EGARCH, GARCH (1, 1), GARCH (p, q) and EWMA models since they take into consideration time series analysis and are heavily used worldwide.

The main objective of this research can be achieved by answering the below questions:

- What would be the most appropriate volatility model for each underlying asset?
- Will the best volatility model be the same for all the studied fiat and virtual currencies?
- Will cryptocurrencies behavior and risk be similar to traditional currencies?
- How will the VaR (Value at Risk) performance differ between fiat currencies and cryptocurrencies?

1.3 Purpose of the study

Upon completion of this research, several answers will be provided. The findings will be used to assess each volatility model implemented on the different currencies and cryptocurrencies as well as the possibility of generalizing this study to the entire digital currency market, since the studied cryptocurrencies (Bitcoin, Ripple) represent more than 62% of the market capitalization as of January 2019. The results will be also used to compare the behavior and properties of cryptocurrencies versus fiat currencies. This might help investors and decision makers to have a better understanding of the digital currency market.

1.4 Brief overview of all chapters

This thesis is structured as follows. Chapter 2 is a review of the existing literature on the performance of different volatility models on cryptocurrencies and other asset classes. A significant number of studies tackled cryptocurrencies' Value at Risk calculation in order to compare their behavior to currencies, commodities and indices. The findings related to cryptocurrencies' volatility and VaR under many approaches are presented.

Chapter 3 explains the adopted methodology. First, the descriptive statistics of the selected currencies and cryptocurrencies' returns over the period March 01, 2016 through February 28, 2019 are presented along with the stationarity tests for the returns. Then, the selected volatility models are discussed: EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1). The maximum likelihood method used for parameters estimation is described as well. Additionally, incorporating volatility updating into historical simulation model is represented to calculate VaR. Lastly, the back-testing methodology used to assess VaR accuracy is explained, whereby the actual returns are compared to the VaR estimates.

Chapter 4 summarizes the main findings. First the parameters are estimated for the selected currencies and cryptocurrencies during the in-sample period ranging from March 01, 2016 through February 28, 2018. The in-sample parameters are then used in the out-of-sample period extending from March 01, 2018 till February 28, 2019 period to determine the predictive ability of the selected models. The selected optimal models in the out-of-sample period are employed in VaR calculation using the volatility update into historical simulation model. A total of 399 scenarios of future returns are generated over 250 days. Finally, Kupiec test results are presented to conclude the accuracy of VaR for the selected currencies and cryptocurrencies.

Chapter 5 discusses the results and highlights the implications of this research on investors and decision makers. The main conclusions are compared to previous studies. Finally, the limitations and the recommendations for future researches related to this thesis are discussed.

Chapter 2

Review of Literature

2.1 State of Knowledge and Previous Research

The purpose of this chapter is to review the previous studies that tackled and assessed the volatility of cryptocurrencies. However, the number of previous researches related to cryptocurrencies is still limited compared to those related to other types of assets.

Volatility is an extremely important concept, representing uncertainty in the future. The latter is used to monitor the change in the market variables that might affect the pricing and investment decisions in exchange rates, equities, derivatives, interest rates and many other financial instruments. For this reason, its measurement represents a serious concern for firms and decision makers. Many existing models such as EWMA and GARCH have been expanded to measure and predict the volatility of assets. It is important to note that volatility is not constant over time since it tends to increase or decrease as market drivers tend to change often (Hull, 2012).

Also, volatility clustering is another important concern to consider. It indicates that large changes tend to be followed by periods of high volatility and vice versa. This was first noted by Mandelbrot in 1963.

Urquhart (2017) and Bariviera (2017) studied Bitcoin price clustering with periods extending from May 2012 till April 2017 and from August 2011 till February 2017, respectively. Both studies found a persistence in the volatility and evidence of price clustering.

Previous studies focused on the risk of virtual currencies, especially Bitcoin. A research conducted by Chu *et al.* (2017) on seven cryptocurrencies (Bitcoin, Dash, Litecoin, Ripple, Monero, Dogecoin and Maidsafecoin) using twelve GARCH models with a period extending from June 2014 to May 2017, revealed that IGARCH (1, 1) and GJRGARCH (1, 1) models are the best models in modelling the volatility associated with virtual currencies. However, they pointed at the high volatility associated with cryptocurrencies making it a suitable investment for risk seeking portfolios.

Stavroyiannis and Babalos (2017) investigated the variance of negative and positive shocks of virtual currencies versus other asset classes using several GARCH models with a time horizon extending from July 2013 till December 2016. They noticed a persistence in the volatility process using such models and concluded that Bitcoin is highly volatile and violates VaR measures more than other assets, such as the S&P500 index and Gold. Their study suggested that cryptocurrencies are unable to provide investors with benefits such as diversification and hedging when compared to the US market.

Naimy and Hayek (2018) implemented a study on Bitcoin to calculate and estimate its in and out-of-sample volatility using EWMA, GARCH and EGARCH during a period extending from April 2013 till March 2016. The findings showed that EGARCH, which captures the leverage effect, outperformed the other models in both contexts. By comparing the estimated volatility of each model to the realized volatility, using the MAE (mean absolute error) and RMSE (root mean square error), the authors concluded that the models revealed more favorable results while forecasting volatility in the out-of-sample period. The authors also noted that the behavior of Bitcoin is different than the behavior of fiat currencies.

Gkillas and Katsiampa (2018) implemented an extreme value analysis for VaR and Expected Shortfall on five major cryptocurrencies (Bitcoin, Ripple, Ethereum, Litecoin and Bitcoin Cash) to examine the tail behavior of the returns. The used data varied in sample size and ranged between July 2010 and July 2017, depending on each currency's initiation date. Surprisingly, their results revealed that Bitcoin and Litecoin were least risky while Bitcoin cash was the riskiest with the highest recorded volatility. However, it is important to note that the number of observations used for Bitcoin cash was limited, since the latter was still in its early stages.

Stavroyiannis (2018) implemented a study on Bitcoin, S&P 500 index and Gold during the period extending from July 2013 till July 2017. The GJR-GARCH (1, 1) model was implemented to estimate VaR and expected shortfall. The results were backtested by counting the number of violations. All the risk measures applied on the data were less favorable towards Bitcoin which needed a larger capital allocation due to its high volatility. The author stated that the main concern of the Basel Committee on banking supervision was the risk associated with an investment in virtual currencies. This distress has risen after the recent global financial crisis, which forced investors and investment banks to secure their investments from risks and unexpected market conditions.

Bouri *et al.* (2017) implemented a research focusing on the risk measures for Bitcoin and conducted a comparative analysis with other indices taking the S&P500 as a benchmark with Brent oil, Crude oil and the gold spot price. The selected time period extended from July 2011 through December 2015. The GARCH (1, 1) model and other models were used in the study. The findings revealed that the GARCH (1, 1) model was the most effective and efficient in forecasting risk and volatility in the digital currencies. Furthermore, they noted that Bitcoin can be used in a portfolio for diversification purposes. In addition, the latter holds hedging and safe-haven properties against few markets, such as Asia Pacific stocks.

Kim (2017) implemented a study on the volume and spread of Bitcoin versus fifteen fiat currencies such as Euro, US dollar, Chinese yuan and many others. The research was conducted in a time horizon extending from April 2014 till April 2015. The findings indicated that Bitcoin is considered an alternative to the foreign exchange market for international transactions and settlements. The spread of Bitcoin was found typically lower than the retail foreign exchange market.

Dyhrberg (2016b) wanted to inspect if Bitcoin holds similar hedging capabilities of gold when compared to the FTSE 100 index, the dollar-euro and the dollar-sterling exchange rates. The research was conducted using the Threshold GARCH model with a period extending from July 2010 till May 2015. The results showed that Bitcoin can serve as a hedging vehicle against the FTSE 100 index. In addition, Bitcoin can be used as a hedge against the US dollar in the short term.

Dyhrberg (2016a) implemented a volatility analysis on Bitcoin, Gold and the US dollar using GARCH (1, 1) and EGARCH (1, 1) models with a time frame extending from July 2010 till May 2015. The results showed similarities between the variables when using the GARCH (1, 1) model in terms of volatility clustering and persistence; thus, suggesting the hedging capabilities of Bitcoin to the US dollar and Gold. However, with the asymmetric EGARCH (1, 1) model, the author suggested the use of Bitcoin for risk management purposes while proposing a hybrid classification of the cryptocurrency between commodities and fiat currencies.

Baur *et al.* (2018) replicated and extended a research done by Dyhrberg (2016a) on Bitcoin, Gold and the US dollar. The applied models were the GARCH (1, 1) and EGARCH (1, 1) with the same time horizon extending from July 2010 till May 2015. However, their findings differed from Dyhrberg as they concluded that Bitcoin characteristics such as volatility, correlation and returns are neither similar to Gold nor to the US dollar.

Katsiampa (2017) wanted to evaluate the volatility of Bitcoin while using six different GARCH models with a period extending from July 2010 till October 2016. The author revealed that Bitcoin market is highly speculative. The results showed that the AR-CGARCH model outperformed other models, which in turn demonstrates the added value of using both a long-run and a short-run conditional variances.

Bouoiyour and Selmi (2016) implemented a study on Bitcoin using several GARCH models in a period extending from December 2010 till July 2016; their findings showed that Bitcoin is extremely volatile with a conditional variance trailing an "explosive" process, where positive shocks have less effect than negative shocks. They suggested that the virtual currencies market was still immature even after it reached low levels of volatility.

Phillip *et al.* (2018) implemented a study on 224 virtual currencies using the stochastic volatility model with a time horizon extending from July 2010 till July 2017. The number of observations differed between the selected cryptocurrencies since their initiation dates were dissimilar. The author noted predictable patterns with the presence of volatility clustering. Mild leverage effects were noticed on cryptocurrencies except for Ripple, which has the weakest leverage effect. This prompted most banks to adopt it as their settlement vehicle. They also pointed at the safety of cryptocurrencies represented by the absence of counterparty risk.

Urquhart and Zhang (2018) wanted to inspect Bitcoin properties to see if they can be used as a hedge or portfolio diversifier and whether they hold similarities to safe-haven assets. The research was conducted for a period extending from November 2014 till October 2017. Using several GARCH models on the studied variables (Bitcoin, Australian Dollar, Canadian Dollar, Swiss Franc, Euro, Great Britain Pound and the Japanese Yen), the findings were as follows: Bitcoin can be used as a hedge against the Swiss Franc, Euro and the Great Britain Pound, whereas for the Australian Dollar, Canadian Dollar and the Japanese Yen, Bitcoin can act as a diversifier. Furthermore, the authors noted that Bitcoin can be considered a safe-haven during market turbulence when compared to the Canadian Dollar, Swiss Franc and Great Britain Pound but not when compared to the Australian Dollar, Euro and the Japanese Yen.

An interesting study done by Liu and Tsyvinski (2018) evaluated the return and price drivers of cryptocurrencies (Bitcoin, Ethereum and Ripple) versus certain stocks, currencies (Great Britain Pound, Euro, Canadian Dollar, Australian Dollar and Singaporean Dollar) and precious metals (Gold, Silver and Platinum) while using the CAPM and Fama French models. Multiple time horizons were used ranging between January 2011 and May 2018. Their findings showed low correlation in returns of cryptocurrencies compared to traditional assets, which cast away the storage of value and unit of account observed in commodities and currencies respectively. They also pointed at the behavior of the cryptocurrency market driven by momentum and supply (mining cost).

On the other hand, Trucíos (2019) wanted to determine the best model in forecasting Bitcoin volatility and Value at Risk using twelve GARCH models with a time horizon extending from September 2011 till December 2017. The results showed that AVGARCH model outperformed other GARCH-type models, on the assumption that errors are symmetric and skewed. However, when using robust procedures, thus emphasizing on outliers with GAS (Generalized Autoregressive Score) model, the robust GARCH outperformed all the non-robust models.

Peng *et al.* (2018) implemented a study on Bitcoin volatility using ten GARCH models, such as GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1) and many others. The sampling consisted of two data sets: the first is daily data characterized by low frequency during a period extending from January 2016 till July 2017. The second sample consisted of intraday hourly data considered as high frequency with six different periods, each one consisting of nine months, ranging from January 2016 till July 2017. RMSE and MAE error metrics were used in order to validate the findings. SVR-GARCH (Support Vector Regression-GARCH) outperformed other models in both samples. This is due to its ability to cover nonlinearity and dynamics in the financials series represented by volatility clusters and leptokurtic data distribution.

Cermak (2017) implemented the GARCH (1, 1) model on Bitcoin versus several currencies, namely the Chinese Yuan, Euro and the Japanese Yen during a period extending from August 2010 till March 2017. The author's aim was to test whether Bitcoin can become an alternative to fiat currencies. The findings were that Bitcoin cannot be an alternative to traditional currencies, due to its relatively high volatility and inability to act as an effective medium of exchange or store of value. However, Bitcoin is considered a safe-haven asset in China. Cermak also pointed at the volatility behavior throughout the lifetime of Bitcoin and suggested that Bitcoin's volatility level will coincide with fiat currencies between 2019 and 2020.

Radovanov *et al.* (2018) implemented a study on four cryptocurrencies (Bitcoin, Ethereum, Ripple and Litecoin) while using GARCH (1, 1), EGARCH (1, 1) and GJRGARCH (1, 1) in order to understand the behavior and characteristics of the virtual currency market within a time horizon extending from August 2015 till March 2018. The findings showed a persistence in the volatility for all the variables. However, Ripple and Litecoin exhibited an asymmetry in volatility where good news has larger effect than bad news. They concluded that cryptocurrencies position is still vague between commodities and currencies, due to their decentralized and scarce nature. In addition, they pointed at the legislation challenges facing the cryptocurrency market. Nevertheless, the authors suggested regulating the Blockchain technology for international use, which might benefit worldwide governments and corporations.

2.2 Conclusion

This chapter summarized previous researches' output on cryptocurrencies volatility. Different conclusions were drawn regarding the nature and properties of cryptocurrencies. However, most of the researches pointed at the high volatility of digital currencies, while some studies suggested that an investment in cryptocurrencies is suitable for risk seeking portfolios.

Most of the studies explored in this chapter used GARCH and EGARCH models, which justify our methodology in using those models to assess the volatility of virtual and fiat currencies. Many studies concluded that cryptocurrencies violate VaR measures more than traditional currencies and other types of assets.

To our knowledge and based on the review of literature, this thesis will be the first to provide a comparison between crypto and fiat currencies while suggesting the optimal model for each currency's volatility forecast and testing the accuracy of VaR among our selected assets. The next chapter will discuss the methodology and present the descriptive statistics and tests related to our selected fiat and cryptocurrencies.

Chapter 3

Procedures and Methodology

3.1 Introduction

From the previous researches stated in the review of literature, inconsistencies in the results can be observed. The main causes of these discrepancies are the time frame and the selected volatility models, since volatilities related to cryptocurrencies are not stable when compared to those related to other types of assets.

This chapter presents the descriptive statistics of the data and exposes the selected volatility models; EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1). The studied period ranges from March 01, 2016 till February 28, 2019. Whereas the in-sample period extends from March 01, 2016 till February 28, 2018, the out-of-sample period extends from March 01, 2018 till February 28, 2019. The volatility for each asset under each of the selected models will be calculated for the in-sample and out-of-sample periods. The results will be compared to the realized and implied volatilities in order to determine the best model for each underlying asset. This will be achieved by using the error metrics such as the MAE (Mean Absolute Error), RMSE (Root Mean Square Error) and the MAPE (Mean Absolute Percentage Error) in order to compare the calculated volatilities to the realized and implied volatilities and to rank the models from the most to the least accurate for each asset. However, the implied volatility will not be tested for cryptocurrencies since their options market is still relatively new and immature. Accordingly, this thesis will compare fiat currencies (Euro, Pound, and Chinese Yuan) and cryptocurrencies (Bitcoin and Ripple), while assessing the predictive ability of the selected volatility models.

3.2 Data Description

The data represents the daily closing prices of three years for Bitcoin/USD, Ripple/USD EUR/USD, GBP/USD and CNY/USD. The Bitcoin/USD and Ripple/USD data are downloaded from coinmarketcap.com website, whereas EUR/USD, GBPUSD and CNY/USD data are extracted from the Bloomberg platform. The overall studied period is from March 01, 2016 till February 28, 2019. The in-sample period will cover March 01, 2016 through February 28, 2018 with 489 observations and the out-of-sample period will range from March 01, 2018 till February 28, 2019 with 243 observations.

On December 16, 2017, Bitcoin prices reached a peak of $19,497, a growth of 4380% from the price at the start of the period on March 01, 2016, which was only $435. This is similar to Ripple prices, which witnessed a huge growth of 42150% from $0.008 at the start of the period on March 01, 2016 and reached $3.38 on January 7, 2018. On the other hand, the Brexit period negatively affected all Europe since the referendum on June 23, 2016 and consequently the Euro. Also, during this period, trade tensions have grown dramatically between the US and china, which has led to a devaluation of the Chinese Yuan (CNY).

Since cryptocurrencies prices are quoted daily including the weekends when compared to fiat currencies, which are quoted only on weekdays, the data needs to be filtered using the VLOOKUP[1] function in Excel to match the closing prices for each of the selected fiat currencies and cryptocurrencies.

Hull (2012) describes daily volatility as the standard deviation of the proportional change in the variable during a day. Consequently, daily prices are converted into daily return using the following equation:

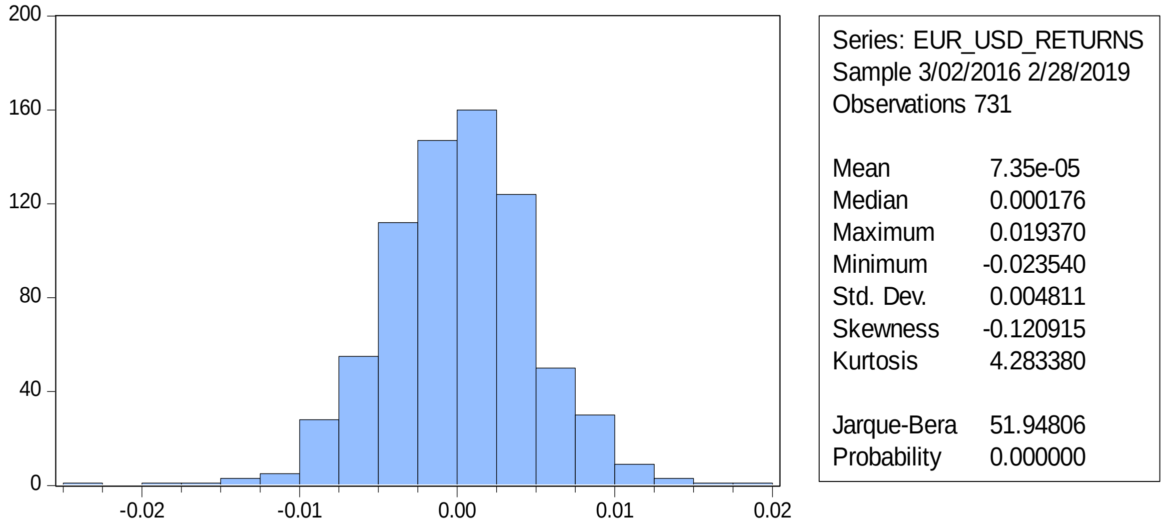
$$u_i = \frac{S_i - S_{i-1}}{S_{i-1}} \tag{1}$$

Where u_i is the return on the i^{it} day, S_i and S_{i-1} are respectively the price of the asset at the end of the i^{it} day and at the end of the previous day i -1.

The descriptive statistics of the daily return of the three currencies and two cryptocurrencies are presented below in Figures 1, 2, 3, 4 and 5; the normality tests were conducted using EViews 8.

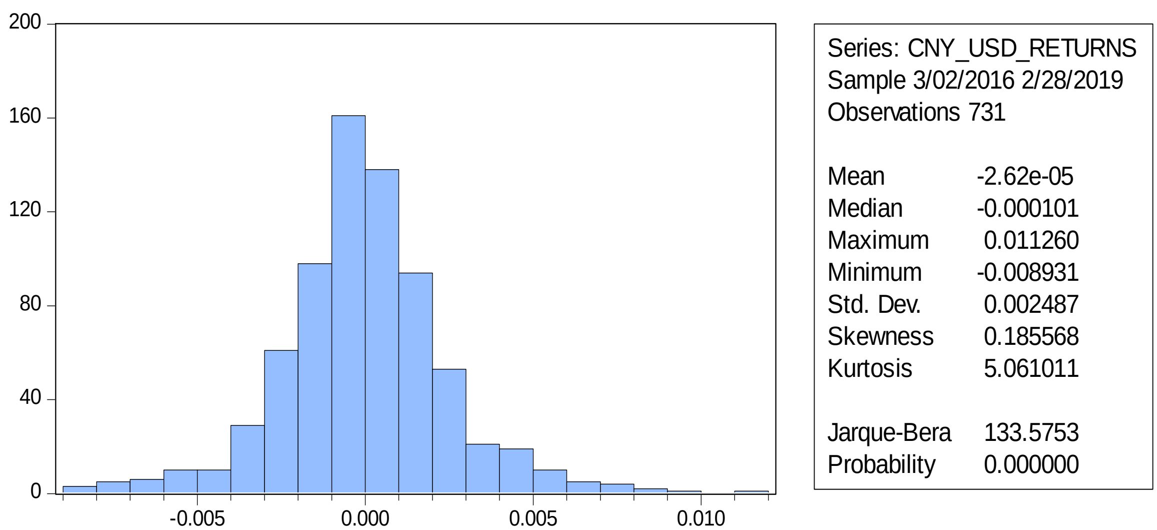
[1] The VLOOKUP function searches for a value (text or number) in a column and returns the value once a match is found. The formula is as follows:
VLOOKUP(lookup_value,table_array,col_index_num,[range_lookup])
The lookup value is the value to look for in the first column of the table. The table array is the table from which the value is retrieved. The col index is the table's column number from which to retrieve a value. The range lookup is set as FALSE to get an exact match.

Figure 1. Descriptive Statistic of "EURUSD" Daily Returns



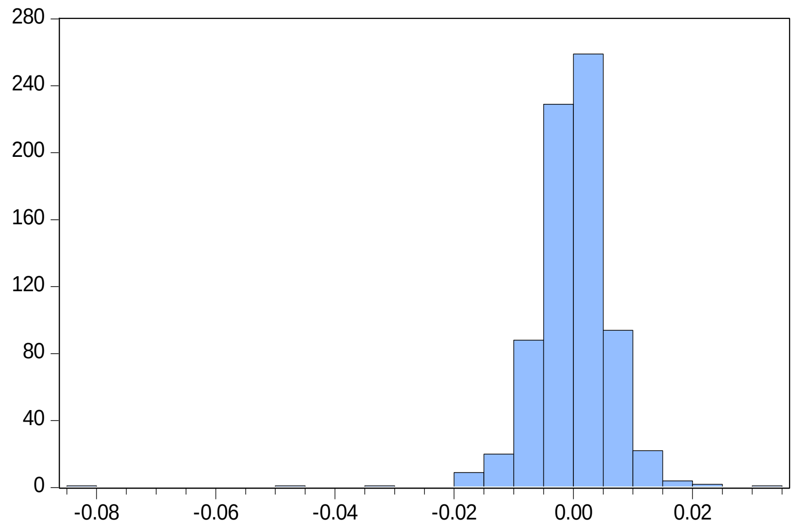
The mean of the daily returns for the "EURUSD" was 0.00735% with a standard deviation of 0.4811%. The minimum return was at -2.354% while the maximum return reached 1.937%. In addition, during the studied period, the minimum Euro rate was 1.0388 while the maximum rate reached 1.2510. The p-value of the Jarque-Bera test is 0, which means that the null hypothesis "the distribution is normal" is rejected at all significance levels. Furthermore, the kurtosis value is 4.28 which is greater than 3 revealing that the returns distribution is leptokurtic relative to the normal distribution.

Figure 2. Descriptive Statistic of "CNYUSD" Daily Returns



The mean of the daily returns for the "CNYUSD" was -0.00262% with a standard deviation of 0.2487%. The minimum return was at -0.8931% while the maximum return reached 1.126%. In addition, during the studied period, the minimum Chinese Yuan rate was 0.143355 while the maximum rate reached 0.159515. The p-value of the Jarque-Bera test is 0, which means that the null hypothesis "the distribution is normal" is rejected at all significance levels. Furthermore, the kurtosis value is 5.06 which is greater than 3 revealing that the returns distribution is leptokurtic relative to the normal distribution.

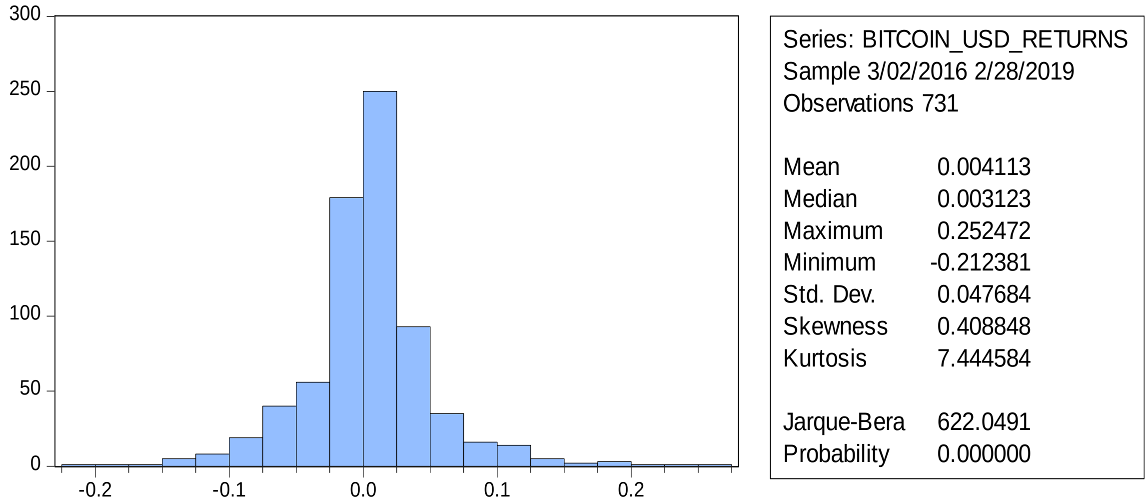
Figure 3. Descriptive Statistic of "GBPUSD" Daily Returns



Series: GBP_USD_RETURNS	
Sample 3/02/2016 2/28/2019	
Observations 731	
Mean	-4.51e-05
Median	0.000224
Maximum	0.030464
Minimum	-0.080527
Std. Dev.	0.006920
Skewness	-2.542613
Kurtosis	31.27080
Jarque-Bera	25131.09
Probability	0.000000

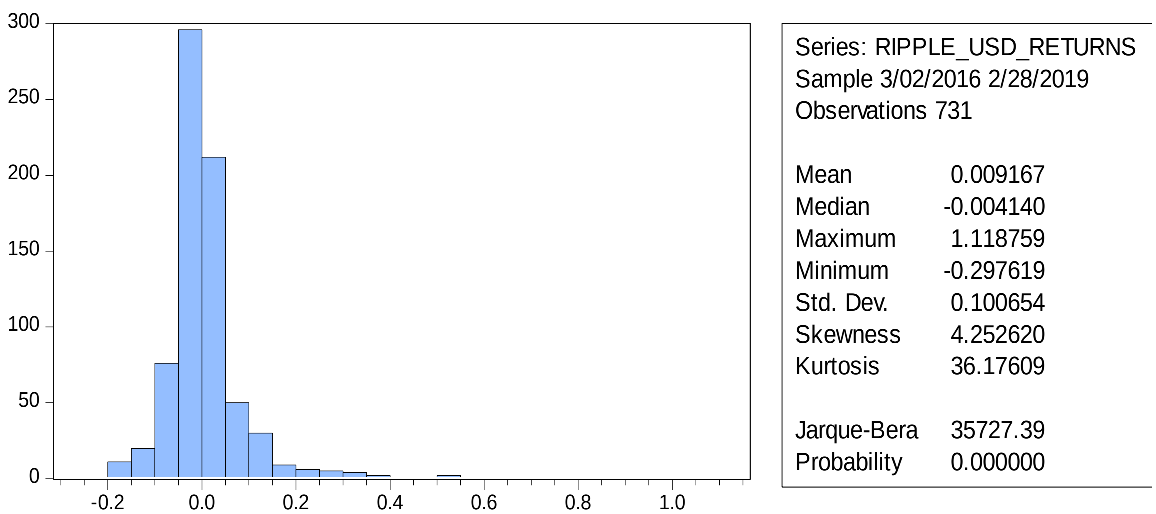
The mean of the daily returns for the "GBPUSD" was -0.00451% with a standard deviation of 0.692%. The minimum return was at -8.0527% while the maximum return reached 3.0464%. In addition, during the studied period, the minimum rate for the British Pound was 1.2047 while the maximum rate reached 1.4877. The p-value of the Jarque-Bera test is 0, which means that the null hypothesis "the distribution is normal" is rejected at all significance levels. Furthermore, the kurtosis value is 31.27 which is greater than 3 revealing that the returns distribution is leptokurtic relative to the normal distribution.

Figure 4. Descriptive Statistic of "BitcoinUSD" Daily Returns



The mean of the daily returns for the "BitcoinUSD" was 0.4113% with a standard deviation of 4.7684%. The minimum return was at -21.2381% while the maximum return reached 25.2472%. In addition, during the studied period, the minimum price for Bitcoin was $409.55 while the maximum price reached $19,497. The p-value of the Jarque-Bera test is 0, which means that the null hypothesis "the distribution is normal" is rejected at all significance levels. Furthermore, the kurtosis value is 7.44 which is greater than 3 revealing that the returns distribution is leptokurtic relative to the normal distribution.

Figure 5. Descriptive Statistic of "RippleUSD" Daily Returns



The mean of the daily returns for the "RippleUSD" was 0.9167% with a standard deviation of 10.0654%. The minimum return was at -29.7619% while the maximum return reached 111.8759%. In addition, during the studied period, the minimum price for Ripple was \$0.0054 while the maximum price reached \$3.38. The p-value of the Jarque-Bera test is 0, which means that the null hypothesis "the distribution is normal" is rejected at all significance levels. Furthermore, the kurtosis value is 36.17 which is greater than 3 revealing that the returns distribution is leptokurtic relative to the normal distribution.

