MODELING THE VOLATILITY AND VALUE AT RISK OF CRYPTOCURRENCIES AND FIAT CURRENCIES USING GARCH MODELS

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ABSTRACT

Purpose: The purpose of this thesis is to investigate and assess the predictive ability of the GARCH (1,1), IGARCH (1,1), EGARCH (1,1), GJR-GARCH (1,1), APARCH (1,1), TGARCH (1,1) and CGARCH (1,1) models in forecasting the volatilities of six major cryptocurrencies: Bitcoin, Ripple, Litecoin, Monero, Dash, Dogecoin and six world currencies: Euro, British Pound, Canadian Dollar, Australian Dollar, Swiss Franc and the Japanese Yen. The optimal volatility model selected for each virtual and hard currency is then integrated into the Volatility Update Historical Simulation approach to evaluate the accuracy of VaR in quantifying the level of downside risk in cryptocurrencies and fiat currencies.

Design/Methodology/Approach: The daily closing prices for each cryptocurrency and fiat currency are collected over a sampled period extending from October 13th 2015 till November 18th 2019. The sampled period is divided into two sub-sample periods: the in-sample period extending from October 13th 2015 till December 3rd 2018, and the out-of-sample period covering the period from December 4th 2018 till November 18th 2019. In-sample returns are calculated from daily closing prices and are used to estimate the parameters of the selected models, subject to the assumptions and constraints of each model. Accordingly, the calculated in-sample parameters are applied to forecast the volatilities for both the in-sample and out-of-sample periods. The three error metrics RMSE, MAE and MAPE are then utilized to determine the optimal model for each currency and cryptocurrency and for each of the in-sample and out-of-sample periods. The Rolling Window procedure is conducted in conjunction with the out-of-sample optimal model's parameters to simulate the variances and volatilities of each cryptocurrency and fiat currency. Using the Volatility Update Historical Simulation method, future return scenarios are generated for each cryptocurrency and fiat currency over each day extending from December 4th 2018 till November 18th 2019. The Value at Risk is then calculated for those 250 days at four confidence levels (90%, 95%, 97.5% and 99% confidence levels) for each cryptocurrency and fiat currency. The Kupeic test is eventually performed to determine the accuracy of the underlying VaR model.

Findings: By comparing the realized volatility to the estimated volatilities, the results show consistency among fiat currencies whereby the Integrated GARCH has proven to be the best performer during both sampled periods for most of the fiat currencies, particularly the British Pound, Australian Dollar, Swiss Franc and the Japanese Yen. The IGARCH model is also found to be the most accurate model for the Canadian Dollar, but only for the out-of-sample period given that the Threshold GARCH performs better during the in-sample period. However, the Component GARCH is the optimal model for the Euro for both the in-sample and out-of-sample contexts. Therefore, the IGARCH has proven to be the prevailing model when modeling foreign exchange markets. Exceptionally and among all cryptocurrencies, the Integrated GARCH is also the best performing model for Monero, for both sampled periods. As for the remaining cryptocurrencies, the GJR-GARCH model proved to be superior during the in-sample period while the CGARCH and TGARCH models proved to be best performers during the out-of-sample period. Specifically, for the in-sample period, the GJR-GARCH model is selected for Bitcoin, Litecoin and Dash, APARCH is selected for Ripple, and SGARCH is selected for Dogecoin. For the out-of-sample period, TGARCH performed best for Bitcoin and Dash while CGARCH is selected for Ripple and Dogecoin and APARCH is selected for Litecoin. The results validate the assumption that advanced GARCH models better model asymmetries in cryptocurrencies' volatility. Finally, the Kupeic test showed that the VaR provides a very accurate measure for the level of downside risk exposing fiat currencies, as the results were accepted at all confidence levels for each of the Euro, British Pound, Canadian Dollar, Australian Dollar, Swiss Franc, and the Japanese Yen, given that the VaR model was only rejected at the 97.5% confidence level for the latter. Dash and Dogecoin provided similar results to fiat currencies where the VaR results were accepted at all confidence levels. As for the remaining cryptocurrencies, the results were different. The VaR results displayed increased accuracy with an increase in confidence level in the case of Litecoin, where the model was accepted at the 95%, 97.5% and 99% confidence levels and was rejected at the 90% significance level. As for Monero, the VaR model was accepted at 90% and 99% confidence levels and rejected at the 95% and 97.5% confidence levels. Nevertheless, it is evident that the VaR provides a poor measure for Bitcoin and Ripple whereby the model was rejected at all confidence levels, noting that it was only accepted at the 99% confidence level. Therefore, it can be deduced that the Value at Risk provides a viable measure of the risk exposure in fiat currencies and some cryptocurrencies, such as Dash and Dogecoin. However, this metric fails in accurately quantifying the level of downside risk in major cryptocurrencies such as the Bitcoin and Ripple.