Figures 6, 7, 8, 9 and 10 below show the plot of return series for the three currencies and two cryptocurrencies, whereby volatility clustering is noticeable, meaning that volatility can be forecasted.

Figure 6. Time Series of "EURUSD" Daily Returns

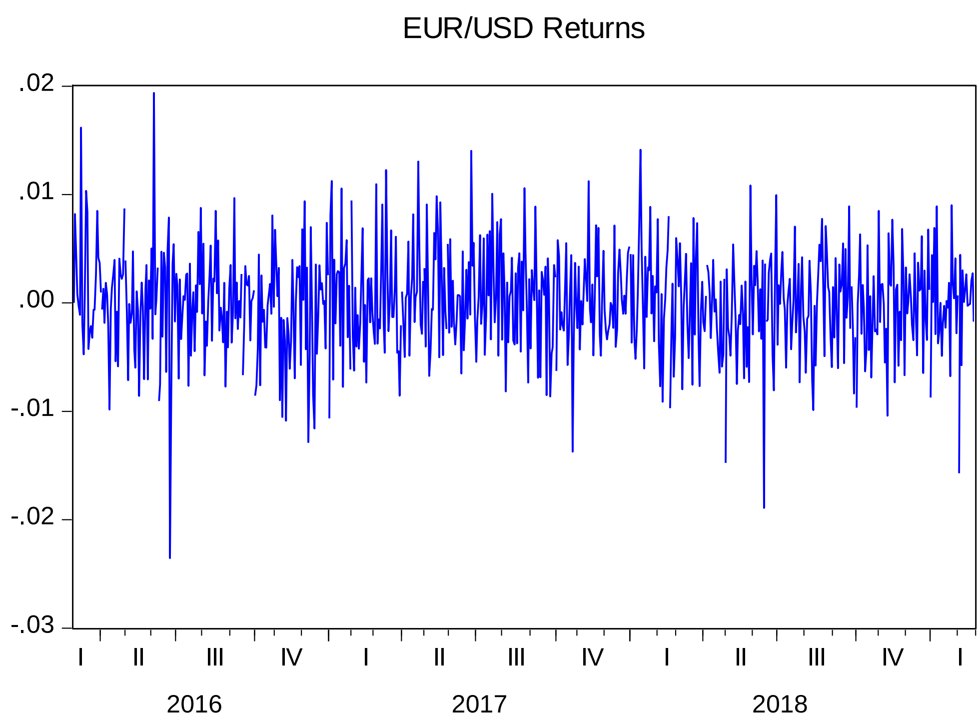


Figure 7. Time Series of "CNYUSD" Daily Returns

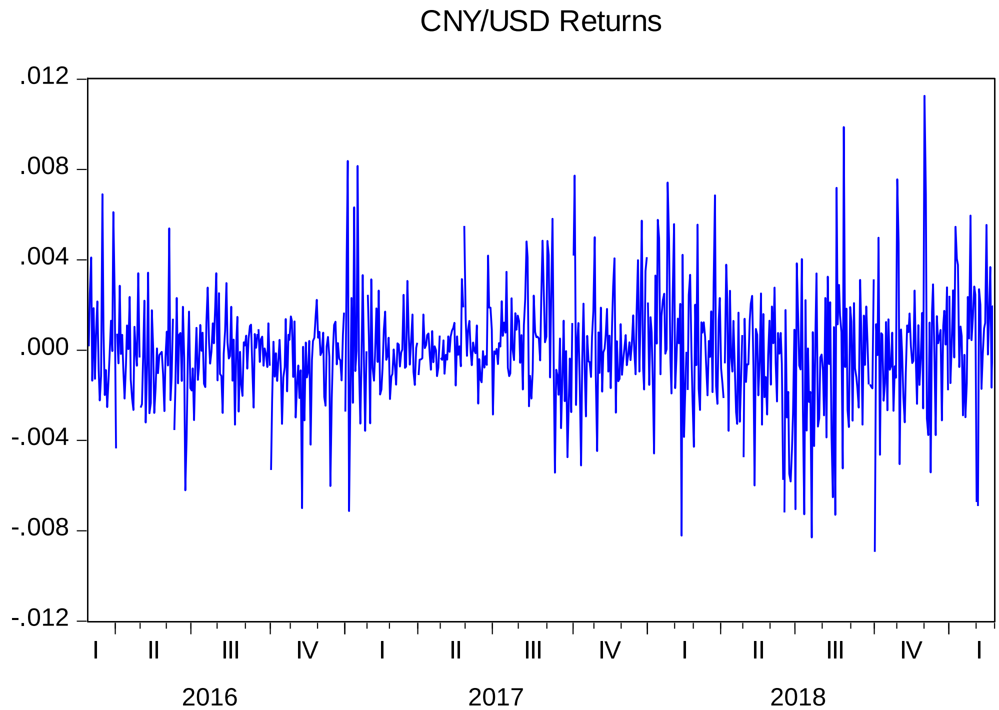


Figure 8. Time Series of "GBPUSD" Daily Returns

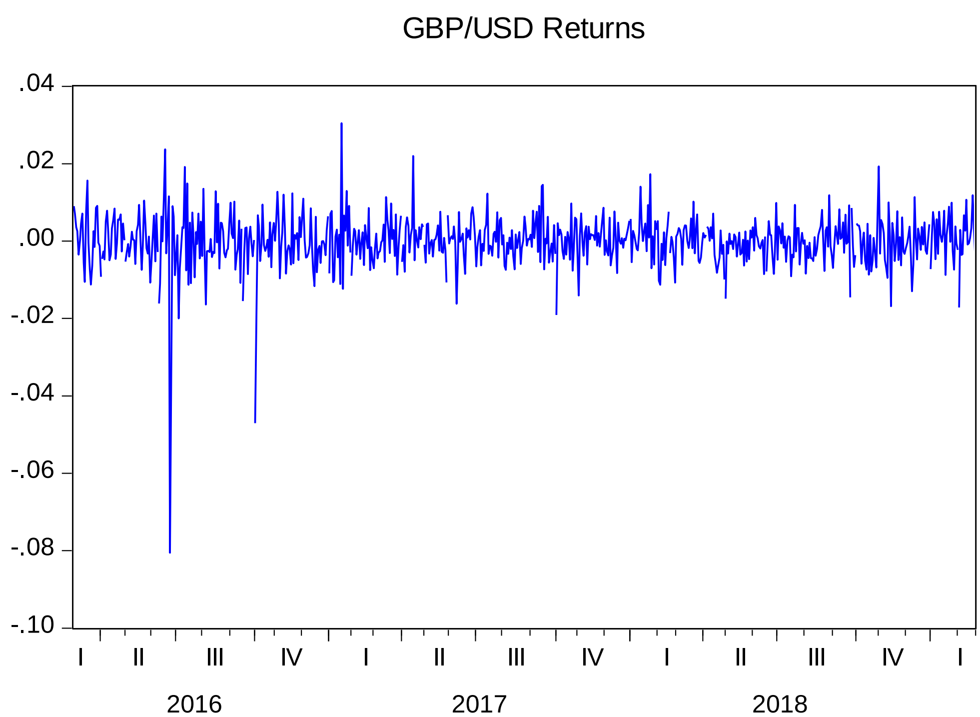


Figure 9. Time Series of "BitcoinUSD" Daily Returns

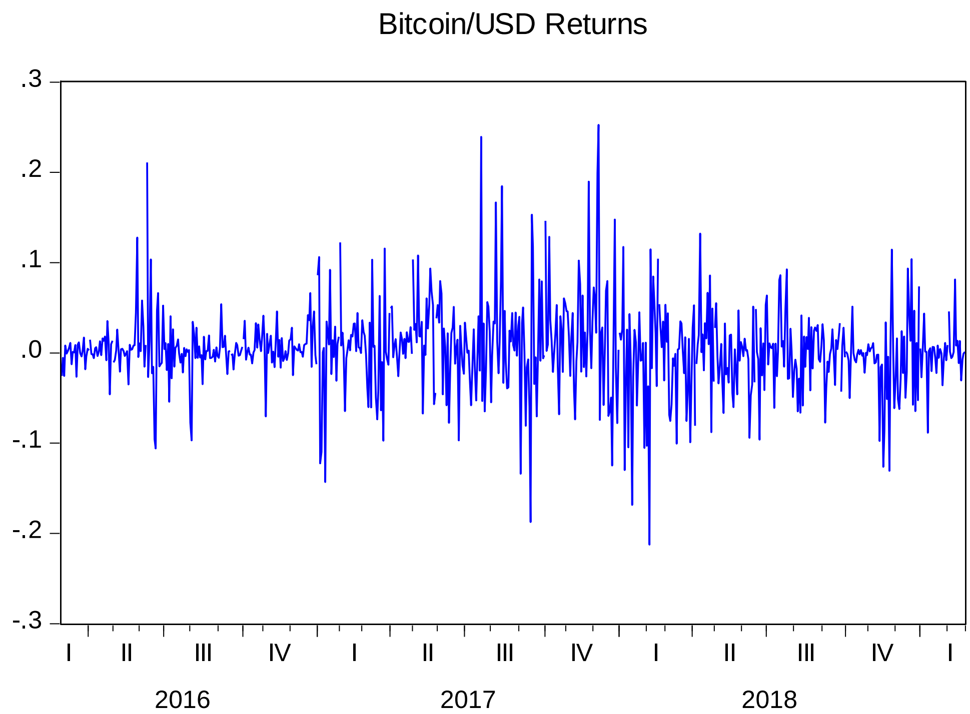
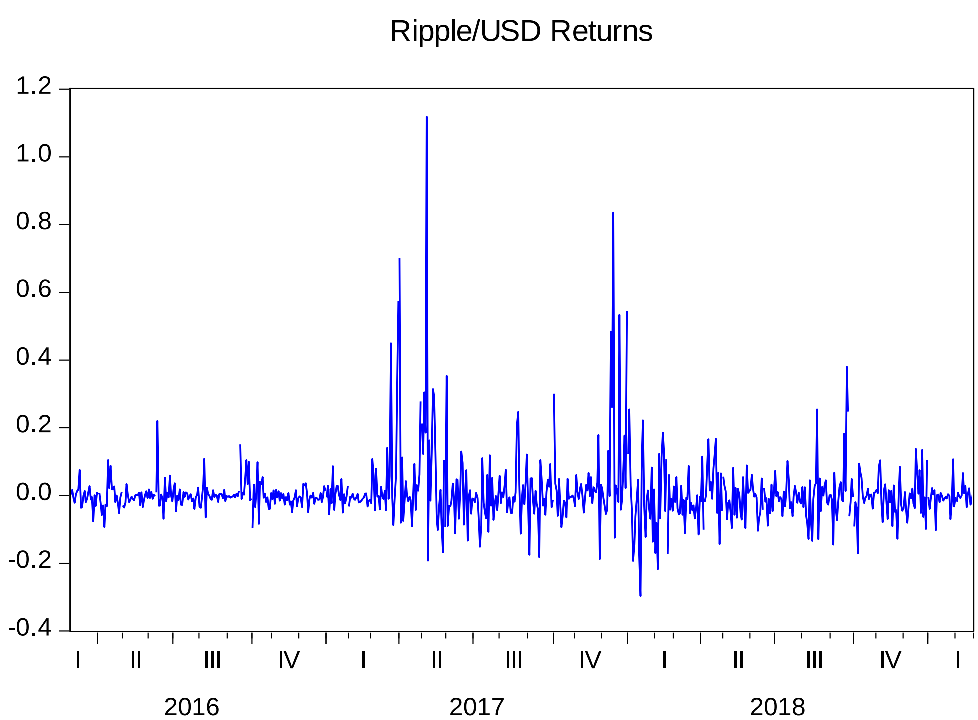


Figure 10. Time Series of "RippleUSD" Daily Returns



To assess the accuracy of the tested models, the realized and implied volatilities will be compared to the estimated volatility. The implied volatility will only be tested for the EURUSD, GBPUSD and CNYUSD since the options market for cryptocurrencies is still immature and the data is not available compared to other asset types. All the volatility data are available on Bloomberg except for the in-sample period for Ripple. Therefore, the in-sample realized volatility for Ripple will be manually calculated. We will use Merton's (1980) approach which was proposed in 1980 and is an estimation of the realized volatility based on the returns sum squared. The formula of the returns standard deviation or realized volatility σ_n is defined as follows:

$$\sigma_n = \sqrt{\sum_{i=1}^{n} u_i^2} \tag{2}$$

Where u_i is the return on day *i,* and *n* is the number of observations.[2]

In order to estimate the parameters of the EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1 ,1) models, the daily returns should be first tested for stationarity. Stationarity is an important aspect of time series analysis. When a process is stationary, the latter has the property of a sample mean and variance which doesn't change over time. If a "unit-root" exists among the observations, this means that the time series sample violates the stationarity assumption. Hyndman and Athanasopoulos (2018) explained that a non-stationary time series is a result of seasonality or existence of trend. Accordingly, the Augmented Dickey Fuller (ADF) test is implemented to check the presence of unit-root.

Table 1 below show the ADF test for the three currencies and two cryptocurrencies returns done using EViews 8.

[2] The realized volatility will be calculated daily in excel and converted into yearly volatility as follows: SQRT(u_i^2)*SQRT(250)

Table 1. Augmented Dickey-Fuller (ADF) test

		Augmented Dickey-Fuller test statistic	Test Critical Values		
		t-Statistic	1% Level	5% Level	10% Level
EURUSD	**t-Statistic**	-28.82819	2.56813	1.94126	-1.61641
	Prob.*	0.0000			
CNYUSD	**t-Statistic**	-26.70239	2.56813	1.94126	-1.61641
	Prob.*	0.0000			
GBPUSD	**t-Statistic**	-26.21038	2.56813	1.94126	-1.61641
	Prob.*	0.0000			
Bitcoin	**t-Statistic**	-25.58975	2.56813	1.94126	-1.61641
	Prob.*	0.0000			
Ripple	**t-Statistic**	-14.1864	2.56814	1.94126	-1.61641
	Prob.*	0.0000			

From the previous Table, the test statistics are less than the critical values at all the significance levels. Moreover, the p-values are 0, thus smaller than 1%, 5% and 10% significance levels. Consequently, the null hypothesis "unit-root existence" is rejected and the data samples are stationary. This means that a transformation of the return series is not needed.

3.3 Selected Models

This section illustrates and describes the methodology that will be used for the volatility assessment and forecast.

3.3.1 Exponentially Weighted Moving Average Model (EWMA)

Since we will be using the Exponentially Weighted Moving Average Model, the below will be an explanation of the model.
The symmetric EWMA model is defined as:

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2 \tag{3}$$

Where, σ_n^2 is the variance of today, σ_{n-1}^2 is the variance of the previous day, u_{n-1}^2 is the square of the previous day's return and λ is the decay factor and smoothing parameter, which ranges between 0 and 1. If $\lambda = 1$ then today's variance is entirely dependent on the most recent variance, whereas if $\lambda = 0$ then the model converges to the Random Walk model.

JP Morgan created the Risk Metrics database and assigned a value of 0.94 for lambda which gave the best variance forecasts when compared to the realized variance rates. When lambda is high, thus emphasizing on σ_{n-1}^2, the variance estimate reacts slowly to new market information. Whereas a low value of lambda giving more weight to u_{n-1}^2, denotes a faster reaction to new market changes. Distinctively, the EWMA model stresses on the most recent observations by assigning a higher weight for them. This weight decreases exponentially when going back to older observations. The main difference between EWMA and GARCH models is the absence of the long-run average variance (v_l) in the EWMA model. The non-existing nature of the long-run average variance means that new market disturbances lead to a perpetual change in volatility (Hull, 2012).

3.3.2 Generalized Autoregressive Conditional Heteroskedastic Models (GARCH)

Engle (1982) introduced the ARCH model, which allowed the conditional variance to change over time as a function of previous errors. The model was generalized by Bollerslev (1986) with an addition of the lagged conditional variance, thus developing the GARCH model.

The GARCH (1,1) model is a narrow form of the more general GARCH (p, q) model. GARCH (1, 1) and GARCH (p, q) are defined as follows, respectively:

$$\sigma_n^2 = \gamma v_l + \beta\sigma_{n-1}^2 + \alpha u_{n-1}^2 \tag{4}$$

$$\sigma_n^2 = \gamma v_l + \sum_{i=1}^{p} \beta\sigma_{n-i}^2 + \sum_{i=1}^{q} \alpha u_{n-i}^2 \tag{5}$$

Where, σ_n^2 is today's variance, v_l is the long run variance rate, σ_{n-1}^2 is the variance of the previous day's return and u_{n-1}^2 is the square of the previous day's return.

The weights assigned to v_l, σ_{n-1}^2, and u_{n-1}^2 are γ, β, and α, respectively. The model is considered stable when the weights sum-up to one. The constraint of $\alpha + \beta < 1$ guarantees the covariance stationarity. A unit root in variance happens when $\alpha + \beta = 1$, then the model would not be considered stable and converges to the Integrated GARCH model. A non-stationarity in variance occurs when $\alpha + \beta > 1$, then the model would have undesirable properties, and instead of “mean reverting”, the process becomes “mean fleeing”. $\alpha + \beta$ represents the persistence of the conditional volatility as their sum determines the pace of the conditional variance slowly moving back to the long-run variance after it deviates from its original value. This can sometimes happen based on the market conditions. The persistence of the shocks is determined by β. The GARCH (1,1) and GARCH (p, q) models can also be respectively defined as:

$$\sigma_n^2 = \omega + \beta\sigma_{n-1}^2 + \alpha u_{n-1}^2 \tag{6}$$

$$\sigma_n^2 = \omega + \sum_{i=1}^{p} \beta\sigma_{n-i}^2 + \sum_{i=1}^{q} \alpha u_{n-i}^2 \tag{7}$$

Where, $\omega = \gamma v_l$. This is done to estimate ω, β, and α, afterwards γ is calculated as 1- β-α, then v_l is calculated as ω/γ. When v_l is computed rather than being estimated, the process is called variance targeting and helps ensuring a stable and robust model. The GARCH (1, 1), which is the simplest, yet the most popular among GARCH models calculates today’s variance depending on the most recent variance and return squared. Whereas the GARCH (p, q) model estimates the variance based on the p^{th} variance and q^{th} return squared (Engle, 2001). Both models will be implemented and are symmetric and leptokurtic, incorporates volatility clustering, skewedness and the presence of extreme values (Hull, 2012). Yet, both models fail to capture the leverage effects; this is tackled by the second generation GARCH models such as the EGARCH (1, 1) which will be detailed in the following section.

3.3.3 Exponentially Generalized Autoregressive Conditional Heteroskedastic (1,1) Model (EGARCH)

This thesis will also make use of the Exponentially Generalized Autoregressive Conditional Heteroskedastic model. Nelson (1991) suggested this model to distinguish between the volatility’s positive and negative shocks.

The EGARCH (1,1) is defined as:

$$log\ \sigma_n^2 = \gamma v_l + \beta g(z_{n-1}) + \alpha log\sigma_{n-1}^2 \tag{8}$$

Where, σ_n^2 is today's variance, v_l is the long run variance rate, σ_{n-1}^2 is the variance of the previous day's return, $g(z_{n-1})$ is an explanatory variable which accounts for the leverage effect.

The weights assigned to v_l, σ_{n-1}^2, and $g(z_{n-1})$ are γ, α, and β respectively. The second generation GARCH models are asymmetric by nature and account for the leverage effect (Hull, 2012). An asymmetric model means that losses distribution have a heavier tail than that of the profit's distribution. This highlights the leverage effect where losses have a larger impact on the future's volatility. Among the most popular and important models is the EGARCH, which automatically incorporates the Maximum Likelihood function. It is similar to the basic GARCH models in terms of mean reversion, skewness, and leptokurtosis. $\beta g(z_{n-1})$, part of the model, aims at capturing the leverage effect. The logarithmic representation of the EGARCH model preserves a positive process, meaning that the model respects the non-negativity constraint. The model maintains its stability if the parameters are positive and less than one. When $\alpha < 1$, the model is considered stable. However, if $\alpha = 1$ (unit root presence), then the model converges to the Integrated EGARCH. If $\alpha > 1$, this indicates the presence of an unstable process characterized by a non-stationary variance with undesirable properties (Radovanov *et al.* 2018).

3.3.4 Maximum Likelihood Methodology

The maximum likelihood method is an approach that estimates the parameters values of econometric models such as GARCH and EWMA by maximizing the likelihood of occurrence of historical data. This method will be adopted in this thesis to estimate the parameters of EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1).

The probability density function is represented by $f(y|\theta)$, where y represents a random variable constrained on a parameters set θ. Furthermore, the likelihood function is known as the joint density function and is mathematically expressed as follows:

$$f(y_{1 \ldots\ldots,} y_n|\theta) = \prod_{i=1}^{n} f(y_i|\theta) = L\,(\theta|y) \tag{9}$$

The vector model parameter is denoted by θ and y is used to indicate the time series at time *i*. In addition, the parameters are constant, and their estimation is based on the data selection.

In finance, the log likelihood[3] function is used to estimate the parameters and is defined as follows:

$$\ln L\,(\theta|y) = \sum_{i=1}^{n} ln\ f(y_i|\theta) \tag{10}$$

3.3.5 Incorporating Volatility Updating into Historical Simulation

Hull and White (1998) suggested an extension to the basic historical simulation approach, which was elaborated by Hull (2012). The model requires incorporating the volatility updating procedure to the returns of historical data. Since the volatility of market variables may vary over time, Hull and White recommend altering past data in order to reflect the variation in volatility at the current market state. This methodology employs the optimal selected model for each currency and cryptocurrency based on the comparison of the calculated volatility results to the realized volatility in order to estimate VaR. Although the parameters could be estimated every 50 days, in this research, the optimal model's parameters are estimated twice using the maximum likelihood method, while the previous estimations are no longer used. The sample period should be 250 days, the closest to the out of sample period. Thus, VaR studied period slightly differs from the out of sample period. More specifically, the parameters are first estimated for the 100-day period, extending from February 13, 2018 till July 16, 2018. Then, the parameters are estimated for the 150-day period, extending from July 17, 2018 till February 28, 2019.

To create 399 scenarios and repeat the procedure 250 times, the studied sample period extends from June 30, 2016 till February 28, 2019 with 649 days of observation. The first sub-sample period, consisting of 400 days of observation from June 30, 2016 till February 13, 2018, is used to create 399 scenarios for the upcoming result on day 400 (February 14, 2018). This procedure is repeated 250 times, reaching the last sub-sample period of 400 days of observation extending from July 10, 2017 through February 28, 2019. By

[3] In excel the log likelihood function is applied as follows: - ln(σ_n^2)-(u_i^2)/(σ_n)

implementing this approach, the value of each currency or cryptocurrency under the *ith* scenario becomes as follows:

$$V_{ith\,scenario} = v_n \frac{v_{i-1}+(v_i-v_{i-1})\sigma_{n+1}/\sigma_i}{v_{i-1}} \quad (11)$$

Where:

v_i is the value of the currency or cryptocurrency on day *i*;

v_n is the value of the currency or cryptocurrency on the last day of the chosen time period;

σ_i is the estimate of the daily volatility on day *i*;

σ_{n+1} is the most recent estimate of the daily volatility.

Using the following equation, the return scenarios will be calculated under each simulation trial leading to 399 return scenarios, to subtract the expected gains and losses on the first day of the out-of-sample period:

$$u_{ith\,scenario} = \frac{(V_{ith\,scenario}-v_n)}{v_n} \quad (12)$$

Where:

$V_{ith\,scenario}$ is the value of the currency or cryptocurrency under the *ith* scenario;

v_n is the value of the currency or cryptocurrency on the last day of the chosen time period.

This procedure is repeated for each sub-sample period in order to estimate VaR for 250 days within a period that extends from February 13, 2018 till February 28, 2019.

3.3.6 Back-Testing Methodology: Kupiec Test

To evaluate the accuracy of VaR a back-test should be performed. The test shows how the selected model used to estimate VaR would perform if it was used in the past. Typically, the number of times where the actual loss is greater than VaR is considered an exception. For example, when testing the model's accuracy for the estimation of a one-day 95% VaR, the maximum number of exceptions would occur on 5% of the days. The Kupiec (1995) test is the most commonly used test for back-testing VaR, and it will be used in this thesis. The Log-likelihood Ratio (LR) suggested by Kupiec (1995) is used to test the accuracy of VaR and is denoted as follows:

$$LR = -2\ln[(1-p)^{T-N}p^N] + 2\ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^N\right] \quad (13)$$

Where:

N is the number of exceptions;

T is the number of trials;

p is the probability of failure.

The above-mentioned equation follows a chi-square distribution with one degree of freedom where there is a 5% probability that the chi-square variable will be more than 3.84. Therefore, the VaR model will be rejected if LR is greater than 3.84. The LR might be greater than 3.84 for either a high or a low number of exceptions, meaning that the model would be rejected in both cases. Furthermore, the probability of failure values *(p)* are 0.01, 0.025, 0.05 and 0.1 which parallel VaR confidence levels of 99%, 97.5%, 95% and 90% respectively. The number of trial value *(T)* is 250 and is constant at all confidence levels. The number of exceptions *(N)* will be obtained by counting the recorded exceptions where the actual loss exceeded VaR on a given day.

3.4 Conclusion

The previously discussed models will be applied on the three currencies and two cryptocurrencies. The studied period will be divided into two sample periods: March 01, 2016 to February 28, 2018 and March 01, 2018 to February 28, 2019. The numerous characteristics and differences of the models were detailed. These models are subject to some limitations. EWMA does not incorporate mean reversion and is likely to overestimate the volatility after sudden large market changes. The GARCH (1, 1) model does not account for the leverage effect and is symmetric by nature. As for GARCH (p, q), the model is subjected to the non-negativity constraint. Accordingly, a new generation of GARCH models originated to counter those limitations such as the Exponential GARCH model. The EGARCH (1, 1), which accounts for the leverage effect, is asymmetric by nature and is not subjected to the non-negativity constraint. Nevertheless, the model’s parameters must be positive with a value less than one in order to be considered stable. By using these approaches, the best model for each currency and cryptocurrency will be applied to calculate VaR. The precision of the models will be distinguished by back-testing VaR results using the Kupiec test.

In the following chapter, the parameters of the volatility models will be estimated by maximizing the log-likelihood function. Then, the volatility will be estimated for each currency and cryptocurrency, and finally the forecasted volatilities will be compared to the realized and implied volatilities to determine the best model for each currency and cryptocurrency in order to measure VaR.

Chapter 4

Findings

4.1 Introduction

In the previous chapter, the data selection procedure was represented along with a general descriptive statistic for each of the selected currencies and cryptocurrencies during the period extending from March 01, 2016 through February 28, 2019. Afterwards, the EWMA, GARCH (1, 1), GARCH (p, q), and EGARCH (1, 1) models were theoretically detailed.

This chapter illustrates the findings under each of the selected models. Then, parameters estimation are performed for the in-sample period (March 01, 2016 till February 28, 2018), under each of the selected volatility models. The in-sample parameters are used to forecast the volatility in the out-of-sample period (March 01, 2018 till February 28, 2019). The realized volatilities related to the three currencies and the two cryptocurrencies are compared to the calculated in-sample volatilities to determine the most accurate model for measuring and predicting volatility. The comparison is addressed using three error metrics, namely: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). The model with the least error difference will be considered the most accurate. Subsequently, to determine the predictive capabilities of the models, the estimated in-sample parameters are applied to the out-of-sample returns. The same error statistics process is performed to check whether the same optimal model for the in-sample returns applies to the out-of-sample returns. The optimal selected models in the out-of-sample period will be used to calculate VaR for each currency and cryptocurrency. Finally, the Kupiec test is applied to measure the VaR accuracy.

4.2 Parameters' Estimation

The Lambda, which is known as the decay factor and smoothing parameter of the EWMA model, is estimated using Excel for the three currencies and the two cryptocurrencies. After computing the daily returns, the variance is calculated using equation (3). The maximum likelihood function described in equation (10) is maximized using the Solver.

The GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1) parameters are also estimated using Excel. First, the daily returns are calculated. Then, the variance is calculated using equations (6), (7) and (8), which represent the GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1) respectively. Similar to the EWMA model, equation (10) is maximized.

4.2.1 EWMA Parameters

The daily in-sample returns of the closing prices for the EURUSD, CNYUSD, GBPUSD, Bitcoin and Ripple are used to obtain the lambda parameters using equation (3). The lambda values and the log likelihood results are detailed in Table 2:

Table 2. EWMA ESTIMATED PARAMETERS

EWMA	Lambda	Likelihood Function
EURUSD	0.938	4287.46
CNYUSD	0.936	5514.94
GBPUSD	0.924	4287.55
Bitcoin	0.911	2508.52
Ripple	0.946	1857.91

The decay factor (λ) ranges between 91% and 95% for the selected currencies and cryptocurrencies. This parameter explains the relative importance of the observations with respect to the returns when determining the current variance rate. Interestingly, Bitcoin's Lambda parameter is smaller than fiat currencies' Lambda parameters whereas Ripple's decay factor is the greatest among the studied market variables.

4.2.2 GARCH (1, 1) Parameters

The daily in-sample returns of the closing prices for the EURUSD, CNYUSD, GBPUSD, Bitcoin and Ripple are used to obtain the GARCH (1, 1) parameters using equation (6). Accordingly, GARCH (1, 1) parameters values and the log likelihood results are presented in Table 3:

Table 3. GARCH (1, 1) Estimated Parameters

GARCH (1, 1)	Omega (ω)	ARCH component (α)	GARCH component (β)	LT Volatility	Likelihood Function
EURUSD	0.00000088	0	0.963	7.76%	4697.09
CNYUSD	0.00000076	0.134	0.715	3.53%	5529.87
GBPUSD	0.00000766	0.187	0.715	13.92%	4327.97
Bitcoin	0.00005507	0.159	0.841	-	2554.31
Ripple	0.00022216	0.235	0.765	-	2139.66

The ARCH component (α) ranges between 0% and 24% for the five selected market variables. This parameter determines the influence of market shocks on the volatility. Curiously, the ARCH component for the EURUSD is 0. This means that market shocks have no effect on the volatility of the EURUSD, unlike the case of the remaining fiat and cryptocurrencies. The GARCH component (β) ranges between 71% and 97%. β, known as the decay factor, explains the relative importance of the observations on the returns, when determining the current variance rate. Interestingly, the GARCH component for the CNYUSD and GBPUSD is found to be the same, while the GARCH component of Bitcoin exceeded that of the Ripple by approximately 10%.

It is important to note that the sum of the ARCH and GARCH components for the Bitcoin and Ripple is equal to 1. This, in turn, points at the presence of a unit root in variance by which the GARCH model converges to the Integrated GARCH model where v_l , known as the long run variance is unachievable. This is mainly due to the same fluctuations in the returns of Bitcoin and Ripple as depicted in figures 9 and 10.

4.2.3 EGARCH (1, 1) Parameters

The daily in-sample returns of the closing prices for the EURUSD, CNYUSD, GBPUSD, Bitcoin and Ripple are used to obtain the EGARCH (1, 1) parameters using equation (8). Accordingly, EGARCH (1, 1) parameters values and the log likelihood results are presented in Table 4:

Table 4. EGARCH (1, 1) Estimated Parameters

EGARCH (1, 1)	Omega (ω)	Leverage coefficient (γ)	ARCH component (α)	GARCH component (β)	LT Volatility	Likelihood Function
EURUSD	-3.190063	-0.01	0.138	0.7	7.71%	2352.44
CNYUSD	-0.992132	0.206	0.09	0.919	3.52%	2773.34
GBPUSD	-0.460894	0.226	0.07	0.951	14.39%	2170.81
Bitcoin	-0.143102	0.243	0.053	0.971	135.50%	1288.9
Ripple	-0.39817	0.313	0.163	0.914	155.76%	1074.3

The leverage coefficient (γ) ranges between -1% and 31%. However, the leverage coefficient exhibits a negative value only for the EURUSD indicating the presence of a leverage effect. This means that the negative shocks for the EURUSD surpass the positive shocks. Nevertheless, its value is not significant since it is relatively small and close to 0. Whereas, for the remaining currencies and cryptocurrencies, the leverage effect is positive and greater than 20% meaning that positive shocks have a higher influence than negative shocks. On the other hand, The GARCH component (β) ranges between 70% and 97%. β, known as the decay factor, explains the relative importance of the observations on the returns, when determining the current variance rate. The EURUSD exhibits a GARCH component of 70%, whereas for the remaining fiat and cryptocurrencies the GARCH coefficient is above 90%.

The long term volatility of the currencies ranges between 3% and 14%. The results are approximately similar to the GARCH (1, 1) long term volatility. However, cryptocurrencies long term volatilities are 135% and 156% for the Bitcoin and Ripple, respectively. This justifies the unachievable calculation of the long term volatility in the GARCH (1, 1) model. There is a huge difference in the long term volatility results emphasizing the increased volatility of cryptocurrencies with respect to fiat currencies. For instance, among the selected currencies, the GBPUSD showed the highest long term volatility of 14.39%. Whereas, for the cryptocurrencies, Bitcoin’s long term volatility was at 135.5% which is 15% lower than that of the Ripple. This implies that Bitcoin’s long term volatility is eight times greater than the Sterling Pound long term volatility.

4.2.4 GARCH (p, q) Parameters

The daily in-sample returns of the closing prices for the EURUSD, CNYUSD, GBPUSD, Bitcoin and Ripple are used to obtain the GARCH (p, q) parameters using equation (7). Several combinations of the ARCH components (p) and the GARCH components (q) are considered. The ARCH components (p) are taken up to 6 and the GARCH components (q) are taken up to 9. Surprisingly, when estimating the GARCH (p, q) parameters for the EURUSD, the log likelihood function couldn't attain any higher value while manipulating the parameters. This means that the GARCH (1, 1) parameters, which were stated earlier, maximized the log likelihood function and no better parameters were found from the models ranging from GARCH (1, 2) through GARCH (6, 9), which led to a total of 53 models. This issue might be attributed to the data frequency which is daily. Whereas an intraday frequency such as hourly might have produced more significant results. The GARCH (p, q) parameters values and the log likelihood results are presented.

Since we are studying three currencies and two cryptocurrencies, we will only present the results related to Bitcoin. The remaining results are illustrated in Appendix A.

Tables 5, 6, 7, 8, 9 and 10 show the GARCH (p, q) parameters for the Bitcoin.

Table 5. GARCH (1, 2) through GARCH (1, 9) Estimated Parameters (Bitcoin)

GARCH	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(1, 5)**	**(1, 6)**	**(1, 7)**	**(1, 8)**	**(1, 9)**
ω	5.00E-05	6.63E-05	7.61E-05	7.61E-05	7.61E-05	7.23E-05	7.23E-05	7.23E-05
α1	0.166	0.235	0.270	0.270	0.270	0.294	0.294	0.294
β1	0.348	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2	0.486	0.260	0.048	0.048	0.048	0.054	0.054	0.054
β3		0.505	0.454	0.454	0.454	0.343	0.343	0.343
β4			0.227	0.227	0.227	0.025	0.025	0.025
β5				0.000	0.000	0.000	0.000	0.000
β6					0.000	0.000	0.000	0.000
β7						0.283	0.283	0.283
β8							0.000	0.000
β9								0.000
LLF	2563.51	2582.41	2589.44	2589.44	2589.44	2598.90	2598.90	2598.90

Table 6. GARCH (2, 1) through GARCH (2, 9) Estimated Parameters (Bitcoin)

GARCH	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	(2, 7)	(2, 8)	(2, 9)
ω	5.53E-05	5.96E-05	6.87E-05	7.61E-05	7.61E-05	7.61E-05	7.22E-05	7.22E-05	7.22E-05
α1	0.159	0.068	0.242	0.270	0.270	0.270	0.294	0.294	0.294
α2	0.000	0.146	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.841	0.000	0.223	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.787	0.000	0.048	0.048	0.048	0.054	0.054	0.054
β3			0.534	0.454	0.454	0.454	0.344	0.344	0.344
β4				0.227	0.227	0.227	0.025	0.025	0.025
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.283	0.283	0.283
β8								0.000	0.000
β9									0.000
LLF	2554.32	2578.34	2580.52	2589.44	2589.44	2589.44	2598.90	2598.90	2598.90

Table 7. GARCH (3, 1) through GARCH (3, 9) Estimated Parameters (Bitcoin)

GARCH	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(3, 7)	(3, 8)	(3, 9)
ω	6.32E-05	6.31E-05	9.01E-05	8.02E-05	8.28E-05	8.28E-05	8.15E-05	8.15E-05	8.15E-05
α1	0.136	0.136	0.158	0.268	0.266	0.266	0.292	0.292	0.292
α2	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.039	0.039	0.044	0.000	0.023	0.023	0.027	0.027	0.027
β1	0.825	0.825	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β3			0.688	0.483	0.464	0.464	0.352	0.352	0.352
β4				0.249	0.247	0.247	0.040	0.040	0.040
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.289	0.289	0.289
β8								0.000	0.000
β9									0.000
LLF	2555.01	2555.01	2574.23	2587.85	2590.55	2590.55	2599.69	2599.69	2599.69

Table 8. GARCH (4, 1) through GARCH (4, 9) Estimated Parameters (Bitcoin)

GARCH	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	(4, 7)	(4, 8)	(4, 9)
ω	1.14E-04	1.03E-04	1.03E-04	1.54E-04	1.54E-04	1.54E-04	9.91E-05	1.58E-04	1.58E-04
α1	0.110	0.117	0.117	0.152	0.152	0.152	0.230	0.165	0.165
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.000	0.000	0.000	0.042	0.042	0.042	0.000	0.035	0.035
α4	0.185	0.199	0.199	0.280	0.280	0.280	0.156	0.305	0.305
β1	0.705	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.684	0.684	0.024	0.024	0.024	0.067	0.030	0.030
β3			0.000	0.154	0.154	0.154	0.205	0.148	0.148
β4				0.347	0.347	0.347	0.086	0.213	0.213
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.256	0.000	0.000
β8								0.103	0.103
β9									0.000
LLF	2565.30	2587.11	2587.11	2603.93	2603.93	2603.93	2603.98	2605.03	2605.03

Table 9. GARCH (5, 1) through GARCH (5, 9) Estimated Parameters (Bitcoin)

GARCH	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(5, 7)	(5, 8)	(5, 9)
ω	1.14E-04	1.03E-04	1.34E-04	1.54E-04	1.54E-04	1.54E-04	1.05E-04	1.05E-04	1.05E-04
α1	0.111	0.117	0.175	0.152	0.152	0.152	0.225	0.225	0.225
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.000	0.000	0.000	0.042	0.042	0.042	0.019	0.019	0.019
α4	0.185	0.199	0.228	0.280	0.280	0.280	0.156	0.156	0.156
α5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.704	0.000	0.030	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.684	0.235	0.024	0.024	0.024	0.031	0.031	0.031
β3			0.332	0.154	0.154	0.155	0.217	0.217	0.217
β4				0.348	0.348	0.347	0.102	0.102	0.102
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.251	0.251	0.251
β8								0.000	0.000
β9									0.000
LLF	2565.30	2587.11	2589.74	2603.93	2603.93	2603.93	2604.20	2604.20	2604.20

Table 10. GARCH (6, 1) through GARCH (6, 9) Estimated Parameters (Bitcoin)

GARCH	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	(6, 7)	(6, 8)	(6, 9)
ω	1.35E-04	1.03E-04	1.33E-04	1.54E-04	1.54E-04	1.54E-04	9.91E-05	1.65E-04	1.65E-04
α1	0.115	0.117	0.175	0.152	0.152	0.152	0.230	0.166	0.166
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.000	0.000	0.000	0.042	0.042	0.042	0.000	0.038	0.038
α4	0.161	0.199	0.228	0.280	0.280	0.280	0.156	0.308	0.308
α5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α6	0.082	0.000	0.000	0.000	0.000	0.000	0.000	0.017	0.017
β1	0.642	0.000	0.030	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.684	0.235	0.024	0.024	0.024	0.067	0.000	0.000
β3			0.332	0.154	0.154	0.154	0.205	0.146	0.146
β4				0.348	0.348	0.348	0.086	0.215	0.215
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.256	0.000	0.000
β8								0.111	0.111
β9									0.000
LLF	2567.10	2587.11	2589.74	2603.93	2603.93	2603.93	2603.98	2605.38	2605.38

From the GARCH (p, q) parameters tables, many ARCH (α) and GARCH (β) components have a value of zero. This means that any assigned trial value for those components was unable to maximize the loglikelihood function. Surprisingly, when estimating the GARCH (p, q) parameters for the Bitcoin and Ripple, the sum of the ARCH and GARCH components was 1. This means that in the 53 combinations of the ARCH (p) and GARCH (q), the models converged the IGARCH (p, q) model where the long run variance is unattainable.

The LT volatility of the GARCH (p, q) models for the GBPUSD ranges between 12.57% and 17.34% compared to 13.92% and 14.39% for the GARCH (1, 1) and EGARCH (1, 1) models, respectively. As for the CNYUSD, the LT volatility of the GARCH (p, q) models ranges between 3.51% and 3.88% compared to 3.53% and 3.52% for the GARCH (1, 1) and EGARCH (1, 1), respectively.

4.3 In-Sample Results

The optimal in-sample model will be selected based on the three error metrics as previously indicated. The Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) are performed using NumXL[4]. These error metrics determine the best model by subtracting the calculated volatility for each model from the realized volatility for each currency and cryptocurrency. The chosen model will have the least errors, therefore ranking first.

The theoretical equation for each of the previously mentioned error metrics is as follows:

i. $RMSE = \sqrt{(\sum_{t=1}^{n}(f-Y)\hat{}2)/n}$

ii. $MAE = \frac{1}{n}\sum_{t=1}^{n}/(f-Y)/$

iii. $MAPE = \frac{100}{n}\sum_{t=1}^{n}/(\frac{Y-f}{Y})/$

Where n is the number of periods, Y is the true value and f is the prediction value.

Before selecting the optimal model for each currency and cryptocurrency, the same comparison using the error metrics is applied on the GARCH (p, q) model to select the best ARCH (p) and GARCH (q) components.

However, since the GARCH (1, 1) parameters maximized the log likelihood function for the EURUSD and no better combination of ARCH (p) and GARCH (q) components were found, only the GARCH (1, 1) model is considered.

The implied volatility was used in the comparison of the currencies' volatility only, since cryptocurrencies' implied volatility data is not available. However, the realized volatility was compared with the calculated volatility for the currencies and cryptocurrencies.

Table 11 shows the ranking between the GARCH (p, q) models compared to the realized volatility for the Bitcoin. The detailed results of the CNYUSD, GBPUSD and Ripple will appear in Appendix B.

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[4] Numerical Analysis for Excel (NumXL) is an econometrics and time series analysis add-in for Microsoft Excel which provides a wide variety of statistical and time series analysis techniques, including linear and nonlinear time series modeling, statistical tests and others.

Table 11. Error Statistics of GARCH (p, q) vs. the Realized Volatility In-Sample (Bitcoin)

Bitcoin	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.41363	11	0.33579	22	0.01722	50
GARCH (1, 3)	0.42003	25	0.33661	26	0.01670	32
GARCH (1, 4)	0.41758	18	0.33242	15	0.01646	19
GARCH (1, 5)	0.41758	17	0.33242	14	0.01646	18
GARCH (1, 6)	0.41758	19	0.33242	16	0.01646	20
GARCH (1, 7)	0.41340	6	0.32854	4	0.01618	3
GARCH (1, 8)	0.41340	7	0.32854	5	0.01618	4
GARCH (1, 9)	0.41340	5	0.32854	3	0.01618	2
GARCH (2, 1)	0.40633	3	0.33122	13	0.01737	51
GARCH (2, 2)	0.41053	4	0.33022	12	0.01625	11
GARCH (2, 3)	0.40335	2	0.32431	2	0.01628	12
GARCH (2, 4)	0.41759	20	0.33242	17	0.01646	21
GARCH (2, 5)	0.41759	21	0.33242	18	0.01646	22
GARCH (2, 6)	0.41759	22	0.33242	19	0.01646	23
GARCH (2, 7)	0.41342	8	0.32856	6	0.01618	5
GARCH (2, 8)	0.41342	9	0.32856	7	0.01618	6
GARCH (2, 9)	0.41342	10	0.32856	8	0.01618	7
GARCH (3, 1)	0.41705	16	0.33860	31	0.01752	53
GARCH (3, 2)	0.41698	15	0.33852	30	0.01751	52
GARCH (3, 3)	0.39973	**1**	0.32150	**1**	0.01607	**1**
GARCH (3, 4)	0.42161	26	0.33644	23	0.01672	33
GARCH (3, 5)	0.41852	23	0.33334	20	0.01650	24
GARCH (3, 6)	0.41852	24	0.33334	21	0.01650	25
GARCH (3, 7)	0.41474	12	0.32983	9	0.01624	8
GARCH (3, 8)	0.41474	13	0.32983	10	0.01624	9
GARCH (3, 9)	0.41474	14	0.32983	11	0.01624	10
GARCH (4, 1)	0.43718	34	0.34650	34	0.01710	49
GARCH (4, 2)	0.44604	38	0.35384	41	0.01685	44
GARCH (4, 3)	0.44604	39	0.35384	42	0.01685	45
GARCH (4, 4)	0.45350	49	0.35470	46	0.01684	36
GARCH (4, 5)	0.45350	50	0.35470	47	0.01684	37
GARCH (4, 6)	0.45350	51	0.35470	48	0.01684	38
GARCH (4, 7)	0.42778	27	0.33658	24	0.01631	13
GARCH (4, 8)	0.45200	41	0.35042	39	0.01666	28
GARCH (4, 9)	0.45200	42	0.35042	40	0.01666	29
GARCH (5, 1)	0.43686	33	0.34618	33	0.01708	48

Bitcoin	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (5, 2)	0.44603	37	0.35385	43	0.01685	46
GARCH (5, 3)	0.44466	35	0.34834	35	0.01667	30
GARCH (5, 4)	0.45352	52	0.35474	52	0.01684	42
GARCH (5, 5)	0.45353	53	0.35474	53	0.01684	43
GARCH (5, 6)	0.45349	45	0.35469	45	0.01684	35
GARCH (5, 7)	0.42939	29	0.33819	27	0.01639	15
GARCH (5, 8)	0.42939	30	0.33819	28	0.01639	16
GARCH (5, 9)	0.42939	31	0.33819	29	0.01639	17
GARCH (6, 1)	0.43513	32	0.34490	32	0.01681	34
GARCH (6, 2)	0.44607	40	0.35390	44	0.01686	47
GARCH (6, 3)	0.44468	36	0.34835	36	0.01667	31
GARCH (6, 4)	0.45350	46	0.35470	49	0.01684	39
GARCH (6, 5)	0.45350	47	0.35470	50	0.01684	40
GARCH (6, 6)	0.45350	48	0.35470	51	0.01684	41
GARCH (6, 7)	0.42778	28	0.33658	25	0.01631	14
GARCH (6, 8)	0.45213	43	0.35031	37	0.01663	26
GARCH (6, 9)	0.45213	44	0.35031	38	0.01663	27

From Table 11, when comparing the GARCH (p, q) models to the realized volatility, the RMSE, MAE and MAPE error metrics had the smallest values for the GARCH (3, 3). Therefore, the GARCH (3, 3) will be selected for further calculations. By conducting similar analysis for the selected fiat and cryptocurrencies, the optimal GARCH (p, q) models are summarized in Table 12.

It is important to note that while trying to attain the best volatility model throughout this thesis, and where an inconclusiveness is found based on the three error metrics, the optimal model is chosen based on the smallest value among the error metrics. This applies to GARCH (p, q) models as well as the optimal selected models for each of the selected market variables when comparing EWMA, GARCH (1 ,1), GARCH (p, q) and EGARCH (1, 1).

Table 12. GARCH (P, Q) Model Selection (In-Sample)

GARCH (P, Q)	Realized Volatility	LT Vol	Implied Volatility	LT Vol
CNYUSD	GARCH (4, 6)	3.76%	GARCH (6, 6)	3.60%
GBPUSD	GARCH (4, 1)	12.62%	GARCH (5, 1)	12.57%
Bitcoin	GARCH (3, 3)	-	-	-
Ripple	GARCH (5, 7)	-	-	-

By comparing the long term volatility of the GARCH (1, 1) to the optimal GARCH (p, q) models, it can be noticed that the CNYUSD shows a higher LT volatility compared to the GARCH (1, 1) volatility, which is 3.53% (Table 3). As, for the GBPUSD, the GARCH (1, 1) LT volatility was 13.92% (Table 3), which is higher than the LT volatility of the selected GARCH (p, q) models. Curiously, the long term volatility of the GARCH (p, q) and GARCH (1, 1) is unobtainable for the cryptocurrencies, since the sum of their ARCH and GARCH components is 1. This means that the models converged to the IGARCH, where a unit root exists in the GARCH process. The shift from GARCH to IGARCH model might be caused by irregular shifts in volatility where the impact of past shocks is persistent.

After selecting the best ARCH (p) and GARCH (q) component for each currency and cryptocurrency, the GARCH (1, 1), GARCH (p, q), EWMA and EGARCH (1, 1) are compared to determine the best in-sample model.

Table 13 shows the error statistics of the realized and implied volatilities during the in-sample period for the EURUSD, CNYUSD and GBPUSD.

Table 14 shows the error statistics of the realized volatility during the in-sample period for the Bitcoin and Ripple.

Table 13. Volatility Models vs. Realized and Implied Volatility In-Sample (Fiat Currencies)

EURUSD						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.01666	2	0.01357	2	0.00177	2
EWMA	0.0155	**1**	0.01073	**1**	0.00134	**1**
EGARCH (1, 1)	0.01873	3	0.01468	3	0.00191	3
Implied Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.01638	**1**	0.01199	**1**	0.001419	2
EWMA	0.01868	3	0.01251	2	0.001416	**1**
EGARCH (1, 1)	0.01862	2	0.01412	3	0.001681	3
CNYUSD						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.00848	3	0.00704	4	0.00288	4
GARCH (4, 6)	0.00824	2	0.00651	2	0.00253	2
EWMA	0.00681	**1**	0.00526	**1**	0.0019	**1**
EGARCH (1, 1)	0.00885	4	0.00674	3	0.00255	3
Implied Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.01148	2	0.00932	2	0.00207	2
GARCH (6, 6)	0.01144	**1**	0.0092	**1**	0.00204	**1**
EWMA	0.01186	3	0.00969	3	0.00229	4
EGARCH (1, 1)	0.01283	4	0.01031	4	0.00229	3
GBPUSD						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.04333	4	0.02876	4	0.00302	4
GARCH (4, 1)	0.0389	3	0.02484	3	0.00259	2
EWMA	0.02547	**1**	0.01723	**1**	0.00165	**1**
EGARCH (1, 1)	0.03183	2	0.02463	2	0.00271	3
Implied Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.05038	4	0.03069	4	0.00308	4
GARCH (5, 1)	0.04925	3	0.02757	**1**	0.00272	2
EWMA	0.04904	2	0.02869	3	0.0025	**1**
EGARCH (1, 1)	0.03647	**1**	0.02777	2	0.00286	3

Table 14. Volatility Models vs. Realized Volatility In-Sample (Cryptocurrencies)

Bitcoin						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.4064	3	0.3313	4	0.0174	4
GARCH (3, 3)	0.3997	**1**	0.3215	2	0.0161	3
EWMA	0.4042	2	0.3108	**1**	0.0134	**1**
EGARCH (1, 1)	0.4133	4	0.3311	3	0.0159	2
Ripple						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	1.555	3	0.943	3	0.1378	3
GARCH (5, 7)	1.5453	2	0.9205	**1**	0.1341	2
EWMA	1.6564	4	0.9861	4	0.118	**1**
EGARCH (1, 1)	1.503	**1**	0.9207	2	0.1489	4

Tables 13 and 14 are an illustration representing the best model in reducing the errors using RMSE, MAE and MAPE. The EWMA model proved to be the best model among all the currencies and cryptocurrencies when compared to the realized volatility. However, when comparing the implied volatility of the currencies (EURUSD, CNYUSD and GBPUSD) to the calculated volatilities, the EWMA was shown as the optimal model only for the GBPUSD. As for the EURUSD and CNYUSD, the GARCH (1, 1) and GARCH (6, 6) were the best models respectively. Surprisingly, the EGARCH model, which is considered superior to the first generation GARCH models and captures the impact of negative volatility was one of the worst performer among all the fiat and virtual currencies.

Figures 11, 13, 15, 17 and 18 represent the fluctuation of the volatility models with respect to the realized volatility for the EURUSD, CNYUSD, GBPUSD, Bitcoin and Ripple, respectively.

Figures 12, 14 and 16 represent the fluctuation of the volatility models with respect to the implied volatility for the EURUSD, CNYUSD and GBPUSD.

Figure 11. Realized Volatility vs. GARCH (1, 1), EWMA and EGARCH (1, 1) In-Sample (EURUSD)

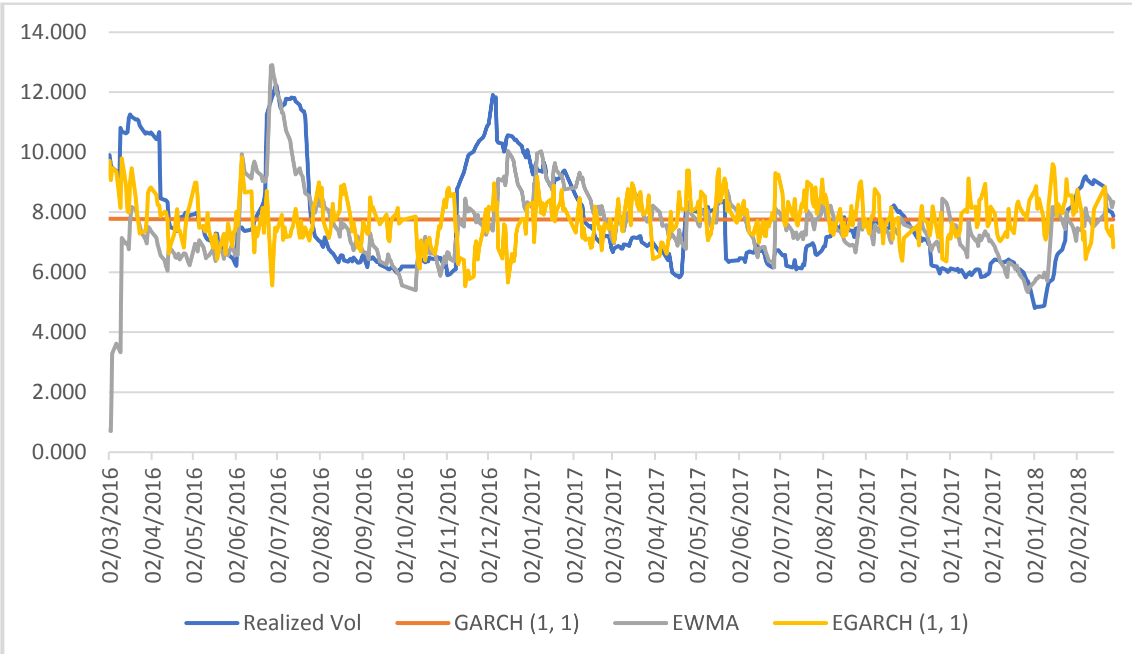


Figure 12. Implied Volatility vs. GARCH (1, 1), EWMA and EGARCH (1, 1) In-Sample (EURUSD)

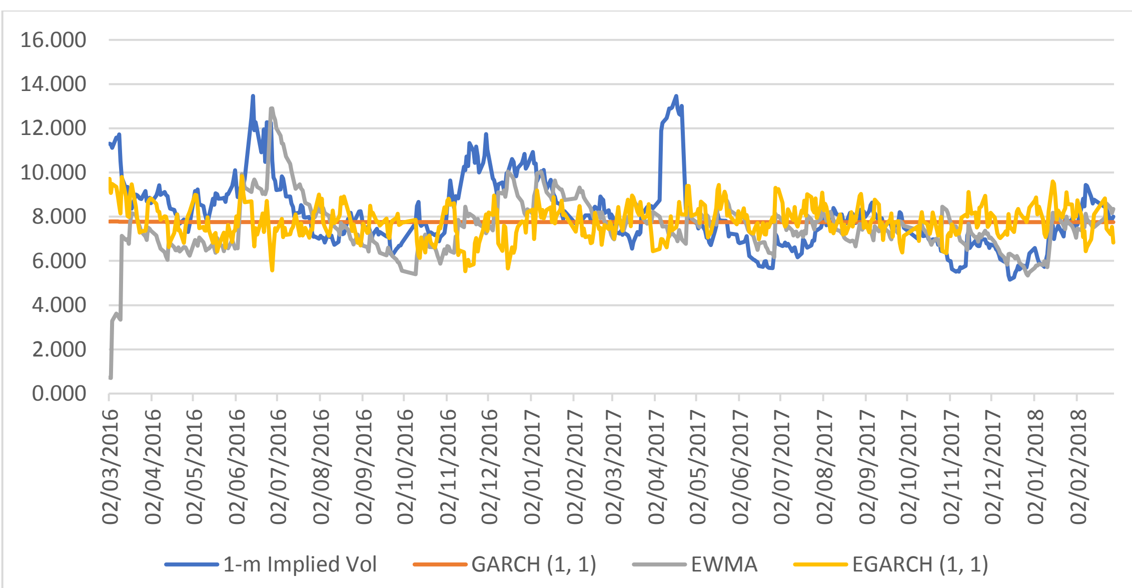


Figure 13. Realized Volatility vs. GARCH (1, 1), GARCH (4, 6), EWMA and EGARCH (1, 1) In-Sample (CNYUSD)

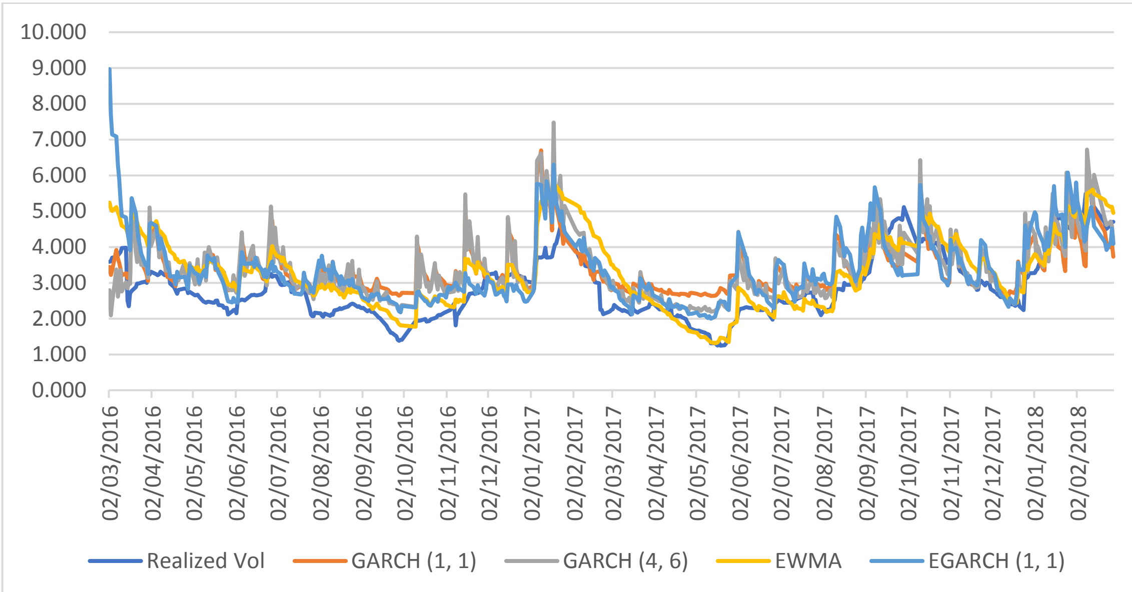


Figure 14. Implied Volatility vs. GARCH (1, 1), GARCH (6, 6), EWMA and EGARCH (1, 1) In-Sample (CNYUSD)

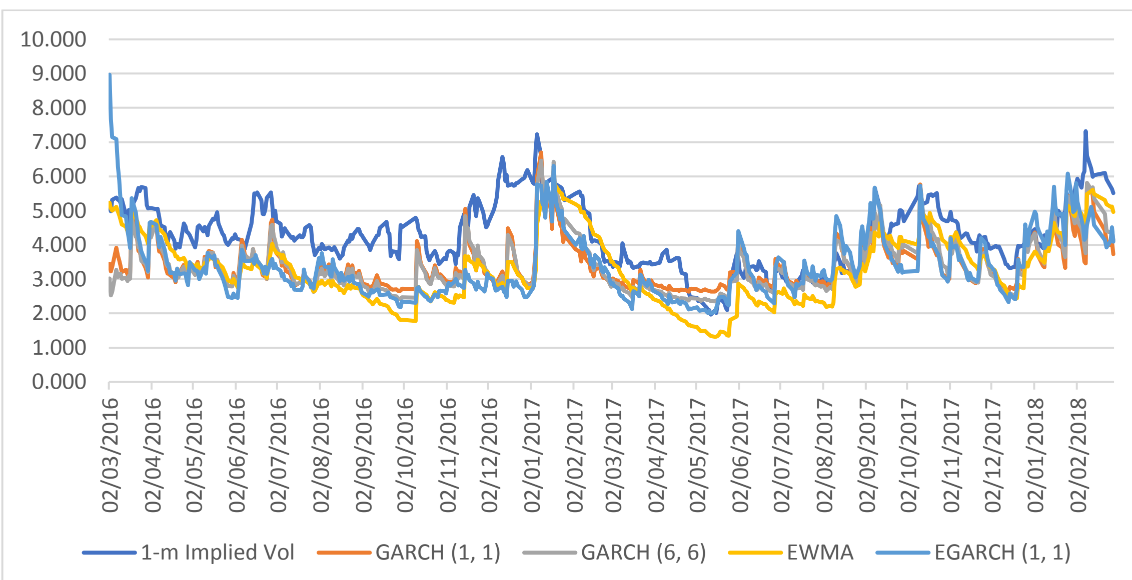


Figure 15. Realized Volatility vs. GARCH (1, 1), GARCH (4, 1), EWMA and EGARCH (1, 1) In-Sample (GBPUSD)

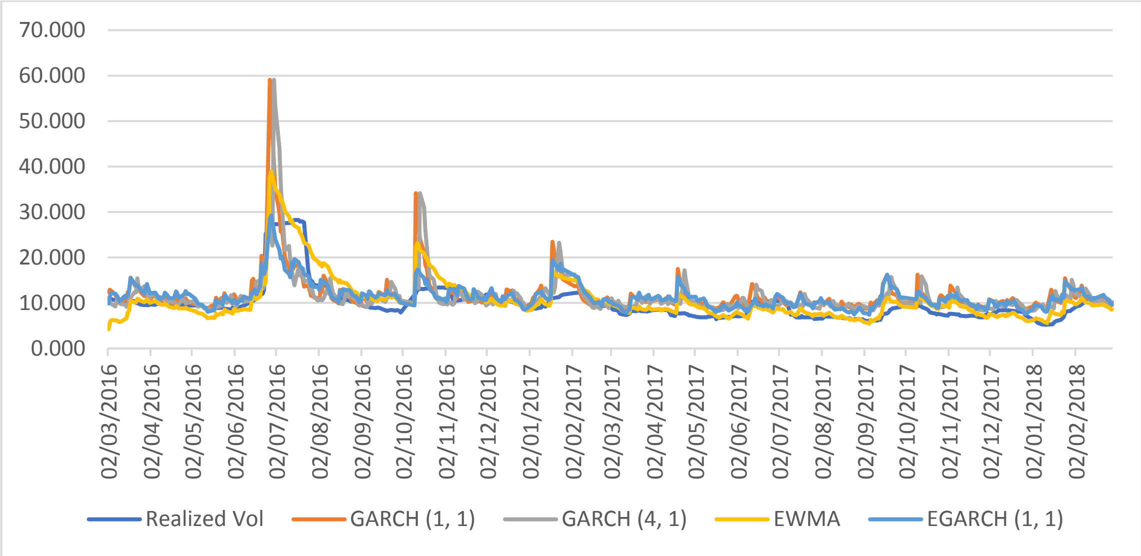


Figure 16. Implied Volatility vs. GARCH (1, 1), GARCH (5, 1), EWMA and EGARCH (1, 1) In-Sample (GBPUSD)

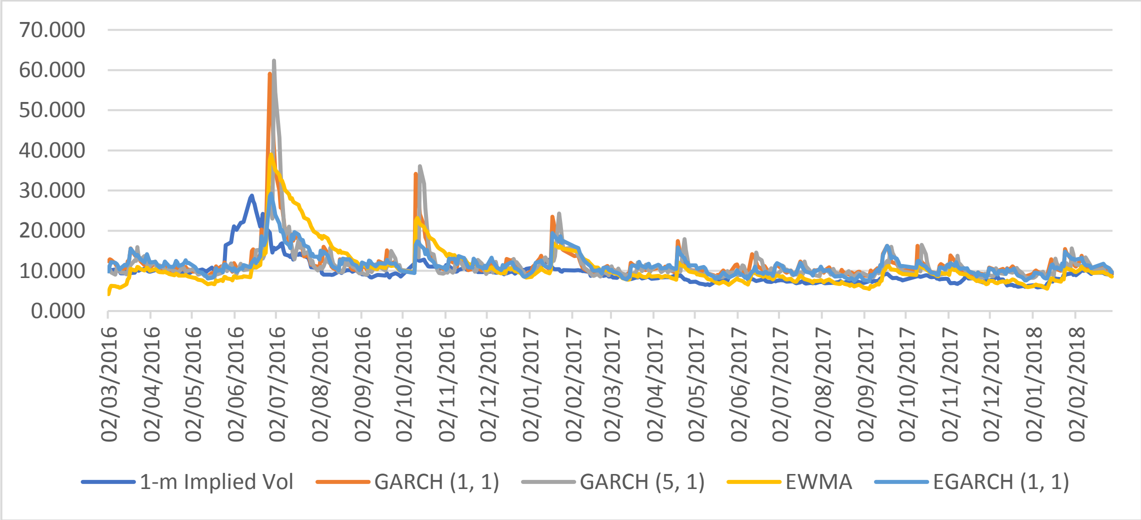


Figure 17. Realized Volatility vs. GARCH (1, 1), GARCH (3, 3), EWMA and EGARCH (1, 1) In-Sample (Bitcoin)

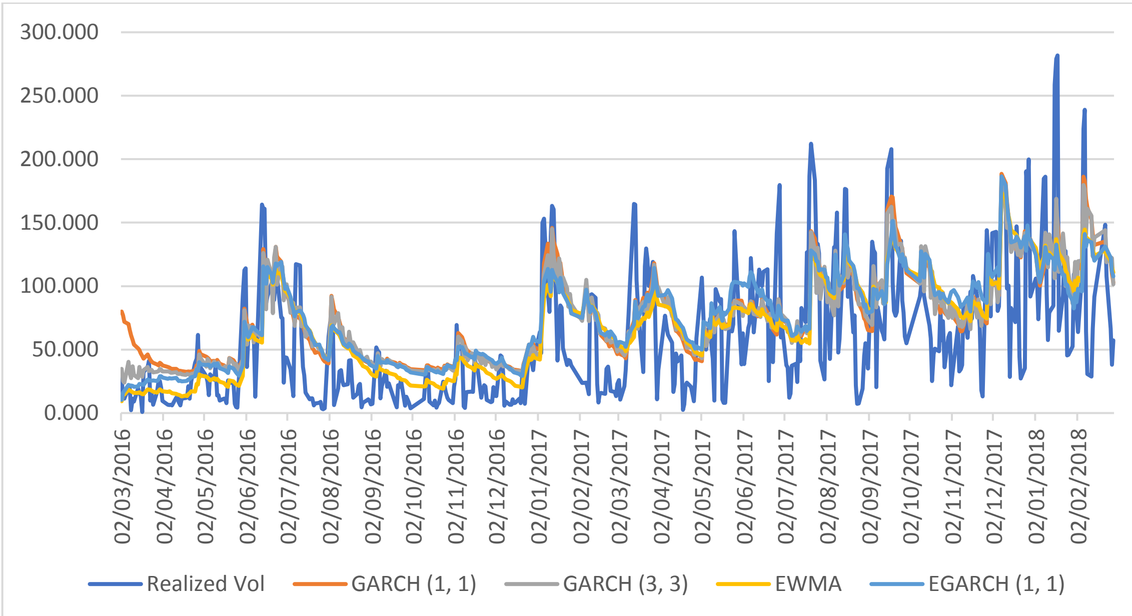
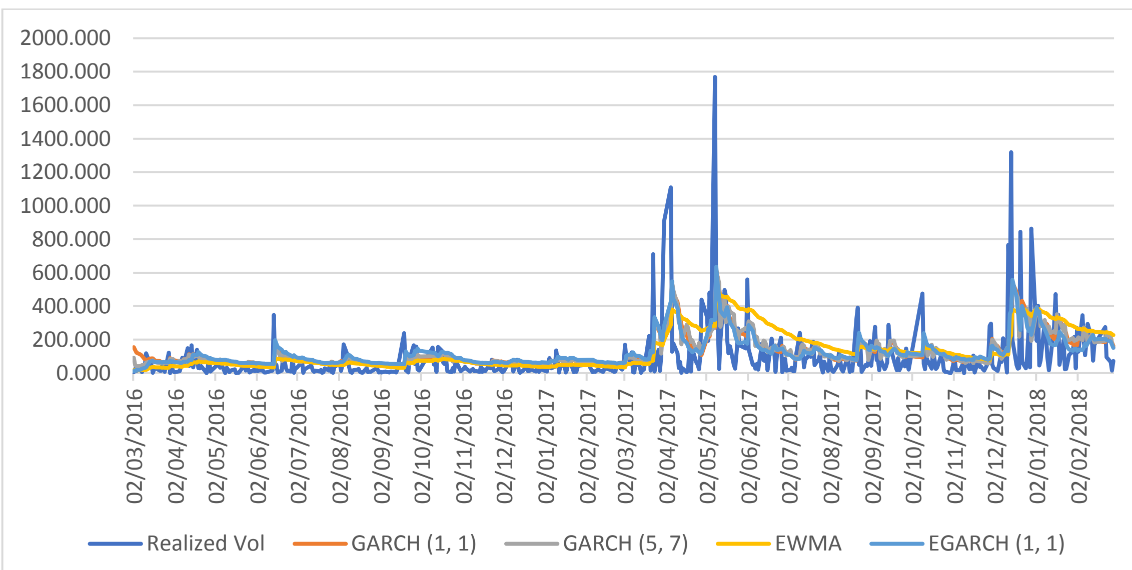


Figure 18. Realized Volatility vs. GARCH (1, 1), GARCH (5, 7), EWMA and EGARCH (1, 1) In-Sample (Ripple)



4.4 Out-Of-Sample Results

The previously discussed methodology based on the three error metrics (RMSE, MAE and MAPE) will be implemented in this section to determine the optimal model for each currency and cryptocurrency for the out-of-sample period. The parameters that were estimated will be used for the out-of-sample period to test their predictive ability.

Before selecting the optimal model for each currency and cryptocurrency, the same comparison using the error metrics is applied on the GARCH (p, q) model to select the best ARCH (p) and GARCH (q) components. The GARCH (p, q) model will be omitted for the EURUSD since the GARCH (1, 1) parameters that were previously stated maximized the loglikelihood and no better combination of the ARCH (p) and GARCH (q) components were found.

The implied volatility was used in the comparison of the currencies' volatility only. However, the realized volatility was compared to the calculated volatility for the currencies and cryptocurrencies.

Since we are studying five market variables, only the Ripple results will be depicted within the thesis, the remaining tables and results are represented in Appendix C.

Table 15 shows the ranking between the GARCH (p, q) models compared to the realized volatility for the Ripple.

Table 15. Error Statistics of GARCH (p, q) vs the Realized Volatility Out-Of-Sample (Ripple)

Ripple	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 2)	0.52055	19	0.42060	11	0.01964	21
GARCH (1, 3)	0.52055	20	0.42060	12	0.01964	22
GARCH (1, 4)	0.54442	51	0.44737	51	0.01995	48
GARCH (1, 5)	0.54442	52	0.44737	52	0.01995	49
GARCH (1, 6)	0.54442	53	0.44737	53	0.01995	50
GARCH (1, 7)	0.50101	3	0.41137	3	0.01814	7
GARCH (1, 8)	0.49912	**1**	0.41090	**1**	0.01798	5
GARCH (1, 9)	0.49912	2	0.41090	2	0.01798	6
GARCH (2, 1)	0.52058	21	0.42064	13	0.01964	23
GARCH (2, 2)	0.52777	39	0.42532	26	0.01981	33
GARCH (2, 3)	0.52994	45	0.42694	33	0.01986	36
GARCH (2, 4)	0.52227	26	0.42628	30	0.01968	27
GARCH (2, 5)	0.52488	33	0.42577	27	0.01970	28
GARCH (2, 6)	0.52488	34	0.42577	28	0.01970	29
GARCH (2, 7)	0.51199	8	0.41777	7	0.01853	12
GARCH (2, 8)	0.51510	11	0.42469	25	0.01860	13
GARCH (2, 9)	0.52620	37	0.43586	50	0.01827	8
GARCH (3, 1)	0.52068	23	0.42075	17	0.01964	25
GARCH (3, 2)	0.52923	43	0.42644	31	0.01984	34
GARCH (3, 3)	0.53009	50	0.42707	39	0.01987	38
GARCH (3, 4)	0.52479	27	0.42709	41	0.01990	42
GARCH (3, 5)	0.52479	28	0.42709	42	0.01990	43
GARCH (3, 6)	0.52479	29	0.42709	43	0.01990	44
GARCH (3, 7)	0.51592	13	0.42070	16	0.01910	18
GARCH (3, 8)	0.51393	9	0.42164	19	0.01878	14
GARCH (3, 9)	0.51710	16	0.42704	36	0.01844	10
GARCH (4, 1)	0.52060	22	0.42068	14	0.01964	24
GARCH (4, 2)	0.52735	38	0.42450	24	0.01978	30
GARCH (4, 3)	0.53009	49	0.42708	40	0.01987	41
GARCH (4, 4)	0.52487	30	0.42715	44	0.01990	45
GARCH (4, 5)	0.52487	31	0.42715	45	0.01990	46
GARCH (4, 6)	0.52487	32	0.42715	46	0.01990	47
GARCH (4, 7)	0.51591	12	0.42069	15	0.01910	17
GARCH (4, 8)	0.51393	10	0.42165	20	0.01878	15
GARCH (4, 9)	0.51710	17	0.42704	37	0.01844	11
GARCH (5, 1)	0.52045	18	0.42054	10	0.01963	20

Ripple	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (5, 2)	0.52924	44	0.42650	32	0.01984	35
GARCH (5, 3)	0.53004	46	0.42704	34	0.01987	39
GARCH (5, 4)	0.53004	47	0.42704	35	0.01987	40
GARCH (5, 5)	0.52592	35	0.42419	22	0.01980	31
GARCH (5, 6)	0.52592	36	0.42419	23	0.01980	32
GARCH (5, 7)	0.51638	15	0.42307	21	0.01837	9
GARCH (5, 8)	0.50754	6	0.41862	9	0.01787	4
GARCH (5, 9)	0.50534	5	0.41734	6	0.01765	2
GARCH (6, 1)	0.52070	24	0.42077	18	0.01964	26
GARCH (6, 2)	0.51594	14	0.41464	4	0.01920	19
GARCH (6, 3)	0.53008	48	0.42707	38	0.01987	37
GARCH (6, 4)	0.52910	40	0.42988	47	0.01998	51
GARCH (6, 5)	0.52910	41	0.42988	48	0.01998	52
GARCH (6, 6)	0.52910	42	0.42988	49	0.01998	53
GARCH (6, 7)	0.52220	25	0.42602	29	0.01885	16
GARCH (6, 8)	0.50754	7	0.41861	8	0.01787	3
GARCH (6, 9)	0.50532	4	0.41733	5	0.01765	**1**

From Table 15, when comparing the GARCH (p, q) models to the realized volatility, the RMSE and MAE error metrics had the smallest values for the GARCH (1, 8) whereas the MAPE was the smallest for the GARCH (6, 9). Therefore, the GARCH (1, 8) will be selected for further calculations. By conducting similar analysis for the selected fiat and cryptocurrencies, the optimal GARCH (p, q) models are summarized in Table 16.

Table 16. GARCH (P, Q) Model Selection (Out-Of-Sample)

GARCH (P, Q)	Realized Volatility	LT Vol	Implied Volatility	LT Vol
CNYUSD	GARCH (6, 2)	3.51%	GARCH (4, 6)	3.76%
GBPUSD	GARCH (1, 8)	14.66%	GARCH (3, 1)	17.34%
Bitcoin	GARCH (3, 3)	-	-	-
Ripple	GARCH (1, 8)	-	-	-

The GARCH (1, 1) long term volatility is 3.53% for the CNYUSD and it falls between the LT volatility of the GARCH (6, 2) model (3.51%) and GARCH (4, 6) model (3.76%). In contrast to the in-sample results, the GARCH (1, 1) LT volatility of the GBPUSD is 13.92% which is lower than that of the GARCH (p, q) models which fall between 14.66% and 17.34% for the GARCH (1, 8) and GARCH (3, 1), respectively. Similar to the in-sample results, virtual currencies LT volatility is unattainable since the GARCH models converged to the Integrated GARCH models. Curiously, the optimal number of ARCH (p) and GARCH (q) components for the Bitcoin were found the same during the in-sample and out-of-sample periods.

After selecting the best ARCH (p) and GARCH (q) component for each currency and cryptocurrency, the GARCH (1, 1), GARCH (p, q), EWMA and EGARCH (1, 1) are compared to determine the best out-of-sample model.

Table 17 shows the error statistics of the realized and implied volatilities during the out-of-sample period for the EURUSD, CNYUSD and GBPUSD.

Table 18 shows the error statistics of the realized volatility during the out-of-sample period for the Bitcoin and Ripple.

Table 17. Volatility Models vs. Realized and Implied Volatility Out-Of-Sample (Fiat Currencies)

EURUSD						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.01079	2	0.00915	2	0.00143	2
EWMA	0.0096	**1**	0.00723	**1**	0.0011	**1**
EGARCH (1, 1)	0.01248	3	0.0104	3	0.00159	3
Implied Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.00964	**1**	0.00831	2	0.00127	2
EWMA	0.00977	2	0.00745	**1**	0.0011	**1**
EGARCH (1, 1)	0.01262	3	0.0108	3	0.00162	3
CNYUSD						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.01196	2	0.00874	2	0.00208	2
GARCH (6, 2)	0.01123	**1**	0.00836	**1**	0.00202	**1**
EWMA	0.01231	3	0.01083	4	0.00291	4
EGARCH (1, 1)	0.01325	4	0.00962	3	0.00225	3
Implied Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.0156	3	0.01347	4	0.00248	4
GARCH (4, 6)	0.01388	2	0.01168	2	0.00217	2
EWMA	0.00857	**1**	0.00683	**1**	0.00126	**1**
EGARCH (1, 1)	0.01561	4	0.01308	3	0.00236	3
GBPUSD						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.02899	4	0.02508	4	0.00329	4
GARCH (1, 8)	0.02686	3	0.02036	2	0.00263	2
EWMA	0.01843	**1**	0.01314	**1**	0.00162	**1**
EGARCH (1, 1)	0.02533	2	0.02097	3	0.00273	3
Implied Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.02369	4	0.02056	4	0.00237	4
GARCH (3, 1)	0.02081	2	0.01715	3	0.00192	3
EWMA	0.02198	3	0.01655	2	0.00173	**1**
EGARCH (1, 1)	0.01882	**1**	0.01563	**1**	0.00174	2

Table 18. Volatility Models vs. Realized Volatility Out-Of-Sample (Cryptocurrencies)

Bitcoin						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.3305	3	0.276	2	0.0154	3
GARCH (3, 3)	0.3302	2	0.2768	3	0.0154	2
EWMA	0.3255	**1**	0.2682	**1**	0.0137	**1**
EGARCH (1, 1)	0.3858	4	0.3287	4	0.0174	4
Ripple						
Realized Volatility	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (1, 1)	0.5206	2	0.4206	2	0.0196	2
GARCH (1, 8)	0.4991	**1**	0.4109	**1**	0.018	**1**
EWMA	0.5672	3	0.4543	3	0.0202	3
EGARCH (1, 1)	0.5952	4	0.4863	4	0.0224	4

Tables 17 and 18 are an illustration representing the best model in reducing the errors using RMSE, MAE and MAPE. The EWMA model proved to be the best model for the EURUSD, GBPUSD and Bitcoin when compared to the realized volatility. In the same context, GARCH (6, 2) and GARCH (1, 8) were shown as the optimal models for the CNYUSD and Ripple, respectively. However, when comparing the implied volatility of the currencies (EURUSD, CNYUSD and GBPUSD) to the calculated volatilities, the EWMA was shown as the optimal model for the EURUSD and CNYUSD. As for the GBPUSD, the EGARCH (1, 1) model, which captures the leverage effect, was superior to the rest of the selected volatility models.

Figures 19, 21, 23, 25 and 26 represent the fluctuation of the volatility models with respect to the realized volatility for the EURUSD, CNYUSD, GBPUSD, Bitcoin and Ripple, respectively.

Figures 20, 22 and 24 represent the fluctuation of the volatility models with respect to the implied volatility for the EURUSD, CNYUSD and GBPUSD.

Figure 19. Realized Volatility vs. GARCH (1, 1), EWMA and EGARCH (1, 1) Out-Of-Sample (EURUSD)

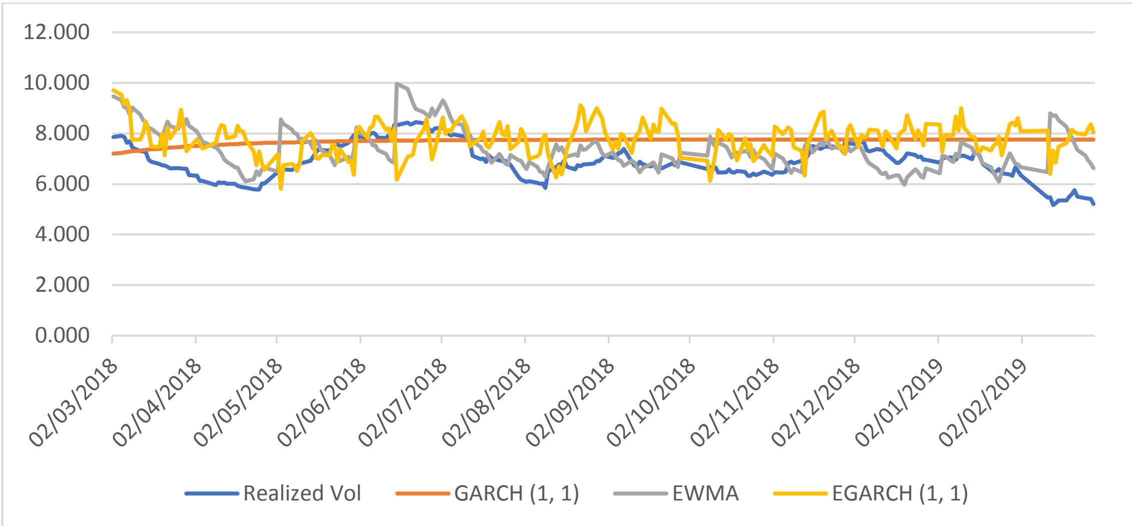


Figure 20. Implied Volatility vs. GARCH (1, 1), EWMA and EGARCH (1, 1) Out-Of-Sample (EURUSD)

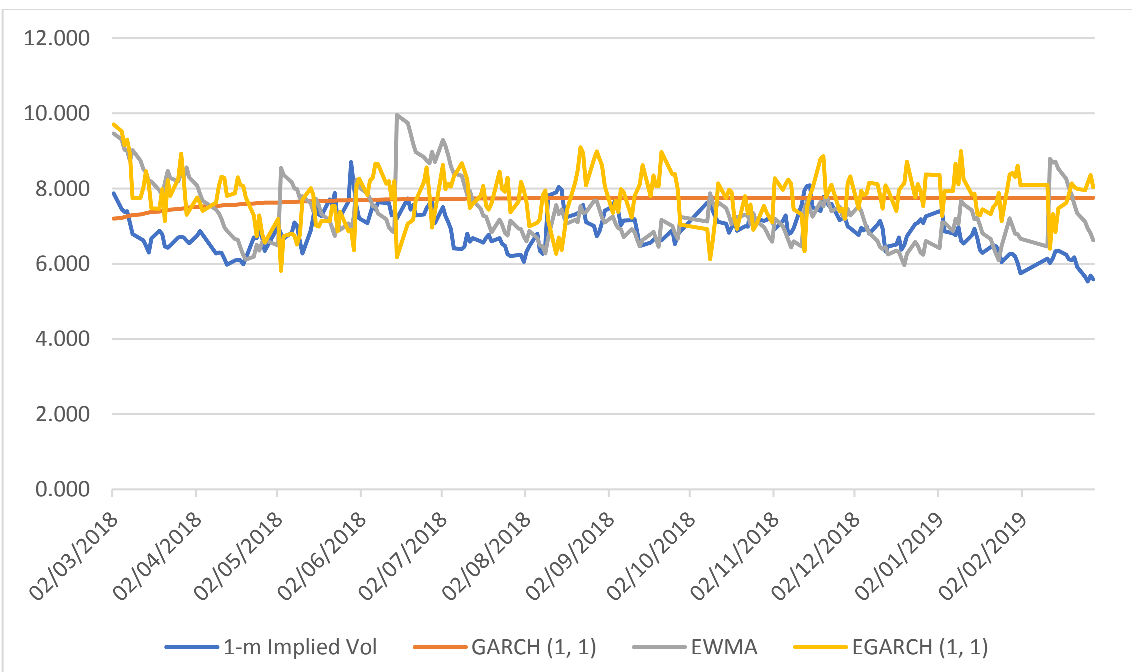


Figure 21. Realized Volatility vs. GARCH (1, 1), GARCH (6, 2), EWMA and EGARCH (1, 1) Out-Of-Sample (CNYUSD)

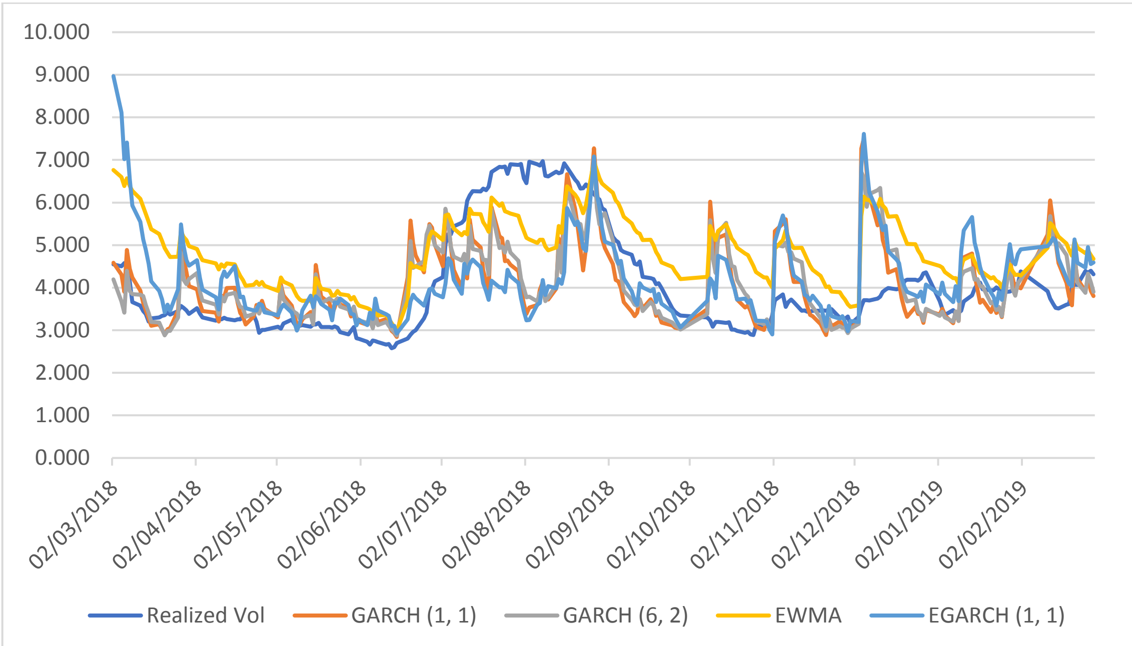


Figure 22. Implied Volatility vs. GARCH (1, 1), GARCH (4, 6), EWMA and EGARCH (1, 1) Out-Of-Sample (CNYUSD)

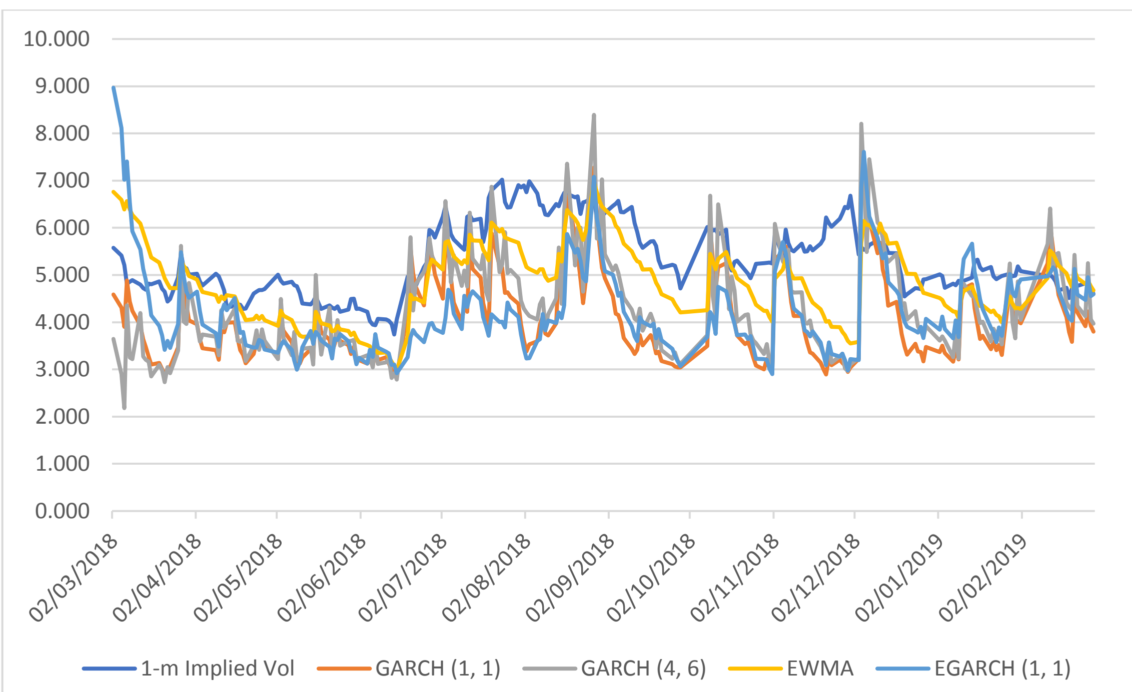


Figure 23. Realized Volatility vs. GARCH (1, 1), GARCH (1, 8), EWMA and EGARCH (1, 1) Out-Of-Sample (GBPUSD)

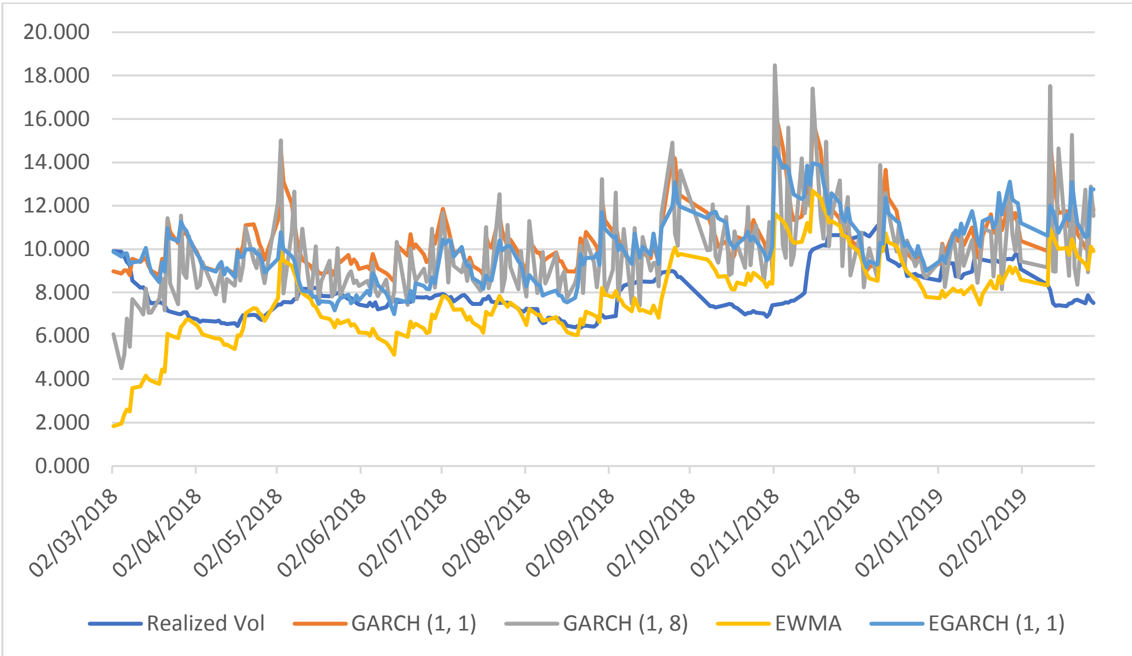


Figure 24. Implied Volatility vs. GARCH (1, 1), GARCH (3, 1), EWMA and EGARCH (1, 1) Out-Of-Sample (GBPUSD)

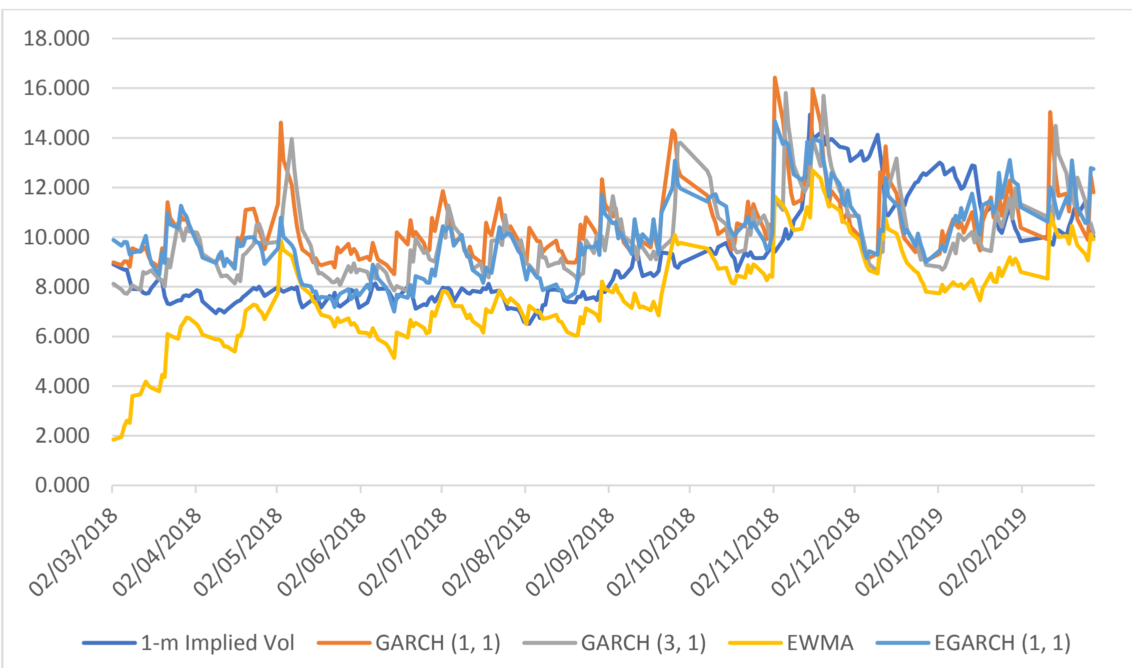


Figure 25. Realized Volatility vs. GARCH (1, 1), GARCH (3, 3), EWMA and EGARCH (1, 1) Out-Of-Sample (Bitcoin)

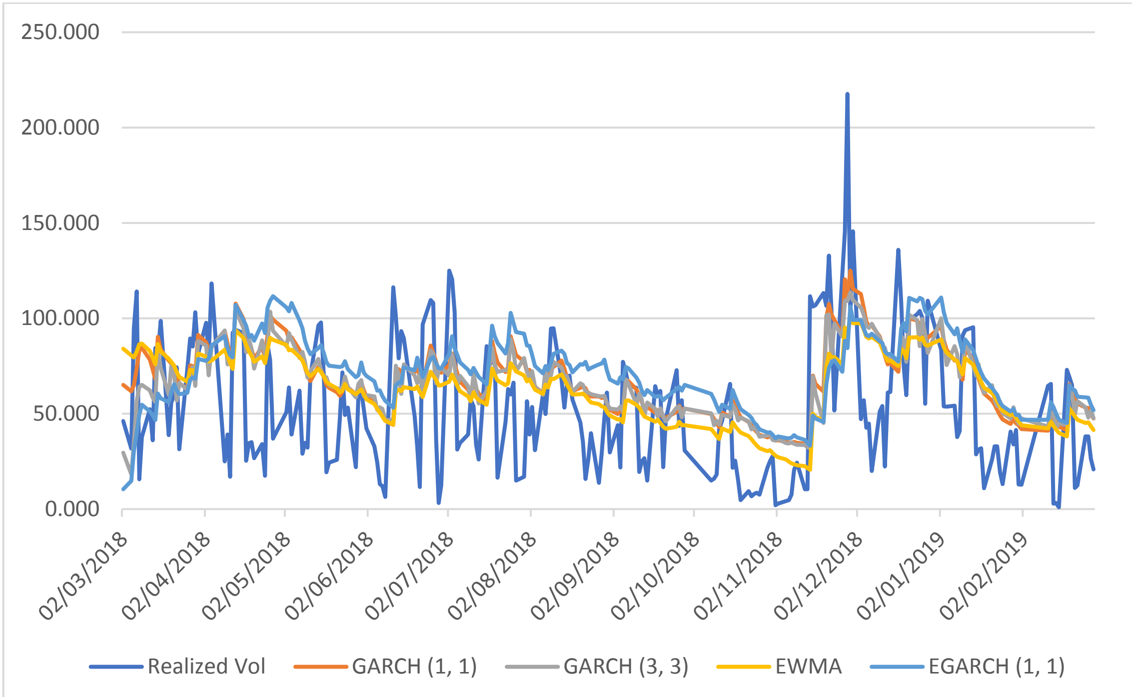


Figure 26. Realized Volatility vs. GARCH (1, 1), GARCH (1, 8), EWMA and EGARCH (1, 1) Out-Of-Sample (Ripple)

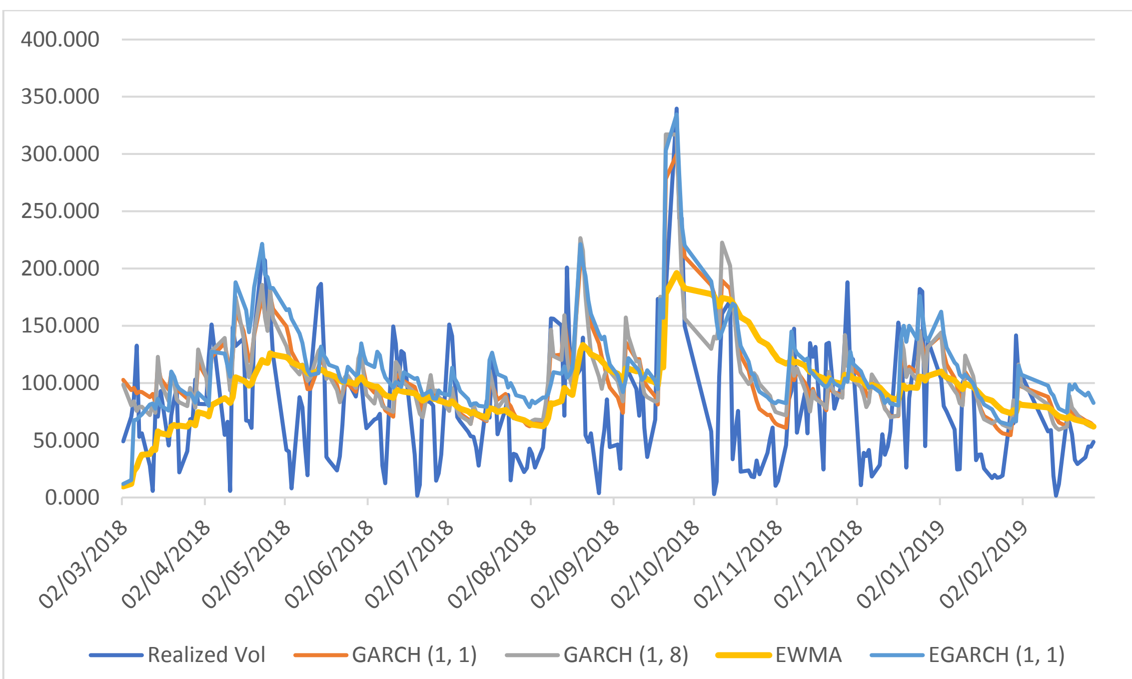


Table 19 summarizes the results of the selected optimal models based on the realized volatility for the in-sample and out-of-sample periods for the selected currencies and cryptocurrencies.

Table 19. Optimal Selected Models Based on the Realized Volatility Comparison

Optimal Model	**EURUSD**	**CNYUSD**	**GBPUSD**	**Bitcoin**	**Ripple**
In-Sample	EWMA	EWMA	EWMA	EWMA	EWMA
Out-Of-Sample	EWMA	GARCH (6, 2)	EWMA	EWMA	GARCH (1, 8)

Table 20 summarizes the results of the selected optimal models based on the implied volatility during the in-sample and out-of-sample periods for the selected currencies.

Table 20. Optimal Selected Models Based on the Implied Volatility Comparison

Optimal Model	**EURUSD**	**CNYUSD**	**GBPUSD**
In-Sample	GARCH (1, 1)	GARCH (6, 6)	EWMA
Out-Of-Sample	EWMA	EWMA	EGARCH (1, 1)

From Tables 19 and 20, the EWMA model was shown as the optimal model among most of the studied market variables in both contexts, despite the absence of a long-run average variance. The EGARCH (1, 1), which is superior to the first generation GARCH models did not prove its advantage and had the least desirable results between the selected models.

Distinctively, when comparing the calculated volatilities to the realized volatility, the EWMA proved to be the best model in both contexts for the EURUSD, GBPUSD and Bitcoin. Whereas, for the CNYUSD and Ripple, the EWMA was the optimal model during the in-sample period while the GARCH (p, q) yielded better results in the out-of-sample period.

Furthermore, when comparing the calculated volatilities to the implied volatility for the currencies, the results were mixed. The GARCH models produced preferable output in the in-sample period for the EURUSD and CNYUSD, whereas in the out-of-sample period, the EWMA revealed better outcome. However, for the GBPUSD, the EWMA and EGARCH (1, 1) were the best models during the in-sample and out-of-sample period, respectively.

Additionally, the optimal selected models from Table 19 during the out-of-sample period will be used in the next section to calculate VaR.

4.5 VaR Results

The procedure of incorporating volatility updating into historical simulation, which was proposed by Hull and White, is used in our research. The previously discussed optimal selected models based on the comparison between the realized volatility and calculated volatilities for each currency and cryptocurrency during the out-of-sample period are used for the VaR calculations.

The models' parameters are estimated twice for each of the selected market variables. EWMA's Lambda parameter (λ) is calculated for the EURUSD, GBPUSD and Bitcoin. Whereas, GARCH (6, 2) and GARCH (1, 8) parameters are computed for the CNYUSD and Ripple, respectively. The parameters are first estimated for a sample period of 100 days extending from February 13, 2018 till July 16, 2018. The second set of parameters is estimated for a sample period of 150 days extending from July 17, 2018 till February 28, 2019.

As previously explained in section 3.3.5, the data consists of 649 daily observations extending from June 30, 2016 to February 28, 2019. This sample is divided into 250 sub-samples each sub-sample including 400 observations. From the estimated parameters and using the returns of each currency and cryptocurrency, the daily variances are calculated 399 times for each sub-sample using equation (3) for the EURUSD, GBPUSD and Bitcoin. Using equation (7), the daily variances are also calculated 399 times for each sub-sample period for the CNYUSD and Ripple. This results in a total of 99,750 (399*250) values of variance for each currency and cryptocurrency, or a total of 498,750 (399*250*5) values of variances. Accordingly, the daily volatilities are calculated as the square root of the variances. The next step consists in plugging the volatilities values in equation (11), resulting in different price scenarios for each currency and cryptocurrency. Then, the price scenarios are fitted into equation (12) to determine the return scenarios. Finally, by taking the 90^{th}, 95^{th}, 97.5^{th} and 99^{th} percentiles of the loss/profit probability distribution, 250 VaR estimates are obtained for each confidence level and for each of the selected market variables. VaR estimates under each confidence level are compared to the actual returns to determine the number of exceptions. Exceptions are denoted where the actual loss is greater than the VaR loss value.

4.5.1 Parameters' Estimation

The parameters are estimated twice for each currency and cryptocurrency based on the optimal selected model in the out-of-sample period. The EWMA model proved to be the best model for the EURUSD, GBPUSD and Bitcoin. Whereas the GARCH (6, 2) and GARCH (1, 8) were the best models for the CNYUSD and Ripple, respectively.

Table 21 summarizes the EWMA parameters for the EURUSD, GBPUSD and Bitcoin. Table 22 summarizes the GARCH (6, 2) parameters for the CNYUSD and Table 23 summarizes the GARCH (1, 8) parameters for the Ripple.

Table 21. EWMA estimated parameters (EURUSD, GBPUSD, Bitcoin)

	EURUSD		GBPUSD		Bitcoin	
EWMA	**Lambda**	**LLF**	**Lambda**	**LLF**	**Lambda**	**LLF**
13/02/2018 16/07/2018	0.959	926.83	0.931	950.11	0.922	497.96
17/07/2018 28/02/2019	0.935	1419.60	0.960	1362.19	0.921	747.68

The decay factor (λ) ranges between 92% and 96% between the selected market variables. This parameter explains the relative importance of the observations with respect to the returns when determining the current variance rate. Interestingly, Bitcoin's Lambda parameter was almost the same in the two periods, meaning that Bitcoin's behavior was similar in the two time frames.

Table 22. GARCH (6, 2) estimated parameters (CNYUSD)

GARCH (6, 2)	ω	α1	α2	α3	α4	α5	α6	β1	β2	LLF
13/02/2018 16/07/2018	0.00000073	0.051	0.000	0.000	0.003	0.011	0.003	0.521	0.334	1069.31
17/07/2018 28/02/2019	0.00000147	0.002	0.050	0.000	0.000	0.000	0.000	0.472	0.351	1555.18

Table 23. GARCH (1, 8) estimated parameters (Ripple)

GARCH (1, 8)	ω	α1	β1	β2	β3	β4	β5	β6	β7	β8	LLF
13/02/2018 16/07/2018	0.00247983	0.274	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.119	454.17
17/07/2018 28/02/2019	0.00032670	0.396	0.369	0.000	0.000	0.000	0.000	0.235	0.000	0.000	684.21

Tables 22 and 23 show that many ARCH (α) and GARCH (β) components have a value of zero. This means that any assigned trial value for those components was unable to maximize the loglikelihood function. Curiously, Ripple's estimated parameters had a long term volatility of 101.12% in the first sub-sample extending from February 13, 2018 through July 16, 2018. Whereas in the second sub-sample which range from July 17, 2018 till February 28, 2019 the long term volatility was unattainable, meaning that the model converged to the IGARCH model.

4.5.2 VaR Calculations

Following the estimation of the models' parameters, the volatilities are calculated for each currency and cryptocurrency. These volatilities will be used to generate different price scenarios, from which the return scenarios are deduced.

Table 24 displays part of the results representing the first 20 days and the last 3 days for the EURUSD. The exceptions (Exp) are denoted by "0" in the following tables. Exceptions occur when the VaR loss value is less than the actual return. However, when the actual return is smaller than the VaR loss value, no exception is recorded and a value of "1" is allocated.

Similar tables representing parts of the results for the CNYUSD, GBPUSD, Bitcoin and Ripple are shown in Appendix D.

Table 24. VaR Calculations for the EURUSD using EWMA Volatility-weighted Historical Simulation

Day	Date	Actual Returns	VaR 90%	Exp	VaR 95%	Exp	VaR 97.5%	Exp	VaR 99%	Exp
1	2/13/2018	0.0049	-0.0053	1	-0.0076	1	-0.0092	1	-0.0110	1
2	2/14/2018	0.0080	-0.0057	1	-0.0080	1	-0.0096	1	-0.0116	1
3	2/22/2018	-0.0097	-0.0056	0	-0.0079	0	-0.0096	0	-0.0111	1
4	2/23/2018	-0.0028	-0.0060	1	-0.0084	1	-0.0099	1	-0.0116	1
5	2/26/2018	0.0018	-0.0055	1	-0.0081	1	-0.0096	1	-0.0112	1
6	2/27/2018	-0.0068	-0.0060	0	-0.0089	1	-0.0101	1	-0.0117	1
7	2/28/2018	-0.0032	-0.0060	1	-0.0088	1	-0.0101	1	-0.0116	1
8	3/1/2018	0.0060	-0.0061	1	-0.0089	1	-0.0104	1	-0.0119	1
9	3/2/2018	0.0041	-0.0059	1	-0.0081	1	-0.0098	1	-0.0114	1
10	3/5/2018	0.0015	-0.0058	1	-0.0080	1	-0.0096	1	-0.0112	1
11	3/6/2018	0.0055	-0.0053	1	-0.0072	1	-0.0092	1	-0.0105	1
12	3/7/2018	0.0006	-0.0054	1	-0.0078	1	-0.0091	1	-0.0108	1
13	3/8/2018	-0.0080	-0.0055	0	-0.0080	0	-0.0093	1	-0.0109	1
14	3/9/2018	-0.0004	-0.0058	1	-0.0080	1	-0.0097	1	-0.0115	1
15	3/12/2018	0.0022	-0.0057	1	-0.0078	1	-0.0095	1	-0.0113	1
16	3/13/2018	0.0045	-0.0053	1	-0.0077	1	-0.0089	1	-0.0105	1
17	3/14/2018	-0.0018	-0.0055	1	-0.0076	1	-0.0092	1	-0.0108	1
18	3/15/2018	-0.0051	-0.0056	1	-0.0076	1	-0.0093	1	-0.0112	1
19	3/16/2018	-0.0012	-0.0051	1	-0.0071	1	-0.0086	1	-0.0099	1
20	3/19/2018	0.0037	-0.0054	1	-0.0074	1	-0.0091	1	-0.0107	1
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
248	2/26/2019	0.0027	-0.0057	1	-0.0068	1	-0.0078	1	-0.0109	1
249	2/27/2019	-0.0017	-0.0055	1	-0.0066	1	-0.0076	1	-0.0106	1
250	2/28/2019	0.0001	-0.0053	1	-0.0063	1	-0.0072	1	-0.0102	1

Figures 27, 28, 29, 30 and 31 illustrate VaR results compared to the actual returns for each currency and cryptocurrency.

Figure 27. EURUSD Daily Returns vs. VaR (EWMA Volatility-weighted Historical Simulation)

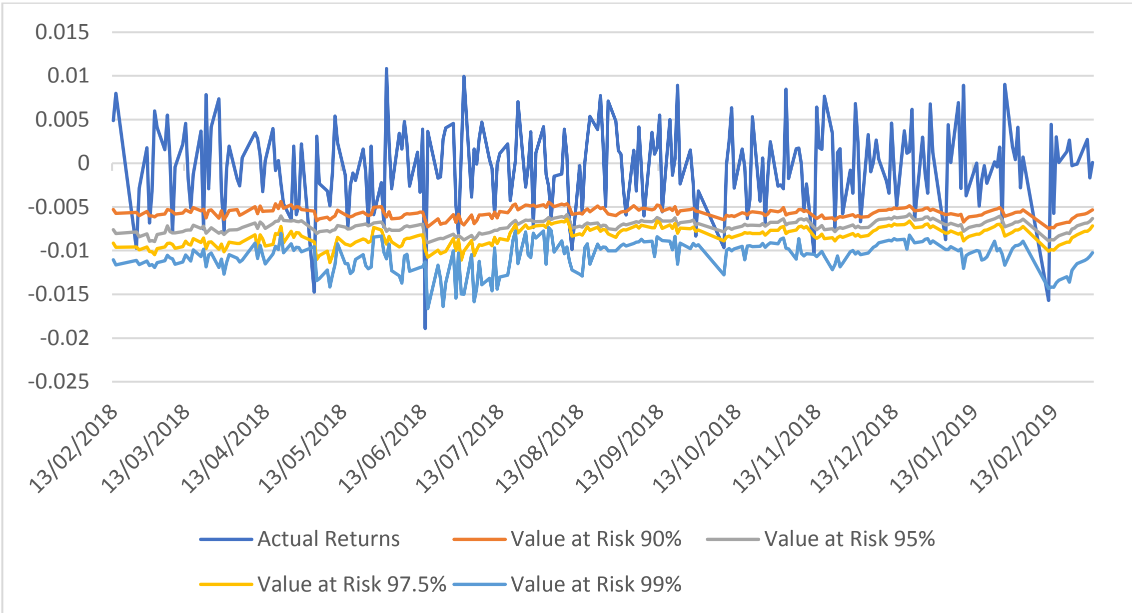


Figure 28. CNYUSD Daily Returns vs. VaR (GARCH (6, 2) Volatility-weighted Historical Simulation)

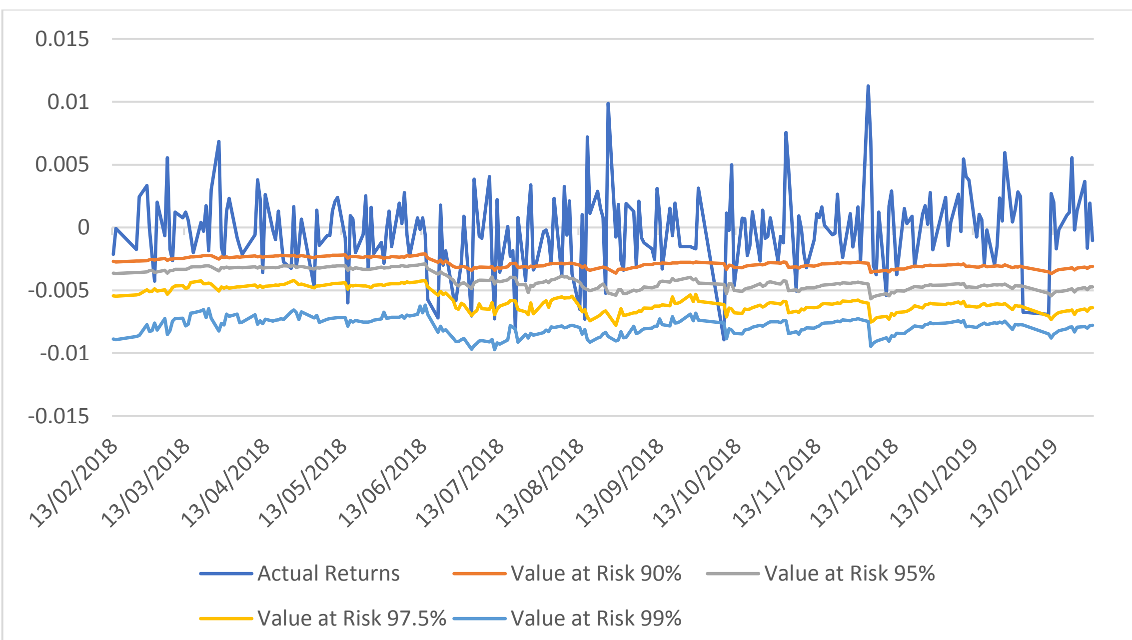


Figure 29. GBPUSD Daily Returns vs. VaR (EWMA Volatility-weighted Historical Simulation)

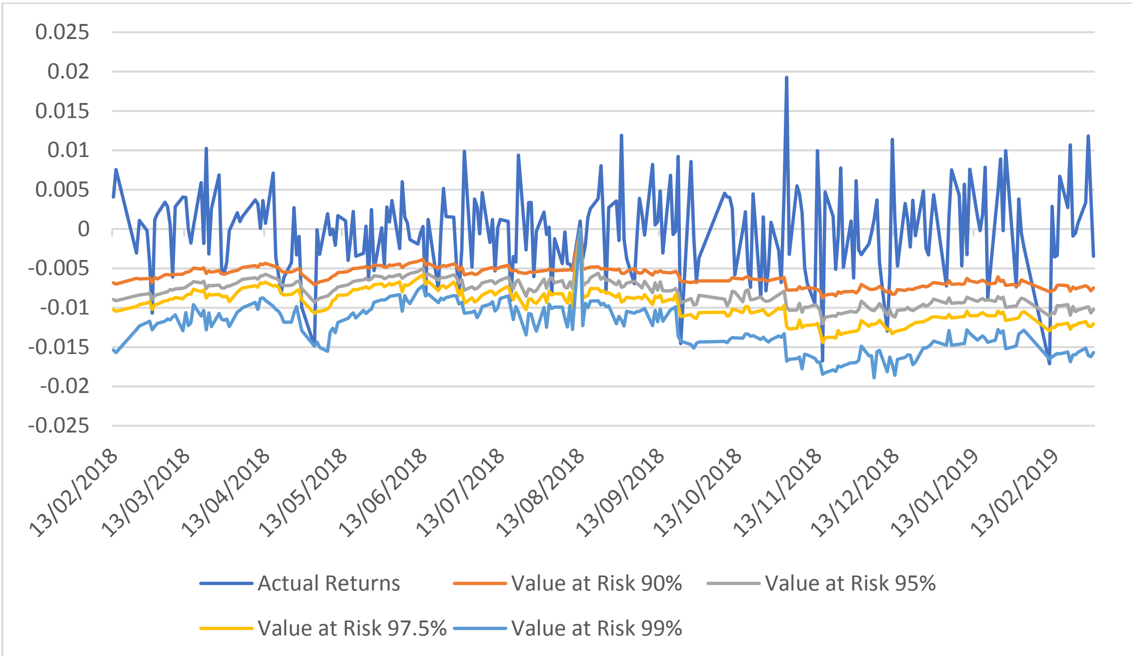


Figure 30. Bitcoin Daily Returns vs. VaR (EWMA Volatility-weighted Historical Simulation)

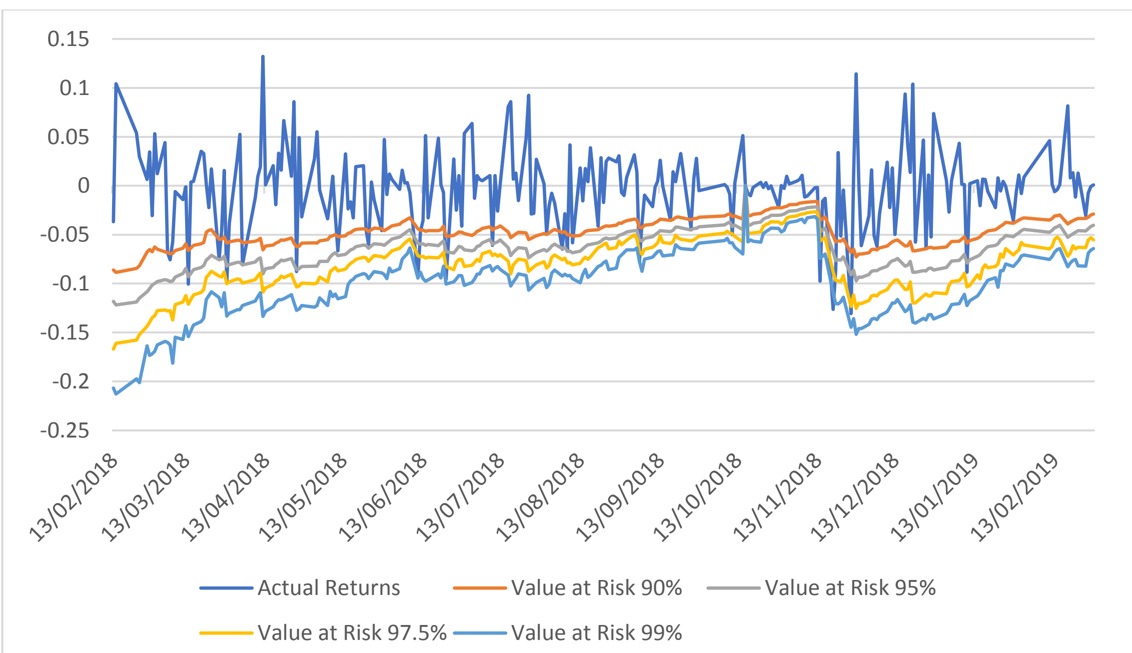
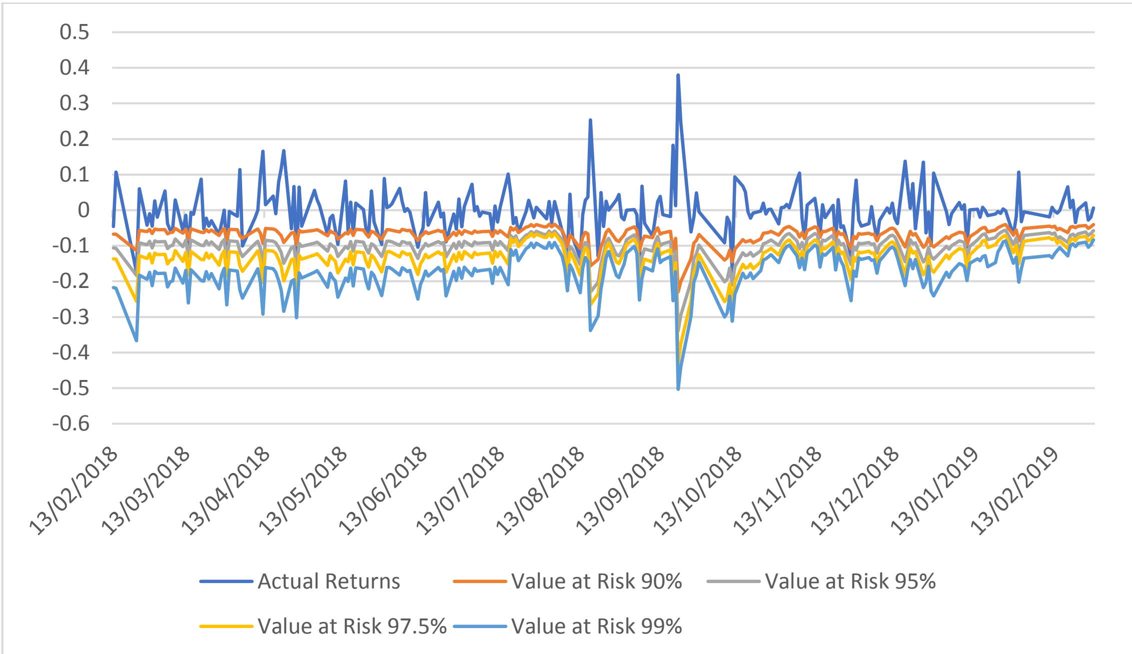


Figure 31. Ripple Daily Returns vs. VaR (GARCH (1, 8) Volatility-weighted Historical Simulation)



4.5.3 Kupiec Test Results

The Kupiec test measures VaR accuracy and it is applied by calculating the likelihood ratio (LR) using equation (13). The number of exceptions *(N)* calculated for each currency and cryptocurrency is fitted into LR. For a 90% VaR, the probability of failure is 10%; for a 95% VaR the probability of failure is 5%; for a 97.5% VaR the probability of failure is 2.5%; and for a 99% VaR the probability of failure is 1%. Moreover, the number of trials is 250, which is equivalent to the number of VaR calculated for each currency and cryptocurrency.

Results of the Kupiec test for the currencies and cryptocurrencies are presented in Table 25.

Table 25. Kupiec Test

	Method Applied	VaR CL	Exceptions Recorded	Allowed Exceptions	LR	95% Critical Value	Test Outcome
EURUSD	Incorporating volatility to historical simulation using **EWMA**	90%	34	[17, 35]	3.27	3.84	**Accept**
		95%	16	[7, 20]	0.95	3.84	**Accept**
		97.5%	11	[2, 11]	3.03	3.84	**Accept**
		99%	3	[0, 5]	0.09	3.84	**Accept**
CNYUSD	Incorporating volatility to historical simulation using **GARCH (6, 2)**	90%	41	[17, 35]	9.73	3.84	**Reject**
		95%	22	[7, 20]	6.26	3.84	**Reject**
		97.5%	10	[2, 11]	1.96	3.84	**Accept**
		99%	2	[0, 5]	0.11	3.84	**Accept**
GBPUSD	Incorporating volatility to historical simulation using **EWMA**	90%	30	[17, 35]	1.05	3.84	**Accept**
		95%	15	[7, 20]	0.5	3.84	**Accept**
		97.5%	9	[2, 11]	1.09	3.84	**Accept**
		99%	3	[0, 5]	0.09	3.84	**Accept**
Bitcoin	Incorporating volatility to historical simulation using **EWMA**	90%	26	[17, 35]	0.04	3.84	**Accept**
		95%	14	[7, 20]	0.18	3.84	**Accept**
		97.5%	7	[2, 11]	0.09	3.84	**Accept**
		99%	2	[0, 5]	0.11	3.84	**Accept**
Ripple	Incorporating volatility to historical simulation using **GARCH (1, 8)**	90%	20	[17, 35]	1.18	3.84	**Accept**
		95%	0	[7, 20]	-	3.84	**Reject**
		97.5%	0	[2, 11]	-	3.84	**Reject**
		99%	0	[0, 5]	-	3.84	**Accept**

The EURUSD[5], GBPUSD and Bitcoin had the most accurate results at all confidence levels, with LR values less than 3.84. However, VaR results for the CNYUSD were rejected at 90% and 95% confidence levels. Interestingly, VaR results were rejected at 95% and 97.5% for the Ripple since the recorded exceptions were less than the allowed amount of exceptions. This means that the model overestimated the risk. Nonetheless, Ripple’s outcomes were accepted at 90% and 99% confidence levels.

[5] Example of LR calculation for the EURUSD at 90% VaR confidence level where the number of recorded exceptions (N) is 34. The number of trials (T) is 250. The probability of failure (p) is 0.1 (1 – 90%).
LR = (-2*LN(((1-0.1)^(250-34))*(0.1^34)))+(2*LN((((1-(34/250))^(250-34))*(34/250)^34))) = 3.27

4.6 Conclusion

The EWMA model proved to be the best model for all the selected currencies and cryptocurrencies, when compared to the realized volatility in the in-sample period. However, for the out-of-sample period, the EWMA appeared to be the optimal model for the EURUSD, GBPUSD and Bitcoin. Whereas the GARCH (p, q) was the best model for the CNYUSD and Ripple. When estimating the GARCH (1, 1) and GARCH (p, q) parameters for the Bitcoin and Ripple, the ARCH and GARCH parameters sum was equal to 1. This indicates the presence of a unit root in the variance whereby the GARCH models converge to the Integrated GARCH models with an unattainable long term volatility. This implies that irregular shifts in volatility occurred where the impact of past shocks is persistent.

Furthermore, when comparing the calculated volatilities of the currencies to their respective implied volatility, the results were mixed. The GARCH (1, 1), GARCH (6, 6) and EWMA were the optimal models for the EURUSD, CNYUSD and GBPUSD, respectively during the in-sample period. On the other hand, in the out-of-sample period, the EWMA was the best model for the EURUSD and CNYUSD, whereas the EGARCH (1, 1) proved to be the best model for the GBPUSD. Surprisingly, the advantage of the EGARCH model over the first generation of GARCH models did not secure its superiority. Notably, the EGARCH (1, 1) was one of the worst performer among the selected volatility models. This might be caused by the stationarity of the returns of the selected currencies and cryptocurrencies, whereby an existence of trend or seasonality in the returns does not exist.

The error statistics had smaller values in the out of sample period for all the selected currencies and cryptocurrencies, except for the CNYUSD. This reflects an increased accuracy in the volatility forecast between the selected market variables besides the Chinese Yuan, where the out-of-sample period resulted in a less favorable outcome.

As for the VaR, the Kupiec test had different results for each of the selected market variables. The results were accepted at all confidence levels for the EURUSD, GBPUSD and Bitcoin. However, the results were rejected at 90% and 95% confidence levels for the CNYUSD. Interestingly, for the Ripple the results were accepted at 90% and 99%

confidence levels, but they were rejected at 95% and 97.5% confidence levels, since the number of exceptions was less than the number of allowed exceptions. The model is overestimating the risk given the high volatility of the Ripple compared to the selected currencies; this means that our model is robust and significant to be used.

Chapter 5

Conclusions and Recommendations

5.1 Introduction

This thesis assessed and compared the predictive ability of the EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1) models on three currencies: EURUSD, CNYUSD and GBPUSD, and two cryptocurrencies, the Bitcoin and the Ripple. The studied period extended from March 01, 2016 to February 28, 2019. The in-sample period was March 01, 2016 through February 28, 2018 while the out-of-sample period covered March 01, 2018 to February 28, 2019. The volatility models' parameters were estimated during the in-sample period by maximizing the log-likelihood function, while respecting the assumptions of each model. The in-sample parameters were used to forecast the volatility related to the out-of-sample period. Accordingly, after calculating the volatility under each model for each of the selected market variables, the results were compared to the realized and implied volatilities to determine the optimal model for each currency and cryptocurrency in both periods by using three error metrics (RMSE, MAE and MAPE). When comparing the calculated volatilities to the realized volatility, the EWMA model was superior and outperformed the rest of the models for all of the selected currencies and cryptocurrencies during the in-sample period. However, for the out-of-sample period, the GARCH (p, q) was the optimal model for the CNYUSD and Ripple. Whereas, the EWMA proved to be the best model for the EURUSD, GBPUSD and Bitcoin. The calculated volatilities were then compared to the implied volatility for the selected fiat currencies only since the implied volatility data for cryptocurrencies is not available. The GARCH (1, 1), GARCH (6, 6) and EWMA were the optimal models for the EURUSD, CNYUSD and GBPUSD, respectively during the in-sample period, and the EWMA model was optimal for the EURUSD and CNYUSD in the out-of-sample period. As for the GBPUSD, the EGARCH (1, 1) was selected as the best model.

We also calculated the Value at Risk for each currency and cryptocurrency by incorporating the volatility update model into historical simulation. The best performing models in the out-of-sample period, which were selected by comparing the realized volatilities to the calculated volatilities for each of the selected market variables, were used to calculate VaR. 649 daily observations ranging from June 30, 2016 to February 28, 2019 were divided into 250 sub-samples, where each sub-sample included 400 observations. The parameters of the best selected model in the out-of-sample period were estimated twice. The parameters were estimated for a sample period of 100 days extending from February 13, 2018 till July 16, 2018. The second set of parameters was estimated for a sample period of 150 days extending from July 17, 2018 till February 28, 2019. Daily variances were calculated 399 times for each sub-sample, which resulted in a total of 498,750 (399*250*5) values of variances. Additionally, the models were evaluated using the back-testing methodology suggested by Kupiec. The back-test was applied on the period February 13, 2018 through February 28, 2019. Kupiec results were accepted for the EURUSD, GBPUSD and Bitcoin at all confidence levels. As for the CNYUSD, the results were rejected at 90% and 95% confidence levels. Ripple's results were only accepted at 90% and 99% confidence levels, since at 95% and 97.5% confidence levels the number of exceptions was less than the number of allowed exceptions.

5.2 Main Findings

The findings of this thesis are original, when compared to those of previous researches. Our research is the first to show the volatility and the behavior of cryptocurrencies versus fiat currencies, and in using the GARCH (p, q) model. We went beyond forecasting the volatility of fiat and cryptocurrencies by measuring the VaR. While, in most of the available literature the VaR was never considered. This study came up with four main conclusions.

The first conclusion relates to the forecasting and predictive ability of the selected volatility models, when applied on Bitcoin. This thesis shows that the EWMA model outperformed other volatility models in both contexts (in-sample and out-of-sample). This contradicts Naimy and Hayek (2018), who found that the EGARCH (1, 1) model outperformed the EWMA and GARCH (1, 1) models in both contexts. Furthermore, our findings also counter Bouri *et al.* (2017), who confirmed that the GARCH (1, 1) model was the most effective in forecasting the volatility of Bitcoin and other virtual currencies. Such contradictions may be related to the selected time period we opted to choose. Our research time horizon extended from March 01, 2016 through February 28, 2019, whereas Naimy and Hayek (2018) chose another period, which ranged from April 2013 through March 2016. As for the study of Bouri *et al. (2017),* the authors selected time frame ranged from July 2011 to December 2015. Another possible cause for such discrepancy might relate to the evolvement of cryptocurrencies behavior.

The second finding is related to the volatility asymmetry of the selected cryptocurrencies. Our research concludes that Bitcoin and Ripple exhibit an asymmetry in their volatility. In fact, good news have a larger effect than bad news. For instance, the values of the leverage coefficient for the Bitcoin and Ripple are 0.24 and 0.31, respectively. This is in line with Radovanov *et al.* (2018) results when they studied the Ripple. However, this contradicts Phillip *et al.* (2018) findings who revealed the presence of mild leverage effects on most of the cryptocurrency market, except for Ripple, where the latter had the weakest leverage effect.

The third observation relates to the behavior of Bitcoin when compared to fiat currencies. This thesis shows that Bitcoin and Ripple volatility is significantly higher than that of the studied fiat currencies (EURUSD, CNYUSD and GBPUSD). This confirms the findings of Cermak (2017) and Naimy and Hayek (2018), who concluded that cryptocurrencies' volatility is relatively higher than fiat currencies and other market variables such as the S&P 500 index and the Gold spot. Intriguingly, during the out-of-sample period, our results revealed that Bitcoin's optimal volatility model was the same as the EURUSD and GBPUSD. As for the Ripple, the GARCH (p, q) was the best performing model, which is similar to the Chinese Yuan. This highlights similarities in the volatility behavior between the studied fiat and cryptocurrencies.

Finally, our results oppose those of Stavroyiannis (2018), who stated that Bitcoin violates VaR and other risk measures. In this thesis, we assessed the VaR using volatility update into the historical simulation model, which is proposed by Hull and White (1998). Our findings showed that Bitcoin does not violate VaR in the studied period at all confidence levels. Two possible reasons might have caused the difference in the results. First, Stavroyiannis (2018) used a different method based on a simple calculation of the daily volatility to estimate VaR, while we used the adjusted historical simulation method through incorporating the optimal volatility model in order to enhance the performance of the historical simulation. Second, we used a different and recent time horizon (up to February 2019), whereas Stavroyiannis (2018) selected another time frame which ranged from July 2013 till July 2017.

5.3 Limitation of the Research

In this thesis, we only considered the EWMA, GARCH (1, 1), GARCH (p, q) and EGARCH (1, 1) models when estimating and forecasting the volatility of the selected fiat and cryptocurrencies. There are other models that could be used such as the IGARCH, GJR-GARCH and many other models. Also, when calculating VaR, we solely examined incorporating volatility update into the historical simulation model, while disregarding other models. For instance, the parametric approach (variance-covariance), Monte Carlo simulation and the basic historical simulation models are options that could have been used. We also failed to use the Extreme Value Theory (EVT), which can produce superior results for highly volatile markets.

Additionally, the selected sample size in our thesis is equivalent to 732 observations which corresponds to three years of data ranging from March 01, 2016 till February 28, 2019. As for the VaR, the back-test was applied on a sample size of 250 observations chosen from the out-of-sample period, where 399 return scenarios were generated for each of the 250 days. Altering the number of generated scenarios and the sample size, when calculating the volatility and VaR, might have led to different results.

Another limitation could be related to the selected virtual and fiat currencies. Our research focused solely on the EURUSD, CNYUSD, GBPUSD, Bitcoin and Ripple, while more currencies and cryptocurrencies could be considered.

Lastly, we only back-tested VaR results using the Kupiec test. Other tests might be applied such as the independence test suggested by Christoffersen, where consecutive and frequent exceptions are taken into consideration.

5.4 Managerial Implications

The results of our research are of utmost importance for decision makers, financial managers, and investors.

First, Bitcoin and generally the cryptocurrencies market cannot act as alternatives to fiat currencies at the moment. This is due to their volatile behavior being significantly different from the fiat currencies behavior.

Second, financial managers and investors need to be prudent when considering an investment in cryptocurrencies, given their high risk and unexpected and extremely volatile behavior.

Finally, market participants aiming at diversifying their portfolios or seeking a risky position could consider cryptocurrencies, given their unique behavior compared to other instruments.

5.5 Recommendations

Many complementary studies can be done based on our results. This involves including more econometric models such as the GJR-GARCH and IGARCH. Also, VaR was only examined using the non-parametric approach, however it could be beneficial to test the parametric approach and Monte Carlo simulation to estimate the VaR.

Also, extending our study's time period or selecting different sample sizes might produce complementary results. This is important to back test the accuracy of our results.

Finally, a dedicated research aiming at studying the impact of two major upcoming events on the cryptocurrencies market is needed to complement our study. In September 23, 2019, Bitcoin Futures contracts will be launched on the New York Stock Exchange (NYSE) where the payout will be in Bitcoin tokens. And in 2020, Facebook will launch a new cryptocurrency called "Libra. The latter will be backed up by US treasury securities and a basket of currencies.

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APPENDIX A

Table 1. GARCH (1, 2) through GARCH (1, 9) Estimated Parameters (CNYUSD)

GARCH	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(1, 7)	(1, 8)	(1, 9)
ω	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07
α1	0.163	0.202	0.202	0.202	0.248	0.248	0.248	0.248
β1	0.340	0.310	0.310	0.310	0.000	0.000	0.000	0.000
β2	0.348	0.000	0.000	0.000	0.285	0.285	0.285	0.285
β3		0.345	0.345	0.345	0.000	0.000	0.000	0.000
β4			0.000	0.000	0.321	0.321	0.321	0.321
β5				0.000	0.000	0.000	0.000	0.000
β6					0.012	0.012	0.012	0.012
β7						0.000	0.000	0.000
β8							0.000	0.000
β9								0.000
LLF	5532.20	5533.31	5533.31	5533.31	5536.65	5536.65	5536.65	5536.65

Table 2. GARCH (2, 1) through GARCH (2, 9) Estimated Parameters (CNYUSD)

GARCH	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	(2, 7)	(2, 8)	(2, 9)
ω	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07	7.59E-07
α1	0.134	0.164	0.176	0.176	0.176	0.217	0.217	0.217	0.217
α2	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.020	0.020
β1	0.715	0.337	0.355	0.355	0.355	0.268	0.268	0.288	0.288
β2		0.349	0.135	0.135	0.135	0.085	0.085	0.097	0.097
β3			0.185	0.185	0.185	0.130	0.130	0.132	0.132
β4				0.000	0.000	0.042	0.042	0.001	0.001
β5					0.000	0.000	0.000	0.000	0.000
β6						0.116	0.116	0.105	0.105
β7							0.000	0.000	0.000
β8								0.002	0.002
β9									0.000
LLF	5529.87	5532.20	5532.88	5532.88	5532.88	5533.24	5533.24	5533.29	5533.29

Table 3. GARCH (3, 1) through GARCH (3, 9) Estimated Parameters (CNYUSD)

GARCH	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(3, 7)	(3, 8)	(3, 9)
ω	7.31E-07	7.31E-07	7.31E-07	7.31E-07	7.31E-07	7.31E-07	7.31E-07	7.31E-07	7.31E-07
α1	0.115	0.150	0.170	0.155	0.155	0.155	0.155	0.155	0.155
α2	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.030	0.013	0.000	0.049	0.049	0.049	0.049	0.049	0.049
β1	0.708	0.377	0.314	0.336	0.336	0.336	0.336	0.336	0.336
β2		0.315	0.239	0.171	0.171	0.171	0.171	0.171	0.171
β3			0.129	0.000	0.000	0.000	0.000	0.000	0.000
β4				0.149	0.149	0.149	0.149	0.149	0.149
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.000	0.000	0.000
β8								0.000	0.000
β9									0.000
LLF	5530.92	5532.54	5532.75	5533.44	5533.44	5533.44	5533.44	5533.44	5533.44

Table 4. GARCH (4, 1) through GARCH (4, 9) Estimated Parameters (CNYUSD)

GARCH	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	(4, 7)	(4, 8)	(4, 9)
ω	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07
α1	0.113	0.120	0.153	0.153	0.153	0.182	0.186	0.190	0.190
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.000
α3	0.000	0.000	0.000	0.000	0.000	0.013	0.008	0.037	0.037
α4	0.049	0.039	0.049	0.049	0.049	0.100	0.095	0.076	0.076
β1	0.690	0.578	0.424	0.424	0.424	0.325	0.338	0.356	0.356
β2		0.114	0.000	0.000	0.000	0.007	0.003	0.000	0.000
β3			0.230	0.230	0.230	0.021	0.002	0.000	0.000
β4				0.000	0.000	0.005	0.000	0.000	0.000
β5					0.000	0.073	0.000	0.000	0.000
β6						0.144	0.111	0.111	0.111
β7							0.123	0.000	0.000
β8								0.102	0.102
β9									0.000
LLF	5532.43	5533.13	5534.51	5534.51	5534.51	5535.26	5535.63	5536.74	5536.74

Table 5. GARCH (5, 1) through GARCH (5, 9) Estimated Parameters (CNYUSD)

GARCH	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(5, 7)	(5, 8)	(5, 9)
ω	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	9.12E-07	9.12E-07	9.12E-07	9.12E-07
α1	0.114	0.132	0.148	0.151	0.151	0.224	0.224	0.224	0.224
α2	0.000	0.000	0.000	0.000	0.000	0.004	0.004	0.004	0.004
α3	0.000	0.000	0.000	0.008	0.008	0.063	0.063	0.063	0.063
α4	0.028	0.001	0.052	0.021	0.021	0.038	0.038	0.038	0.038
α5	0.024	0.046	0.000	0.035	0.035	0.110	0.110	0.110	0.110
β1	0.687	0.415	0.422	0.316	0.316	0.000	0.000	0.000	0.000
β2		0.259	0.030	0.146	0.146	0.000	0.000	0.000	0.000
β3			0.204	0.000	0.000	0.000	0.000	0.000	0.000
β4				0.178	0.178	0.000	0.000	0.000	0.000
β5					0.000	0.000	0.000	0.000	0.000
β6						0.410	0.410	0.410	0.410
β7							0.000	0.000	0.000
β8								0.000	0.000
β9									0.000
LLF	5532.65	5533.99	5534.47	5534.56	5534.56	5540.94	5540.94	5540.94	5540.94

Table 6. GARCH (6, 1) through GARCH (6, 9) Estimated Parameters (CNYUSD)

GARCH	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	(6, 7)	(6, 8)	(6, 9)
ω	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07	7.34E-07
α1	0.114	0.112	0.112	0.112	0.112	0.122	0.160	0.160	0.160
α2	0.000	0.000	0.000	0.000	0.000	0.025	0.000	0.000	0.000
α3	0.000	0.000	0.000	0.000	0.000	0.034	0.021	0.021	0.021
α4	0.028	0.024	0.024	0.024	0.024	0.023	0.068	0.068	0.068
α5	0.024	0.025	0.025	0.025	0.025	0.017	0.000	0.000	0.000
α6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.687	0.586	0.586	0.586	0.586	0.498	0.447	0.447	0.447
β2		0.105	0.105	0.105	0.105	0.013	0.001	0.001	0.001
β3			0.000	0.000	0.000	0.003	0.001	0.001	0.001
β4				0.000	0.000	0.001	0.000	0.000	0.000
β5					0.000	0.021	0.000	0.000	0.000
β6						0.102	0.040	0.040	0.040
β7							0.122	0.122	0.122
β8								0.000	0.000
β9									0.000
LLF	5532.65	5533.33	5533.33	5533.33	5533.33	5534.52	5536.22	5536.22	5536.22

Table 7. GARCH (1, 2) through GARCH (1, 9) Estimated Parameters (GBPUSD)

GARCH	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(1, 7)	(1, 8)	(1, 9)
ω	5.74E-06	7.74E-06	7.74E-06	6.96E-06	6.96E-06	6.96E-06	6.96E-06	6.96E-06
α1	0.197	0.280	0.280	0.271	0.271	0.271	0.284	0.284
β1	0.378	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2	0.359	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β3		0.627	0.627	0.622	0.622	0.622	0.610	0.610
β4			0.000	0.000	0.000	0.000	0.000	0.000
β5				0.022	0.022	0.022	0.010	0.010
β6					0.000	0.000	0.000	0.000
β7						0.000	0.005	0.005
β8							0.009	0.009
β9								0.000
LLF	4331.69	4368.16	4368.16	4369.13	4369.13	4369.13	4369.28	4369.28

Table 8. GARCH (2, 1) through GARCH (2, 9) Estimated Parameters (GBPUSD)

GARCH	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	(2, 7)	(2, 8)	(2, 9)
ω	7.66E-06	6.72E-06	7.76E-06	7.76E-06	7.76E-06	7.76E-06	7.76E-06	6.98E-06	6.98E-06
α1	0.187	0.164	0.284	0.284	0.284	0.284	0.284	0.294	0.294
α2	0.000	0.041	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.715	0.352	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.349	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β3			0.620	0.620	0.620	0.620	0.620	0.618	0.618
β4				0.000	0.000	0.000	0.000	0.000	0.000
β5					0.000	0.000	0.000	0.013	0.013
β6						0.000	0.000	0.000	0.000
β7							0.000	0.000	0.000
β8								0.003	0.003
β9									0.000
LLF	4327.97	4330.93	4368.13	4368.13	4368.13	4368.13	4368.13	4369.10	4369.10

Table 9. GARCH (3, 1) through GARCH (3, 9) Estimated Parameters (GBPUSD)

GARCH	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(3, 7)	(3, 8)	(3, 9)
ω	4.18E-06	4.18E-06	9.06E-06	9.06E-06	9.06E-06	9.06E-06	9.06E-06	6.98E-06	6.98E-06
α1	0.034	0.034	0.277	0.277	0.277	0.277	0.277	0.294	0.294
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.152	0.152	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.780	0.780	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β3			0.598	0.598	0.598	0.598	0.598	0.618	0.618
β4				0.000	0.000	0.000	0.000	0.000	0.000
β5					0.000	0.000	0.000	0.013	0.013
β6						0.000	0.000	0.000	0.000
β7							0.000	0.000	0.000
β8								0.003	0.003
β9									0.000
LLF	4333.97	4333.97	4367.93	4367.93	4367.93	4367.93	4367.93	4369.10	4369.10

Table 10. GARCH (4, 1) through GARCH (4, 9) Estimated Parameters (GBPUSD)

GARCH	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	(4, 7)	(4, 8)	(4, 9)
ω	8.35E-06	9.53E-06	1.27E-05	1.27E-05	1.27E-05	1.27E-05	1.27E-05	1.51E-05	1.51E-05
α1	0.029	0.045	0.088	0.088	0.088	0.088	0.088	0.086	0.086
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α4	0.178	0.271	0.310	0.310	0.310	0.310	0.310	0.373	0.373
β1	0.662	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.556	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β3			0.435	0.435	0.435	0.435	0.435	0.000	0.000
β4				0.000	0.000	0.000	0.000	0.000	0.000
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.270	0.270
β7							0.000	0.000	0.000
β8								0.053	0.053
β9									0.000
LLF	4357.44	4371.79	4377.78	4377.78	4377.78	4377.78	4377.78	4389.22	4389.22

Table 11. GARCH (5, 1) through GARCH (5, 9) Estimated Parameters (GBPUSD)

GARCH	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(5, 7)	(5, 8)	(5, 9)
ω	9.76E-06	9.53E-06	1.19E-05	1.19E-05	1.10E-05	1.52E-05	1.52E-05	1.52E-05	1.52E-05
α1	0.037	0.045	0.092	0.092	0.113	0.087	0.087	0.087	0.087
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α4	0.201	0.271	0.270	0.270	0.249	0.366	0.366	0.366	0.366
α5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.607	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.556	0.019	0.019	0.000	0.000	0.000	0.000	0.000
β3			0.450	0.450	0.456	0.000	0.000	0.000	0.000
β4				0.000	0.000	0.000	0.000	0.000	0.000
β5					0.028	0.000	0.000	0.000	0.000
β6						0.324	0.324	0.324	0.324
β7							0.000	0.000	0.000
β8								0.000	0.000
β9									0.000
LLF	4358.02	4371.79	4378.55	4378.55	4379.40	4386.38	4386.38	4386.38	4386.38

Table 12. GARCH (6, 1) through GARCH (6, 9) Estimated Parameters (GBPUSD)

GARCH	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	(6, 7)	(6, 8)	(6, 9)
ω	9.76E-06	9.53E-06	1.19E-05	1.19E-05	1.10E-05	1.52E-05	1.35E-05	1.44E-05	1.44E-05
α1	0.037	0.045	0.092	0.092	0.113	0.087	0.098	0.090	0.090
α2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α4	0.201	0.271	0.270	0.270	0.249	0.366	0.345	0.373	0.373
α5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.607	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.556	0.019	0.019	0.000	0.000	0.000	0.000	0.000
β3			0.450	0.450	0.456	0.000	0.000	0.000	0.000
β4				0.000	0.000	0.000	0.000	0.000	0.000
β5					0.028	0.000	0.000	0.000	0.000
β6						0.324	0.341	0.277	0.277
β7							0.027	0.000	0.000
β8								0.058	0.058
β9									0.000
LLF	4358.02	4371.79	4378.55	4378.55	4379.40	4386.38	4386.52	4389.11	4389.11

Table 13. GARCH (1, 2) through GARCH (1, 9) Estimated Parameters (Ripple)

GARCH	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(1, 5)**	**(1, 6)**	**(1, 7)**	**(1, 8)**	**(1, 9)**
ω	2.22E-04	2.22E-04	3.69E-04	3.69E-04	3.69E-04	2.88E-04	2.97E-04	2.97E-04
α1	0.236	0.236	0.378	0.378	0.378	0.318	0.328	0.328
β1	0.764	0.764	0.173	0.173	0.173	0.584	0.567	0.567
β2	0.000	0.000	0.156	0.156	0.156	0.000	0.000	0.000
β3		0.000	0.000	0.000	0.000	0.000	0.000	0.000
β4			0.292	0.292	0.292	0.000	0.000	0.000
β5				0.000	0.000	0.000	0.000	0.000
β6					0.000	0.000	0.000	0.000
β7						0.098	0.063	0.063
β8							0.042	0.042
β9								0.000
LLF	2139.66	2139.66	2146.34	2146.34	2146.34	2149.60	2149.91	2149.91

Table 14. GARCH (2, 1) through GARCH (2, 9) Estimated Parameters (Ripple)

GARCH	**(2, 1)**	**(2, 2)**	**(2, 3)**	**(2, 4)**	**(2, 5)**	**(2, 6)**	**(2, 7)**	**(2, 8)**	**(2, 9)**
ω	2.22E-04	3.64E-04	3.48E-04	4.58E-04	4.12E-04	4.12E-04	4.00E-04	3.82E-04	4.98E-04
α1	0.236	0.237	0.248	0.298	0.295	0.295	0.312	0.320	0.452
α2	0.000	0.148	0.129	0.167	0.140	0.140	0.142	0.119	0.075
β1	0.764	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.615	0.566	0.289	0.378	0.378	0.411	0.373	0.182
β3			0.056	0.000	0.000	0.000	0.000	0.000	0.000
β4				0.246	0.138	0.138	0.000	0.006	0.079
β5					0.048	0.048	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.135	0.075	0.000
β8								0.107	0.121
β9									0.091
LLF	2139.66	2147.84	2148.36	2149.77	2150.07	2150.07	2154.85	2155.84	2158.08

Table 15. GARCH (3, 1) through GARCH (3, 9) Estimated Parameters (Ripple)

GARCH	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(3, 7)	(3, 8)	(3, 9)
ω	2.2E-04	3.6E-04	3.5E-04	5.1E-04	5.1E-04	5.1E-04	5.4E-04	5.7E-04	5.7E-04
α1	0.236	0.239	0.248	0.261	0.261	0.261	0.292	0.326	0.375
α2	0.000	0.142	0.129	0.169	0.169	0.169	0.183	0.165	0.119
α3	0.000	0.000	0.000	0.110	0.110	0.110	0.122	0.126	0.128
β1	0.764	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.619	0.567	0.000	0.000	0.000	0.000	0.000	0.000
β3			0.057	0.118	0.118	0.118	0.000	0.000	0.000
β4				0.342	0.342	0.342	0.238	0.186	0.139
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.165	0.112	0.024
β8								0.084	0.102
β9									0.112
LLF	2139.66	2147.86	2148.36	2154.84	2154.84	2154.84	2160.49	2160.93	2162.13

Table 16. GARCH (4, 1) through GARCH (4, 9) Estimated Parameters (Ripple)

GARCH	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	(4, 7)	(4, 8)	(4, 9)
ω	2.23E-04	3.25E-04	3.48E-04	5.09E-04	5.09E-04	5.09E-04	5.41E-04	5.71E-04	5.70E-04
α1	0.236	0.248	0.248	0.261	0.261	0.261	0.292	0.326	0.375
α2	0.000	0.103	0.129	0.168	0.168	0.168	0.183	0.165	0.119
α3	0.000	0.000	0.000	0.110	0.110	0.110	0.122	0.126	0.128
α4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β1	0.764	0.142	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.507	0.566	0.000	0.000	0.000	0.000	0.000	0.000
β3			0.057	0.118	0.118	0.118	0.000	0.000	0.000
β4				0.343	0.343	0.343	0.238	0.186	0.139
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.165	0.113	0.024
β8								0.084	0.102
β9									0.112
LLF	2139.66	2146.87	2148.36	2154.84	2154.84	2154.84	2160.49	2160.93	2162.13

Table 17. GARCH (5, 1) through GARCH (5, 9) Estimated Parameters (Ripple)

GARCH	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(5, 7)	(5, 8)	(5, 9)
ω	2.23E-04	3.62E-04	3.48E-04	3.48E-04	6.01E-04	6.01E-04	5.60E-04	7.04E-04	6.26E-04
α1	0.236	0.239	0.248	0.248	0.201	0.201	0.211	0.257	0.278
α2	0.000	0.142	0.129	0.129	0.222	0.222	0.182	0.217	0.195
α3	0.000	0.000	0.000	0.000	0.118	0.118	0.142	0.176	0.157
α4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α5	0.000	0.000	0.000	0.000	0.094	0.094	0.121	0.105	0.088
β1	0.764	0.000	0.000	0.000	0.000	0.000	0.085	0.000	0.000
β2		0.619	0.566	0.566	0.125	0.125	0.000	0.000	0.000
β3			0.057	0.057	0.000	0.000	0.000	0.000	0.000
β4				0.000	0.000	0.000	0.000	0.000	0.000
β5					0.240	0.240	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.259	0.120	0.075
β8								0.125	0.066
β9									0.141
LLF	2139.66	2147.86	2148.36	2148.36	2153.52	2153.52	2166.91	2168.92	2171.53

Table 18. GARCH (6, 1) through GARCH (6, 9) Estimated Parameters (Ripple)

GARCH	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	(6, 7)	(6, 8)	(6, 9)
ω	2.23E-04	4.89E-04	3.48E-04	5.32E-04	5.32E-04	5.32E-04	6.03E-04	7.04E-04	6.26E-04
α1	0.236	0.244	0.248	0.238	0.238	0.238	0.198	0.257	0.278
α2	0.000	0.196	0.129	0.169	0.169	0.169	0.196	0.217	0.195
α3	0.000	0.050	0.000	0.118	0.118	0.118	0.164	0.176	0.157
α4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α5	0.000	0.000	0.000	0.039	0.039	0.039	0.118	0.105	0.088
α6	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.000	0.000
β1	0.764	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
β2		0.510	0.567	0.000	0.000	0.000	0.000	0.000	0.000
β3			0.057	0.119	0.119	0.119	0.000	0.000	0.000
β4				0.317	0.317	0.317	0.000	0.000	0.000
β5					0.000	0.000	0.000	0.000	0.000
β6						0.000	0.000	0.000	0.000
β7							0.294	0.120	0.075
β8								0.125	0.066
β9									0.141
LLF	2139.66	2145.56	2148.36	2155.21	2155.21	2155.21	2167.83	2168.92	2171.53

APPENDIX B

Table 1. Error Statistics of GARCH (p, q) vs. the Realized Volatility In-Sample (CNYUSD)

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.00834	36	0.00686	47	0.00277	47
GARCH (1, 3)	0.00858	39	0.00685	44	0.00273	39
GARCH (1, 4)	0.00858	40	0.00685	45	0.00273	40
GARCH (1, 5)	0.00858	41	0.00685	46	0.00273	41
GARCH (1, 6)	0.00923	50	0.00722	50	0.00279	49
GARCH (1, 7)	0.00923	51	0.00722	51	0.00279	50
GARCH (1, 8)	0.00923	52	0.00722	52	0.00279	51
GARCH (1, 9)	0.00923	53	0.00722	53	0.00279	52
GARCH (2, 1)	0.00848	38	0.00704	49	0.00288	53
GARCH (2, 2)	0.00836	37	0.00687	48	0.00278	48
GARCH (2, 3)	0.00833	33	0.00678	36	0.00273	42
GARCH (2, 4)	0.00833	34	0.00678	37	0.00273	43
GARCH (2, 5)	0.00833	35	0.00678	38	0.00273	44
GARCH (2, 6)	0.00859	42	0.00681	40	0.00270	36
GARCH (2, 7)	0.00859	43	0.00681	41	0.00270	37
GARCH (2, 8)	0.00871	44	0.00683	42	0.00269	34
GARCH (2, 9)	0.00871	45	0.00683	43	0.00269	35
GARCH (3, 1)	0.00821	27	0.00680	39	0.00277	46
GARCH (3, 2)	0.00820	26	0.00676	35	0.00273	45
GARCH (3, 3)	0.00826	29	0.00675	34	0.00272	38
GARCH (3, 4)	0.00811	20	0.00663	24	0.00265	20
GARCH (3, 5)	0.00811	21	0.00663	25	0.00265	21
GARCH (3, 6)	0.00811	22	0.00663	26	0.00265	22
GARCH (3, 7)	0.00811	23	0.00663	27	0.00265	23
GARCH (3, 8)	0.00811	24	0.00663	28	0.00265	24
GARCH (3, 9)	0.00811	25	0.00663	29	0.00265	25
GARCH (4, 1)	0.00800	15	0.00660	23	0.00268	33
GARCH (4, 2)	0.00791	11	0.00656	17	0.00267	32
GARCH (4, 3)	0.00807	17	0.00657	18	0.00263	16
GARCH (4, 4)	0.00807	18	0.00657	19	0.00263	17
GARCH (4, 5)	0.00807	19	0.00657	20	0.00263	18
GARCH (4, 6)	0.00824	28	0.00651	14	0.00253	**1**
GARCH (4, 7)	0.00826	30	0.00655	16	0.00255	2

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (4, 8)	0.00832	31	0.00658	21	0.00255	3
GARCH (4, 9)	0.00832	32	0.00658	22	0.00255	4
GARCH (5, 1)	0.00790	9	0.00651	12	0.00265	26
GARCH (5, 2)	0.00784	8	0.00650	11	0.00263	19
GARCH (5, 3)	0.00802	16	0.00655	15	0.00262	15
GARCH (5, 4)	0.00781	5	0.00639	**1**	0.00255	8
GARCH (5, 5)	0.00781	6	0.00639	2	0.00255	9
GARCH (5, 6)	0.00877	46	0.00675	30	0.00256	10
GARCH (5, 7)	0.00877	47	0.00675	31	0.00256	11
GARCH (5, 8)	0.00877	48	0.00675	32	0.00256	12
GARCH (5, 9)	0.00877	49	0.00675	33	0.00256	13
GARCH (6, 1)	0.00790	10	0.00651	13	0.00265	27
GARCH (6, 2)	0.00779	**1**	0.00650	7	0.00266	28
GARCH (6, 3)	0.00779	2	0.00650	8	0.00266	29
GARCH (6, 4)	0.00779	3	0.00650	9	0.00266	30
GARCH (6, 5)	0.00779	4	0.00650	10	0.00266	31
GARCH (6, 6)	0.00781	7	0.00645	3	0.00257	14
GARCH (6, 7)	0.00799	12	0.00647	4	0.00255	5
GARCH (6, 8)	0.00799	13	0.00647	5	0.00255	6
GARCH (6, 9)	0.00799	14	0.00647	6	0.00255	7

Table 2. Error Statistics of GARCH (p, q) vs. the Implied Volatility In-Sample (CNYUSD)

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.01159	26	0.00940	29	0.00208	22
GARCH (1, 3)	0.01172	37	0.00954	41	0.00211	43
GARCH (1, 4)	0.01172	38	0.00954	42	0.00211	44
GARCH (1, 5)	0.01172	39	0.00954	43	0.00211	45
GARCH (1, 6)	0.01229	50	0.00990	50	0.00219	46
GARCH (1, 7)	0.01229	51	0.00990	51	0.00219	47
GARCH (1, 8)	0.01229	52	0.00990	52	0.00219	48
GARCH (1, 9)	0.01229	53	0.00990	53	0.00219	49
GARCH (2, 1)	0.01148	10	0.00932	13	0.00207	13
GARCH (2, 2)	0.01158	25	0.00939	27	0.00208	20
GARCH (2, 3)	0.01160	28	0.00944	37	0.00209	33
GARCH (2, 4)	0.01160	29	0.00944	38	0.00209	34

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (2, 5)	0.01160	27	0.00944	36	0.00209	32
GARCH (2, 6)	0.01179	44	0.00957	44	0.00211	39
GARCH (2, 7)	0.01179	45	0.00957	45	0.00211	40
GARCH (2, 8)	0.01177	42	0.00954	39	0.00211	41
GARCH (2, 9)	0.01177	43	0.00954	40	0.00211	42
GARCH (3, 1)	0.01147	8	0.00930	12	0.00206	12
GARCH (3, 2)	0.01155	19	0.00936	18	0.00207	16
GARCH (3, 3)	0.01158	24	0.00940	28	0.00208	21
GARCH (3, 4)	0.01154	13	0.00930	2	0.00206	6
GARCH (3, 5)	0.01154	14	0.00930	3	0.00206	7
GARCH (3, 6)	0.01154	15	0.00930	4	0.00206	8
GARCH (3, 7)	0.01154	16	0.00930	5	0.00206	9
GARCH (3, 8)	0.01154	17	0.00930	6	0.00206	10
GARCH (3, 9)	0.01154	18	0.00930	7	0.00206	11
GARCH (4, 1)	0.01147	9	0.00933	16	0.00207	18
GARCH (4, 2)	0.01149	11	0.00937	19	0.00208	19
GARCH (4, 3)	0.01157	21	0.00939	24	0.00209	27
GARCH (4, 4)	0.01157	22	0.00939	25	0.00209	28
GARCH (4, 5)	0.01157	23	0.00939	26	0.00209	29
GARCH (4, 6)	0.01170	35	0.00943	35	0.00211	38
GARCH (4, 7)	0.01171	36	0.00941	30	0.00210	35
GARCH (4, 8)	0.01173	40	0.00943	31	0.00210	36
GARCH (4, 9)	0.01173	41	0.00943	32	0.00210	37
GARCH (5, 1)	0.01146	6	0.00933	14	0.00207	14
GARCH (5, 2)	0.01151	12	0.00936	17	0.00207	17
GARCH (5, 3)	0.01156	20	0.00939	23	0.00208	23
GARCH (5, 4)	0.01162	30	0.00943	33	0.00209	30
GARCH (5, 5)	0.01162	31	0.00943	34	0.00209	31
GARCH (5, 6)	0.01225	46	0.00986	46	0.00221	50
GARCH (5, 7)	0.01225	47	0.00986	47	0.00221	51
GARCH (5, 8)	0.01225	48	0.00986	48	0.00221	52
GARCH (5, 9)	0.01225	49	0.00986	49	0.00221	53
GARCH (6, 1)	0.01146	7	0.00933	15	0.00207	15
GARCH (6, 2)	0.01141	**1**	0.00930	8	0.00206	2
GARCH (6, 3)	0.01141	2	0.00930	9	0.00206	3
GARCH (6, 4)	0.01141	3	0.00930	10	0.00206	4
GARCH (6, 5)	0.01141	4	0.00930	11	0.00206	5
GARCH (6, 6)	0.01144	5	0.00920	**1**	0.00204	**1**
GARCH (6, 7)	0.01163	32	0.00938	20	0.00209	24

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (6, 8)	0.01163	33	0.00938	21	0.00209	25
GARCH (6, 9)	0.01163	34	0.00938	22	0.00209	26

Table 3. Error Statistics of GARCH (p, q) vs. the Realized Volatility In-Sample (GBPUSD)

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.03905	3	0.02553	10	0.00271	36
GARCH (1, 3)	0.04544	28	0.02686	41	0.00274	46
GARCH (1, 4)	0.04544	29	0.02686	42	0.00274	47
GARCH (1, 5)	0.04330	11	0.02560	11	0.00263	12
GARCH (1, 6)	0.04330	12	0.02560	12	0.00263	13
GARCH (1, 7)	0.04330	13	0.02560	13	0.00263	14
GARCH (1, 8)	0.04408	17	0.02568	16	0.00263	10
GARCH (1, 9)	0.04408	18	0.02568	17	0.00263	11
GARCH (2, 1)	0.04333	14	0.02876	53	0.00302	53
GARCH (2, 2)	0.03796	**1**	0.02491	2	0.00261	4
GARCH (2, 3)	0.04553	34	0.02666	36	0.00271	31
GARCH (2, 4)	0.04553	33	0.02666	38	0.00271	33
GARCH (2, 5)	0.04553	30	0.02666	40	0.00271	35
GARCH (2, 6)	0.04553	31	0.02666	37	0.00271	32
GARCH (2, 7)	0.04553	32	0.02666	39	0.00271	34
GARCH (2, 8)	0.04557	35	0.02660	32	0.00272	42
GARCH (2, 9)	0.04557	36	0.02660	33	0.00272	43
GARCH (3, 1)	0.04026	4	0.02562	14	0.00268	24
GARCH (3, 2)	0.04026	5	0.02562	15	0.00268	25
GARCH (3, 3)	0.04472	24	0.02655	27	0.00272	37
GARCH (3, 4)	0.04472	25	0.02655	28	0.00272	38
GARCH (3, 5)	0.04472	26	0.02655	29	0.00272	39
GARCH (3, 6)	0.04472	27	0.02655	30	0.00272	40
GARCH (3, 7)	0.04472	23	0.02655	31	0.00272	41
GARCH (3, 8)	0.04557	37	0.02660	34	0.00272	44
GARCH (3, 9)	0.04557	38	0.02660	35	0.00272	45
GARCH (4, 1)	0.03890	2	0.02484	**1**	0.00259	**1**
GARCH (4, 2)	0.04216	8	0.02539	5	0.00262	5
GARCH (4, 3)	0.04731	47	0.02720	51	0.00276	51
GARCH (4, 4)	0.04731	48	0.02720	52	0.00276	52
GARCH (4, 5)	0.04731	44	0.02720	48	0.00276	48

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (4, 6)	0.04731	45	0.02720	49	0.00276	49
GARCH (4, 7)	0.04731	46	0.02720	50	0.00276	50
GARCH (4, 8)	0.04645	42	0.02624	24	0.00266	22
GARCH (4, 9)	0.04645	43	0.02624	25	0.00266	23
GARCH (5, 1)	0.04063	6	0.02547	8	0.00263	8
GARCH (5, 2)	0.04216	9	0.02539	6	0.00262	6
GARCH (5, 3)	0.04417	21	0.02574	20	0.00263	15
GARCH (5, 4)	0.04417	22	0.02574	21	0.00263	16
GARCH (5, 5)	0.04355	16	0.02533	4	0.00259	2
GARCH (5, 6)	0.04765	53	0.02705	44	0.00270	26
GARCH (5, 7)	0.04765	52	0.02705	45	0.00270	28
GARCH (5, 8)	0.04765	51	0.02705	46	0.00270	29
GARCH (5, 9)	0.04765	50	0.02705	47	0.00270	30
GARCH (6, 1)	0.04063	7	0.02547	9	0.00263	9
GARCH (6, 2)	0.04216	10	0.02539	7	0.00262	7
GARCH (6, 3)	0.04417	19	0.02574	18	0.00263	17
GARCH (6, 4)	0.04417	20	0.02574	19	0.00263	18
GARCH (6, 5)	0.04355	15	0.02533	3	0.00259	3
GARCH (6, 6)	0.04765	49	0.02705	43	0.00270	27
GARCH (6, 7)	0.04613	39	0.02638	26	0.00266	19
GARCH (6, 8)	0.04639	40	0.02620	22	0.00266	20
GARCH (6, 9)	0.04639	41	0.02620	23	0.00266	21

Table 4. Error Statistics of GARCH (p, q) vs. the Implied Volatility In-Sample (GBPUSD)

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.05075	6	0.03104	51	0.00303	50
GARCH (1, 3)	0.05420	31	0.03016	44	0.00293	44
GARCH (1, 4)	0.05420	32	0.03016	45	0.00293	45
GARCH (1, 5)	0.05333	18	0.02980	29	0.00288	29
GARCH (1, 6)	0.05333	19	0.02980	30	0.00288	30
GARCH (1, 7)	0.05333	20	0.02980	31	0.00288	31
GARCH (1, 8)	0.05404	24	0.02987	37	0.00288	32
GARCH (1, 9)	0.05404	25	0.02987	38	0.00288	33
GARCH (2, 1)	0.05038	5	0.03069	50	0.00308	51
GARCH (2, 2)	0.04844	**1**	0.02902	19	0.00283	19

GBPUSD	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (2, 3)	0.05415	26	0.02983	32	0.00289	34
GARCH (2, 4)	0.05415	29	0.02983	35	0.00289	36
GARCH (2, 5)	0.05415	30	0.02983	36	0.00289	38
GARCH (2, 6)	0.05415	27	0.02983	33	0.00289	35
GARCH (2, 7)	0.05415	28	0.02983	34	0.00289	37
GARCH (2, 8)	0.05544	45	0.03065	46	0.00296	46
GARCH (2, 9)	0.05544	46	0.03065	47	0.00296	47
GARCH (3, 1)	0.05510	41	0.03235	52	0.00313	52
GARCH (3, 2)	0.05510	42	0.03235	53	0.00313	53
GARCH (3, 3)	0.05233	7	0.02922	24	0.00286	24
GARCH (3, 4)	0.05233	8	0.02922	25	0.00286	25
GARCH (3, 5)	0.05233	9	0.02922	26	0.00286	26
GARCH (3, 6)	0.05233	10	0.02922	27	0.00286	27
GARCH (3, 7)	0.05233	11	0.02922	28	0.00286	28
GARCH (3, 8)	0.05544	47	0.03065	48	0.00296	48
GARCH (3, 9)	0.05544	48	0.03065	49	0.00296	49
GARCH (4, 1)	0.04877	2	0.02781	3	0.00274	3
GARCH (4, 2)	0.05334	21	0.02915	20	0.00283	20
GARCH (4, 3)	0.05557	52	0.02992	42	0.00293	42
GARCH (4, 4)	0.05557	53	0.02992	43	0.00293	43
GARCH (4, 5)	0.05557	49	0.02992	39	0.00293	39
GARCH (4, 6)	0.05557	50	0.02992	40	0.00293	40
GARCH (4, 7)	0.05557	51	0.02992	41	0.00293	41
GARCH (4, 8)	0.05475	33	0.02858	4	0.00280	4
GARCH (4, 9)	0.05475	34	0.02858	5	0.00280	5
GARCH (5, 1)	0.04925	3	0.02757	**1**	0.00272	**1**
GARCH (5, 2)	0.05334	22	0.02915	21	0.00283	21
GARCH (5, 3)	0.05319	14	0.02877	11	0.00281	13
GARCH (5, 4)	0.05319	15	0.02877	12	0.00281	14
GARCH (5, 5)	0.05319	12	0.02890	17	0.00281	6
GARCH (5, 6)	0.05508	36	0.02869	6	0.00281	8
GARCH (5, 7)	0.05508	38	0.02869	8	0.00281	10
GARCH (5, 8)	0.05508	39	0.02869	9	0.00281	11
GARCH (5, 9)	0.05508	40	0.02869	10	0.00281	12
GARCH (6, 1)	0.04925	4	0.02757	2	0.00272	2
GARCH (6, 2)	0.05334	23	0.02915	22	0.00283	22
GARCH (6, 3)	0.05319	16	0.02877	13	0.00281	15
GARCH (6, 4)	0.05319	17	0.02877	14	0.00281	16
GARCH (6, 5)	0.05319	13	0.02890	18	0.00281	7

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (6, 6)	0.05508	37	0.02869	7	0.00281	9
GARCH (6, 7)	0.05488	35	0.02920	23	0.00285	23
GARCH (6, 8)	0.05513	43	0.02885	15	0.00282	17
GARCH (6, 9)	0.05513	44	0.02885	16	0.00282	18

Table 5. Error Statistics of GARCH (p, q) vs. the Realized Volatility In-Sample (Ripple)

Ripple	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	1.55504	46	0.94302	47	0.13775	29
GARCH (1, 3)	1.55504	47	0.94302	48	0.13775	30
GARCH (1, 4)	1.52793	**1**	0.92197	10	0.13836	49
GARCH (1, 5)	1.52793	2	0.92197	11	0.13836	50
GARCH (1, 6)	1.52793	3	0.92197	12	0.13836	51
GARCH (1, 7)	1.54574	33	0.93471	41	0.13578	9
GARCH (1, 8)	1.54371	28	0.93384	33	0.13566	7
GARCH (1, 9)	1.54371	29	0.93384	34	0.13566	8
GARCH (2, 1)	1.55505	48	0.94305	49	0.13776	31
GARCH (2, 2)	1.55150	42	0.93541	44	0.13821	48
GARCH (2, 3)	1.55037	36	0.93384	35	0.13800	38
GARCH (2, 4)	1.52972	4	0.92781	28	0.13791	37
GARCH (2, 5)	1.53816	17	0.92965	31	0.13817	46
GARCH (2, 6)	1.53816	18	0.92965	32	0.13817	47
GARCH (2, 7)	1.54758	35	0.92827	29	0.13623	11
GARCH (2, 8)	1.54165	25	0.92889	30	0.13655	17
GARCH (2, 9)	1.53685	14	0.91893	3	0.13605	10
GARCH (3, 1)	1.55510	50	0.94321	51	0.13781	33
GARCH (3, 2)	1.55154	43	0.93526	43	0.13838	52
GARCH (3, 3)	1.55038	37	0.93385	36	0.13802	39
GARCH (3, 4)	1.53098	8	0.92453	23	0.13669	23
GARCH (3, 5)	1.53098	9	0.92453	24	0.13669	24
GARCH (3, 6)	1.53098	10	0.92453	25	0.13669	25
GARCH (3, 7)	1.53808	16	0.92288	16	0.13668	19
GARCH (3, 8)	1.53901	19	0.92270	13	0.13649	15
GARCH (3, 9)	1.54054	23	0.92046	8	0.13685	26
GARCH (4, 1)	1.55511	52	0.94323	53	0.13782	36
GARCH (4, 2)	1.55191	45	0.93514	42	0.13781	35
GARCH (4, 3)	1.55042	41	0.93399	40	0.13805	45

Ripple	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (4, 4)	1.53097	5	0.92451	20	0.13669	20
GARCH (4, 5)	1.53097	6	0.92451	21	0.13669	21
GARCH (4, 6)	1.53097	7	0.92451	22	0.13669	22
GARCH (4, 7)	1.53805	15	0.92284	15	0.13667	18
GARCH (4, 8)	1.53903	20	0.92271	14	0.13649	16
GARCH (4, 9)	1.54054	24	0.92046	7	0.13685	27
GARCH (5, 1)	1.55508	49	0.94310	50	0.13778	32
GARCH (5, 2)	1.55163	44	0.93555	45	0.13846	53
GARCH (5, 3)	1.55041	38	0.93396	38	0.13804	43
GARCH (5, 4)	1.55041	39	0.93396	39	0.13804	44
GARCH (5, 5)	1.54277	26	0.92756	26	0.13803	40
GARCH (5, 6)	1.54277	27	0.92756	27	0.13803	41
GARCH (5, 7)	1.54532	32	0.92045	6	0.13406	**1**
GARCH (5, 8)	1.54442	30	0.91927	5	0.13438	3
GARCH (5, 9)	1.53934	22	0.91565	**1**	0.13460	4
GARCH (6, 1)	1.55510	51	0.94321	52	0.13781	34
GARCH (6, 2)	1.55760	53	0.93602	46	0.13752	28
GARCH (6, 3)	1.55042	40	0.93389	37	0.13803	42
GARCH (6, 4)	1.53209	11	0.92384	17	0.13640	12
GARCH (6, 5)	1.53209	12	0.92384	18	0.13640	13
GARCH (6, 6)	1.53209	13	0.92384	19	0.13640	14
GARCH (6, 7)	1.54610	34	0.92079	9	0.13464	6
GARCH (6, 8)	1.54444	31	0.91927	4	0.13438	2
GARCH (6, 9)	1.53932	21	0.91565	2	0.13460	5

APPENDIX C

Table 1. Error Statistics of GARCH (p, q) vs the Realized Volatility Out-Of-Sample (CNYUSD)

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.01183	32	0.00860	26	0.00207	17
GARCH (1, 3)	0.01219	41	0.00880	41	0.00214	37
GARCH (1, 4)	0.01219	42	0.00880	42	0.00214	38
GARCH (1, 5)	0.01219	43	0.00880	43	0.00214	39
GARCH (1, 6)	0.01338	50	0.00953	46	0.00235	46
GARCH (1, 7)	0.01338	51	0.00953	47	0.00235	47
GARCH (1, 8)	0.01338	52	0.00953	48	0.00235	48
GARCH (1, 9)	0.01338	53	0.00953	49	0.00235	49
GARCH (2, 1)	0.01196	35	0.00874	39	0.00208	20
GARCH (2, 2)	0.01186	33	0.00862	28	0.00208	19
GARCH (2, 3)	0.01182	30	0.00857	24	0.00207	14
GARCH (2, 4)	0.01182	31	0.00857	25	0.00207	15
GARCH (2, 5)	0.01182	29	0.00857	23	0.00207	16
GARCH (2, 6)	0.01215	39	0.00864	33	0.00211	32
GARCH (2, 7)	0.01215	40	0.00864	34	0.00211	33
GARCH (2, 8)	0.01244	44	0.00885	44	0.00218	41
GARCH (2, 9)	0.01244	45	0.00885	45	0.00218	42
GARCH (3, 1)	0.01177	28	0.00863	32	0.00208	18
GARCH (3, 2)	0.01170	26	0.00855	22	0.00207	12
GARCH (3, 3)	0.01175	27	0.00853	21	0.00207	13
GARCH (3, 4)	0.01165	17	0.00849	12	0.00209	21
GARCH (3, 5)	0.01165	18	0.00849	13	0.00209	22
GARCH (3, 6)	0.01165	19	0.00849	14	0.00209	23
GARCH (3, 7)	0.01165	20	0.00849	15	0.00209	24
GARCH (3, 8)	0.01165	21	0.00849	16	0.00209	25
GARCH (3, 9)	0.01165	22	0.00849	17	0.00209	26
GARCH (4, 1)	0.01164	16	0.00865	35	0.00209	30
GARCH (4, 2)	0.01145	9	0.00850	18	0.00205	8
GARCH (4, 3)	0.01169	23	0.00862	29	0.00211	34
GARCH (4, 4)	0.01169	24	0.00862	30	0.00211	35
GARCH (4, 5)	0.01169	25	0.00862	31	0.00211	36
GARCH (4, 6)	0.01203	36	0.00874	40	0.00219	45
GARCH (4, 7)	0.01194	34	0.00866	36	0.00217	40

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (4, 8)	0.01210	37	0.00871	37	0.00219	43
GARCH (4, 9)	0.01210	38	0.00871	38	0.00219	44
GARCH (5, 1)	0.01151	10	0.00853	19	0.00206	10
GARCH (5, 2)	0.01135	8	0.00843	11	0.00204	7
GARCH (5, 3)	0.01162	15	0.00860	27	0.00210	31
GARCH (5, 4)	0.01133	6	0.00829	**1**	0.00203	5
GARCH (5, 5)	0.01133	7	0.00829	2	0.00203	6
GARCH (5, 6)	0.01319	46	0.00954	50	0.00240	50
GARCH (5, 7)	0.01319	47	0.00954	51	0.00240	51
GARCH (5, 8)	0.01319	48	0.00954	52	0.00240	52
GARCH (5, 9)	0.01319	49	0.00954	53	0.00240	53
GARCH (6, 1)	0.01151	11	0.00853	20	0.00206	11
GARCH (6, 2)	0.01123	**1**	0.00836	4	0.00202	**1**
GARCH (6, 3)	0.01123	2	0.00836	5	0.00202	2
GARCH (6, 4)	0.01123	3	0.00836	6	0.00202	3
GARCH (6, 5)	0.01123	4	0.00836	7	0.00202	4
GARCH (6, 6)	0.01131	5	0.00832	3	0.00205	9
GARCH (6, 7)	0.01157	12	0.00842	8	0.00209	27
GARCH (6, 8)	0.01157	13	0.00842	9	0.00209	28
GARCH (6, 9)	0.01157	14	0.00842	10	0.00209	29

Table 2. Error Statistics of GARCH (p, q) vs the Implied Volatility Out-Of-Sample (CNYUSD)

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.01519	47	0.01312	51	0.00242	51
GARCH (1, 3)	0.01496	37	0.01287	38	0.00238	38
GARCH (1, 4)	0.01496	38	0.01287	39	0.00238	39
GARCH (1, 5)	0.01496	39	0.01287	40	0.00238	40
GARCH (1, 6)	0.01555	49	0.01298	46	0.00240	46
GARCH (1, 7)	0.01555	50	0.01298	47	0.00240	47
GARCH (1, 8)	0.01555	51	0.01298	48	0.00240	48
GARCH (1, 9)	0.01555	52	0.01298	49	0.00240	49
GARCH (2, 1)	0.01560	53	0.01347	53	0.00248	53
GARCH (2, 2)	0.01517	46	0.01309	50	0.00242	50
GARCH (2, 3)	0.01500	41	0.01295	43	0.00239	44
GARCH (2, 4)	0.01500	42	0.01295	44	0.00239	45

CNYUSD	**RMSE**	**Rank**	**MAE**	**Rank**	**MAPE**	**Rank**
GARCH (2, 5)	0.01500	40	0.01295	42	0.00239	43
GARCH (2, 6)	0.01478	28	0.01280	32	0.00237	32
GARCH (2, 7)	0.01478	29	0.01280	33	0.00237	33
GARCH (2, 8)	0.01472	26	0.01263	26	0.00234	26
GARCH (2, 9)	0.01472	27	0.01263	27	0.00234	27
GARCH (3, 1)	0.01528	48	0.01313	52	0.00242	52
GARCH (3, 2)	0.01503	44	0.01295	45	0.00239	42
GARCH (3, 3)	0.01488	34	0.01282	34	0.00237	36
GARCH (3, 4)	0.01443	11	0.01231	15	0.00228	15
GARCH (3, 5)	0.01443	12	0.01231	16	0.00228	16
GARCH (3, 6)	0.01443	13	0.01231	17	0.00228	17
GARCH (3, 7)	0.01443	14	0.01231	18	0.00228	18
GARCH (3, 8)	0.01443	15	0.01231	19	0.00228	19
GARCH (3, 9)	0.01443	16	0.01231	20	0.00228	20
GARCH (4, 1)	0.01503	45	0.01286	37	0.00237	37
GARCH (4, 2)	0.01501	43	0.01295	41	0.00239	41
GARCH (4, 3)	0.01458	18	0.01243	22	0.00230	22
GARCH (4, 4)	0.01458	19	0.01243	23	0.00230	23
GARCH (4, 5)	0.01458	20	0.01243	24	0.00230	24
GARCH (4, 6)	0.01388	4	0.01168	**1**	0.00217	**1**
GARCH (4, 7)	0.01382	**1**	0.01171	2	0.00217	2
GARCH (4, 8)	0.01385	2	0.01175	3	0.00218	3
GARCH (4, 9)	0.01385	3	0.01175	4	0.00218	4
GARCH (5, 1)	0.01495	35	0.01283	35	0.00237	34
GARCH (5, 2)	0.01469	21	0.01262	25	0.00233	25
GARCH (5, 3)	0.01457	17	0.01243	21	0.00230	21
GARCH (5, 4)	0.01431	9	0.01223	13	0.00226	13
GARCH (5, 5)	0.01431	10	0.01223	14	0.00226	14
GARCH (5, 6)	0.01470	22	0.01199	8	0.00222	9
GARCH (5, 7)	0.01470	23	0.01199	9	0.00222	10
GARCH (5, 8)	0.01470	24	0.01199	10	0.00222	11
GARCH (5, 9)	0.01470	25	0.01199	11	0.00222	12
GARCH (6, 1)	0.01495	36	0.01283	36	0.00237	35
GARCH (6, 2)	0.01480	30	0.01278	28	0.00236	28
GARCH (6, 3)	0.01480	31	0.01278	29	0.00236	29
GARCH (6, 4)	0.01480	32	0.01278	30	0.00236	30
GARCH (6, 5)	0.01480	33	0.01278	31	0.00236	31
GARCH (6, 6)	0.01404	8	0.01200	12	0.00222	8
GARCH (6, 7)	0.01401	5	0.01197	5	0.00222	5

CNYUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (6, 8)	0.01401	6	0.01197	6	0.00222	6
GARCH (6, 9)	0.01401	7	0.01197	7	0.00222	7

Table 3. Error Statistics of GARCH (p, q) vs the Realized Volatility Out-Of-Sample (GBPUSD)

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.02648	7	0.02230	52	0.00291	52
GARCH (1, 3)	0.02762	33	0.02130	34	0.00276	34
GARCH (1, 4)	0.02762	34	0.02130	35	0.00276	35
GARCH (1, 5)	0.02670	8	0.02047	4	0.00265	3
GARCH (1, 6)	0.02670	9	0.02047	5	0.00265	4
GARCH (1, 7)	0.02670	10	0.02047	6	0.00265	5
GARCH (1, 8)	0.02686	11	0.02036	**1**	0.00263	**1**
GARCH (1, 9)	0.02686	12	0.02036	2	0.00263	2
GARCH (2, 1)	0.02899	53	0.02508	53	0.00329	53
GARCH (2, 2)	0.02525	**1**	0.02131	36	0.00279	36
GARCH (2, 3)	0.02741	23	0.02098	21	0.00271	18
GARCH (2, 4)	0.02741	25	0.02098	23	0.00271	20
GARCH (2, 5)	0.02741	27	0.02098	25	0.00271	22
GARCH (2, 6)	0.02741	24	0.02098	22	0.00271	19
GARCH (2, 7)	0.02741	26	0.02098	24	0.00271	21
GARCH (2, 8)	0.02772	40	0.02106	26	0.00272	23
GARCH (2, 9)	0.02772	41	0.02106	27	0.00272	24
GARCH (3, 1)	0.02615	5	0.02171	45	0.00283	42
GARCH (3, 2)	0.02615	6	0.02171	46	0.00283	43
GARCH (3, 3)	0.02746	28	0.02148	37	0.00279	37
GARCH (3, 4)	0.02746	29	0.02148	38	0.00279	38
GARCH (3, 5)	0.02746	30	0.02148	39	0.00279	39
GARCH (3, 6)	0.02746	31	0.02148	40	0.00279	40
GARCH (3, 7)	0.02746	32	0.02148	41	0.00279	41
GARCH (3, 8)	0.02772	42	0.02106	28	0.00272	25
GARCH (3, 9)	0.02772	43	0.02106	29	0.00272	26
GARCH (4, 1)	0.02555	2	0.02156	42	0.00283	44
GARCH (4, 2)	0.02705	15	0.02095	18	0.00272	27
GARCH (4, 3)	0.02876	51	0.02203	50	0.00286	50
GARCH (4, 4)	0.02876	52	0.02203	51	0.00286	51
GARCH (4, 5)	0.02876	48	0.02203	47	0.00286	47

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (4, 6)	0.02876	49	0.02203	48	0.00286	48
GARCH (4, 7)	0.02876	50	0.02203	49	0.00286	49
GARCH (4, 8)	0.02774	46	0.02067	9	0.00268	9
GARCH (4, 9)	0.02774	47	0.02067	10	0.00268	10
GARCH (5, 1)	0.02590	3	0.02163	43	0.00284	45
GARCH (5, 2)	0.02705	16	0.02095	19	0.00272	28
GARCH (5, 3)	0.02740	19	0.02120	30	0.00275	30
GARCH (5, 4)	0.02740	20	0.02120	31	0.00275	31
GARCH (5, 5)	0.02702	13	0.02077	16	0.00269	16
GARCH (5, 6)	0.02771	35	0.02069	11	0.00268	11
GARCH (5, 7)	0.02771	37	0.02069	13	0.00268	13
GARCH (5, 8)	0.02771	38	0.02069	14	0.00268	14
GARCH (5, 9)	0.02771	39	0.02069	15	0.00268	15
GARCH (6, 1)	0.02590	4	0.02163	44	0.00284	46
GARCH (6, 2)	0.02705	17	0.02095	20	0.00272	29
GARCH (6, 3)	0.02740	21	0.02120	32	0.00275	32
GARCH (6, 4)	0.02740	22	0.02120	33	0.00275	33
GARCH (6, 5)	0.02702	14	0.02077	17	0.00269	17
GARCH (6, 6)	0.02771	36	0.02069	12	0.00268	12
GARCH (6, 7)	0.02722	18	0.02043	3	0.00265	6
GARCH (6, 8)	0.02774	44	0.02059	7	0.00267	7
GARCH (6, 9)	0.02774	45	0.02059	8	0.00267	8

Table 4. Error Statistics of GARCH (p, q) vs the Implied Volatility Out-Of-Sample (GBPUSD)

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.02138	4	0.01813	10	0.00205	27
GARCH (1, 3)	0.02376	24	0.01862	28	0.00206	30
GARCH (1, 4)	0.02376	25	0.01862	29	0.00206	31
GARCH (1, 5)	0.02306	8	0.01799	4	0.00198	5
GARCH (1, 6)	0.02306	9	0.01799	5	0.00198	6
GARCH (1, 7)	0.02306	10	0.01799	6	0.00198	7
GARCH (1, 8)	0.02333	14	0.01803	8	0.00197	3
GARCH (1, 9)	0.02333	15	0.01803	9	0.00197	4
GARCH (2, 1)	0.02369	18	0.02056	53	0.00237	53
GARCH (2, 2)	0.02082	3	0.01761	3	0.00198	8
GARCH (2, 3)	0.02378	26	0.01852	20	0.00204	17

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (2, 4)	0.02378	27	0.01852	22	0.00204	19
GARCH (2, 5)	0.02378	30	0.01852	24	0.00204	21
GARCH (2, 6)	0.02378	28	0.01852	21	0.00204	18
GARCH (2, 7)	0.02378	29	0.01852	23	0.00204	20
GARCH (2, 8)	0.02386	31	0.01842	14	0.00203	10
GARCH (2, 9)	0.02386	32	0.01842	15	0.00203	11
GARCH (3, 1)	0.02081	**1**	0.01715	**1**	0.00192	**1**
GARCH (3, 2)	0.02081	2	0.01715	2	0.00192	2
GARCH (3, 3)	0.02372	19	0.01881	34	0.00209	43
GARCH (3, 4)	0.02372	20	0.01881	35	0.00209	44
GARCH (3, 5)	0.02372	21	0.01881	36	0.00209	45
GARCH (3, 6)	0.02372	22	0.01881	37	0.00209	46
GARCH (3, 7)	0.02372	23	0.01881	38	0.00209	47
GARCH (3, 8)	0.02386	33	0.01842	16	0.00203	12
GARCH (3, 9)	0.02386	34	0.01842	17	0.00203	13
GARCH (4, 1)	0.02145	5	0.01803	7	0.00203	14
GARCH (4, 2)	0.02324	11	0.01857	25	0.00204	22
GARCH (4, 3)	0.02505	52	0.01955	51	0.00216	51
GARCH (4, 4)	0.02505	53	0.01955	52	0.00216	52
GARCH (4, 5)	0.02505	49	0.01955	48	0.00216	48
GARCH (4, 6)	0.02505	50	0.01955	49	0.00216	49
GARCH (4, 7)	0.02505	51	0.01955	50	0.00216	50
GARCH (4, 8)	0.02459	42	0.01877	32	0.00206	32
GARCH (4, 9)	0.02459	43	0.01877	33	0.00206	33
GARCH (5, 1)	0.02198	6	0.01833	11	0.00206	28
GARCH (5, 2)	0.02324	12	0.01857	26	0.00204	23
GARCH (5, 3)	0.02389	36	0.01882	44	0.00207	39
GARCH (5, 4)	0.02389	37	0.01882	45	0.00207	40
GARCH (5, 5)	0.02360	16	0.01851	18	0.00203	15
GARCH (5, 6)	0.02460	44	0.01881	39	0.00206	34
GARCH (5, 7)	0.02460	46	0.01881	41	0.00206	36
GARCH (5, 8)	0.02460	47	0.01881	42	0.00206	37
GARCH (5, 9)	0.02460	48	0.01881	43	0.00206	38
GARCH (6, 1)	0.02198	7	0.01833	12	0.00206	29
GARCH (6, 2)	0.02324	13	0.01857	27	0.00204	24
GARCH (6, 3)	0.02389	38	0.01882	46	0.00207	41
GARCH (6, 4)	0.02389	39	0.01882	47	0.00207	42
GARCH (6, 5)	0.02360	17	0.01851	19	0.00203	16
GARCH (6, 6)	0.02460	45	0.01881	40	0.00206	35

GBPUSD	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (6, 7)	0.02389	35	0.01837	13	0.00201	9
GARCH (6, 8)	0.02454	40	0.01868	30	0.00205	25
GARCH (6, 9)	0.02454	41	0.01868	31	0.00205	26

Table 5. Error Statistics of GARCH (p, q) vs the Realized Volatility Out-Of-Sample (Bitcoin)

Bitcoin	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (1, 2)	0.33667	4	0.28235	6	0.01561	4
GARCH (1, 3)	0.34242	17	0.28779	17	0.01588	25
GARCH (1, 4)	0.34365	19	0.28879	21	0.01578	14
GARCH (1, 5)	0.34365	18	0.28879	20	0.01578	13
GARCH (1, 6)	0.34365	20	0.28879	22	0.01578	15
GARCH (1, 7)	0.34186	9	0.28499	9	0.01579	20
GARCH (1, 8)	0.34186	10	0.28499	10	0.01579	21
GARCH (1, 9)	0.34186	8	0.28499	8	0.01579	19
GARCH (2, 1)	0.33042	2	0.27593	**1**	0.01539	2
GARCH (2, 2)	0.33772	7	0.28306	7	0.01567	7
GARCH (2, 3)	0.33256	3	0.27926	3	0.01541	3
GARCH (2, 4)	0.34366	21	0.28879	23	0.01578	16
GARCH (2, 5)	0.34366	22	0.28879	24	0.01578	17
GARCH (2, 6)	0.34366	23	0.28879	25	0.01578	18
GARCH (2, 7)	0.34188	11	0.28502	11	0.01579	22
GARCH (2, 8)	0.34188	12	0.28502	12	0.01579	23
GARCH (2, 9)	0.34188	13	0.28502	13	0.01579	24
GARCH (3, 1)	0.33752	6	0.28158	5	0.01567	6
GARCH (3, 2)	0.33746	5	0.28152	4	0.01566	5
GARCH (3, 3)	0.33019	**1**	0.27681	2	0.01535	**1**
GARCH (3, 4)	0.34732	26	0.29171	32	0.01606	32
GARCH (3, 5)	0.34404	24	0.28896	26	0.01576	8
GARCH (3, 6)	0.34404	25	0.28896	27	0.01576	9
GARCH (3, 7)	0.34226	14	0.28526	14	0.01577	10
GARCH (3, 8)	0.34226	15	0.28526	15	0.01577	11
GARCH (3, 9)	0.34226	16	0.28526	16	0.01577	12
GARCH (4, 1)	0.35293	34	0.29225	34	0.01622	38
GARCH (4, 2)	0.36098	38	0.29797	41	0.01612	34
GARCH (4, 3)	0.36098	39	0.29797	42	0.01612	35
GARCH (4, 4)	0.36755	46	0.30066	46	0.01653	42

Bitcoin	RMSE	Rank	MAE	Rank	MAPE	Rank
GARCH (4, 5)	0.36755	47	0.30066	47	0.01653	43
GARCH (4, 6)	0.36755	48	0.30066	48	0.01653	44
GARCH (4, 7)	0.34958	27	0.28855	18	0.01594	26
GARCH (4, 8)	0.36625	41	0.29780	39	0.01654	52
GARCH (4, 9)	0.36625	42	0.29780	40	0.01654	53
GARCH (5, 1)	0.35273	33	0.29205	33	0.01621	37
GARCH (5, 2)	0.36097	37	0.29797	43	0.01612	33
GARCH (5, 3)	0.35712	35	0.29372	35	0.01624	40
GARCH (5, 4)	0.36758	52	0.30069	52	0.01653	48
GARCH (5, 5)	0.36758	53	0.30069	53	0.01653	49
GARCH (5, 6)	0.36754	45	0.30065	45	0.01653	41
GARCH (5, 7)	0.35031	30	0.28912	28	0.01597	28
GARCH (5, 8)	0.35031	31	0.28912	29	0.01597	29
GARCH (5, 9)	0.35031	32	0.28912	30	0.01597	30
GARCH (6, 1)	0.34979	29	0.29079	31	0.01597	31
GARCH (6, 2)	0.36101	40	0.29801	44	0.01612	36
GARCH (6, 3)	0.35712	36	0.29372	36	0.01624	39
GARCH (6, 4)	0.36755	49	0.30066	49	0.01653	45
GARCH (6, 5)	0.36755	50	0.30066	50	0.01653	46
GARCH (6, 6)	0.36755	51	0.30066	51	0.01653	47
GARCH (6, 7)	0.34958	28	0.28856	19	0.01594	27
GARCH (6, 8)	0.36646	43	0.29775	37	0.01654	50
GARCH (6, 9)	0.36646	44	0.29775	38	0.01654	51

APPENDIX D

Table 1. VaR Calculations for the CNYUSD using GARCH (6, 2) Volatility-weighted Historical Simulation

Day	Date	Actual Returns	VaR 90%	Exp	VaR 95%	Exp	VaR 97.5%	Exp	VaR 99%	Exp
1	2/13/2018	-0.0021	-0.0027	1	-0.0036	1	-0.0054	1	-0.0089	1
2	2/14/2018	-0.0001	-0.0027	1	-0.0037	1	-0.0055	1	-0.0089	1
3	2/22/2018	-0.0018	-0.0027	1	-0.0036	1	-0.0054	1	-0.0087	1
4	2/23/2018	0.0024	-0.0027	1	-0.0036	1	-0.0053	1	-0.0086	1
5	2/26/2018	0.0033	-0.0026	1	-0.0035	1	-0.0049	1	-0.0077	1
6	2/27/2018	-0.0001	-0.0026	1	-0.0034	1	-0.0051	1	-0.0083	1
7	2/28/2018	-0.0022	-0.0025	1	-0.0034	1	-0.0051	1	-0.0082	1
8	3/1/2018	-0.0043	-0.0026	0	-0.0036	0	-0.0049	1	-0.0076	1
9	3/2/2018	0.0020	-0.0025	1	-0.0035	1	-0.0051	1	-0.0082	1
10	3/5/2018	-0.0007	-0.0025	1	-0.0034	1	-0.0050	1	-0.0072	1
11	3/6/2018	0.0056	-0.0026	1	-0.0036	1	-0.0053	1	-0.0085	1
12	3/7/2018	-0.0017	-0.0025	1	-0.0034	1	-0.0051	1	-0.0082	1
13	3/8/2018	-0.0026	-0.0025	0	-0.0034	1	-0.0048	1	-0.0074	1
14	3/9/2018	0.0012	-0.0025	1	-0.0033	1	-0.0046	1	-0.0072	1
15	3/12/2018	0.0008	-0.0025	1	-0.0033	1	-0.0046	1	-0.0072	1
16	3/13/2018	0.0012	-0.0024	1	-0.0033	1	-0.0049	1	-0.0078	1
17	3/14/2018	0.0006	-0.0024	1	-0.0033	1	-0.0048	1	-0.0077	1
18	3/15/2018	-0.0006	-0.0023	1	-0.0031	1	-0.0044	1	-0.0068	1
19	3/16/2018	-0.0020	-0.0023	1	-0.0031	1	-0.0043	1	-0.0068	1
20	3/19/2018	0.0004	-0.0023	1	-0.0031	1	-0.0043	1	-0.0066	1
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248	2/26/2019	-0.0017	-0.0033	1	-0.0049	1	-0.0067	1	-0.0080	1
249	2/27/2019	0.0019	-0.0031	1	-0.0047	1	-0.0064	1	-0.0078	1
250	2/28/2019	-0.0010	-0.0031	1	-0.0047	1	-0.0064	1	-0.0078	1

Table 2. VaR Calculations for the GBPUSD using EWMA Volatility-weighted Historical Simulation

Day	**Date**	**Actual Returns**	**VaR 90%**	**Exp**	**VaR 95%**	**Exp**	**VaR 97.5%**	**Exp**	**VaR 99%**	**Exp**
1	2/13/2018	0.0040	-0.0068	1	-0.0089	1	-0.0102	1	-0.0154	1
2	2/14/2018	0.0076	-0.0070	1	-0.0091	1	-0.0104	1	-0.0157	1
3	2/22/2018	-0.0031	-0.0063	1	-0.0085	1	-0.0098	1	-0.0128	1
4	2/23/2018	0.0011	-0.0064	1	-0.0084	1	-0.0096	1	-0.0123	1
5	2/26/2018	-0.0002	-0.0063	1	-0.0083	1	-0.0094	1	-0.0119	1
6	2/27/2018	-0.0042	-0.0062	1	-0.0082	1	-0.0092	1	-0.0117	1
7	2/28/2018	-0.0107	-0.0068	0	-0.0089	0	-0.0102	0	-0.0128	1
8	3/1/2018	0.0012	-0.0061	1	-0.0084	1	-0.0096	1	-0.0124	1
9	3/2/2018	0.0019	-0.0064	1	-0.0084	1	-0.0096	1	-0.0120	1
10	3/5/2018	0.0034	-0.0058	1	-0.0080	1	-0.0091	1	-0.0117	1
11	3/6/2018	0.0028	-0.0058	1	-0.0080	1	-0.0091	1	-0.0114	1
12	3/7/2018	0.0006	-0.0058	1	-0.0077	1	-0.0089	1	-0.0116	1
13	3/8/2018	-0.0061	-0.0058	0	-0.0078	1	-0.0089	1	-0.0112	1
14	3/9/2018	0.0028	-0.0058	1	-0.0076	1	-0.0087	1	-0.0109	1
15	3/12/2018	0.0040	-0.0057	1	-0.0075	1	-0.0089	1	-0.0129	1
16	3/13/2018	0.0040	-0.0055	1	-0.0074	1	-0.0085	1	-0.0106	1
17	3/14/2018	0.0000	-0.0055	1	-0.0072	1	-0.0083	1	-0.0123	1
18	3/15/2018	-0.0018	-0.0053	1	-0.0069	1	-0.0083	1	-0.0119	1
19	3/16/2018	0.0004	-0.0050	1	-0.0067	1	-0.0077	1	-0.0096	1
20	3/19/2018	0.0059	-0.0051	1	-0.0069	1	-0.0080	1	-0.0111	1
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248	2/26/2019	0.0118	-0.0074	1	-0.0099	1	-0.0123	1	-0.0161	1
249	2/27/2019	0.0043	-0.0078	1	-0.0107	1	-0.0124	1	-0.0162	1
250	2/28/2019	-0.0035	-0.0075	1	-0.0102	1	-0.0120	1	-0.0157	1

Table 3. VaR Calculations for the Bitcoin using EWMA Volatility-weighted Historical Simulation

Day	Date	Actual Returns	VaR 90%	Exp	VaR 95%	Exp	VaR 97.5%	Exp	VaR 99%	Exp
1	2/13/2018	-0.0368	-0.0860	1	-0.1183	1	-0.1669	1	-0.2067	1
2	2/14/2018	0.1042	-0.0886	1	-0.1218	1	-0.1612	1	-0.2128	1
3	2/22/2018	0.0538	-0.0844	1	-0.1189	1	-0.1576	1	-0.1973	1
4	2/23/2018	0.0296	-0.0819	1	-0.1149	1	-0.1523	1	-0.2011	1
5	2/26/2018	0.0064	-0.0678	1	-0.1083	1	-0.1437	1	-0.1638	1
6	2/27/2018	0.0346	-0.0650	1	-0.1050	1	-0.1393	1	-0.1733	1
7	2/28/2018	-0.0306	-0.0675	1	-0.1016	1	-0.1348	1	-0.1717	1
8	3/1/2018	0.0532	-0.0626	1	-0.1000	1	-0.1327	1	-0.1690	1
9	3/2/2018	0.0124	-0.0650	1	-0.0979	1	-0.1276	1	-0.1625	1
10	3/5/2018	0.0439	-0.0675	1	-0.0960	1	-0.1271	1	-0.1591	1
11	3/6/2018	-0.0686	-0.0686	1	-0.0964	1	-0.1279	1	-0.1600	1
12	3/7/2018	-0.0755	-0.0696	0	-0.0979	1	-0.1276	1	-0.1625	1
13	3/8/2018	-0.0573	-0.0692	1	-0.0972	1	-0.1372	1	-0.1812	1
14	3/9/2018	-0.0061	-0.0663	1	-0.0932	1	-0.1215	1	-0.1547	1
15	3/12/2018	-0.0142	-0.0638	1	-0.0897	1	-0.1189	1	-0.1572	1
16	3/13/2018	-0.0011	-0.0593	1	-0.0846	1	-0.1123	1	-0.1430	1
17	3/14/2018	-0.1006	-0.0660	0	-0.0929	0	-0.1211	1	-0.1542	1
18	3/15/2018	0.0038	-0.0634	1	-0.0892	1	-0.1163	1	-0.1481	1
19	3/16/2018	0.0045	-0.0609	1	-0.0857	1	-0.1117	1	-0.1422	1
20	3/19/2018	0.0351	-0.0594	1	-0.0836	1	-0.1089	1	-0.1388	1
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248	2/26/2019	-0.0073	-0.0322	1	-0.0440	1	-0.0553	1	-0.0687	1
249	2/27/2019	-0.0009	-0.0299	1	-0.0418	1	-0.0531	1	-0.0659	1
250	2/28/2019	0.0010	-0.0288	1	-0.0405	1	-0.0555	1	-0.0643	1

Table 4. VaR Calculations for the Ripple using GARCH (1, 8) Volatility-weighted Historical Simulation

Day	Date	Actual Returns	VaR 90%	Exp	VaR 95%	Exp	VaR 97.5%	Exp	VaR 99%	Exp
1	2/13/2018	-0.0463	-0.0674	1	-0.1063	1	-0.1368	1	-0.2180	1
2	2/14/2018	0.1068	-0.0675	1	-0.1068	1	-0.1373	1	-0.2190	1
3	2/22/2018	-0.1739	-0.1134	0	-0.1833	1	-0.2564	1	-0.3668	1
4	2/23/2018	0.0606	-0.0566	1	-0.0914	1	-0.1281	1	-0.1830	1
5	2/26/2018	-0.0420	-0.0600	1	-0.0970	1	-0.1358	1	-0.1941	1
6	2/27/2018	-0.0100	-0.0539	1	-0.0872	1	-0.1220	1	-0.1745	1
7	2/28/2018	-0.0451	-0.0656	1	-0.1060	1	-0.1483	1	-0.2122	1
8	3/1/2018	0.0263	-0.0532	1	-0.0859	1	-0.1203	1	-0.1720	1
9	3/2/2018	-0.0196	-0.0552	1	-0.0892	1	-0.1249	1	-0.1785	1
10	3/5/2018	0.0541	-0.0548	1	-0.0885	1	-0.1240	1	-0.1772	1
11	3/6/2018	-0.0380	-0.0668	1	-0.1079	1	-0.1511	1	-0.2159	1
12	3/7/2018	-0.0560	-0.0631	1	-0.1004	1	-0.1405	1	-0.2010	1
13	3/8/2018	-0.0521	-0.0623	1	-0.0992	1	-0.1386	1	-0.1985	1
14	3/9/2018	0.0289	-0.0510	1	-0.0813	1	-0.1137	1	-0.1626	1
15	3/12/2018	-0.0573	-0.0645	1	-0.1026	1	-0.1434	1	-0.2054	1
16	3/13/2018	-0.0142	-0.0532	1	-0.0846	1	-0.1183	1	-0.1694	1
17	3/14/2018	-0.1112	-0.0822	0	-0.1306	1	-0.1819	1	-0.2608	1
18	3/15/2018	-0.0066	-0.0526	1	-0.0835	1	-0.1163	1	-0.1668	1
19	3/16/2018	-0.0115	-0.0549	1	-0.0871	1	-0.1214	1	-0.1740	1
20	3/19/2018	0.0868	-0.0624	1	-0.0988	1	-0.1380	1	-0.1976	1
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248	2/26/2019	-0.0275	-0.0510	1	-0.0708	1	-0.0859	1	-0.1046	1
249	2/27/2019	-0.0199	-0.0469	1	-0.0660	1	-0.0822	1	-0.0974	1
250	2/28/2019	0.0063	-0.0406	1	-0.0577	1	-0.0707	1	-0.0834	1