Research Limitations/Implications: Despite the proven significance of the Student's t and General Error Distributions that have been introduced in this thesis, there are several other distributions that could have been considered. Furthermore, even though the selected models: SGARCH, IGARCH, EGARCH, GJR-GARCH, TGARCH, APARCH and CGARCH models have proven their superiority in predicting the volatility of not only fiat currencies and cryptocurrencies but most securities, this thesis could have integrated further models. Moreover, while the expression "Value at Risk" is widely used, the expression does not refer to one particular methodology or approach for quantifying risk. Although this thesis employed the best possible method, the Volatility Updating Historical Simulation Method, there are other few methods that could have been utilized to measure VaR. In addition, another limitation in this thesis is that the VaR failed in accurately quantifying the level of downside risk in highly volatile markets such as in cryptocurrencies, particularly Bitcoin and Ripple which are the leading cryptocurrencies today. For this reason, more refined and sophisticated tools could have been integrated into our research to remedy deficiencies in VaR. Lastly, there are few uncertainties whether the findings of this thesis and the behavior of the selected cryptocurrencies could be theorized on the entire cryptocurrency market as the market prices of the selected cryptocurrencies have changed since the beginning of this research, thereby as has their representative portion from the entire cryptocurrency market.

Practical Implications: The results of this thesis and the assumptions drawn from our findings can be particularly useful for certain parties. For governmental institutions and regulators, it is recommended from authorities to examine the risk enfolding cryptocurrencies. This thesis provided further wisdom concerning the risks conveyed in the cryptocurrency market. Based on those results, governments and regulatory authorities could strengthen regulations and arouse further awareness by enforcing policies and restraining investors from devoting too much investment in cryptocurrencies. Accordingly, financial managers and investors need to be aware before considering an investment in cryptocurrencies, given their extremely volatile behavior. For this reason, investors and senior managers are advised to limit their positions in cryptocurrencies, specifically

during strained conditions. As for academicians, this thesis provides further clarification surrounding the behavior of cryptocurrencies with respect to world currencies, the relative performance of diverse GARCH models, and reliability concerns of the Value at Risk measure. This thesis can be considered the groundwork and motive for further examining and modeling the volatility of cryptocurrencies or employing alternative models to the Value at Risk.

Originality/Value: The findings of this thesis are novel to those of preceding research, as this research is the first and latest to inspect the volatility and the Value at Risk of six major cryptocurrencies along with that of the top six hard currencies, all together, particularly with the use of several GARCH Models and the Volatility Updating Historical Simulation Method.

Keywords: Bitcoin, Ripple, Litecoin, Monero, Dash, Dogecoin, EURUSD, GBPUSD, CADUSD, AUDUSD, CHFUSD, JPYUSD, GARCH(1,1), IGARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1), APARCH(1,1), TGARCH(1,1), CGARCH(1,1), modeling volatility, GARCH models, in sample, out of sample, Value at Risk, VaR, realized volatility, cryptocurrencies, fiat currencies, Kupiec test, rolling window, volatility update historical simulation.

Chapter 1: Introduction

The study of finance is to a large extent a study of volatility and volatility really permeates finance. Since the 2007 credit crisis, the word ‘volatility’ has become even more prevalent and contagious in the lexicon of financial services professionals, the media and, the public at large. This is why volatility has become increasingly conventional over the last decade. Therefore, understanding price volatility dynamics has become of considerable interest to those seeking to understand the price dynamics of financial assets.

What is volatility; it is a statistical measure that gauges the dispersion of returns for a given security or financial instrument. It can either be measured by using the standard deviation or square root of the variance between daily returns from that same instrument (Kuepper, 2020). Practically, volatility has provoked fears of uncertainty and skepticism as it has been a sole indicator of risk. Also, volatility is a major player in the state of the economy where it represents an indicator of investor and consumer’s confidence. Theoretically speaking, therefore, the higher the volatility the riskier the security. For this reason, precision is very eminent. Inaccurate estimates of volatility can have a substantial impact on financial decisions. An understated volatility can provoke a greater risk exposure, and an overstated volatility may incite an opportunity cost (Naimy & Hayek, 2018). As such, efficient and effective risk management frameworks must be braced to accurately calibrate the level of volatility. In this essence, it is vital to ensure that this risk is being optimally monitored and mitigated.

As such, volatility is a key element around which financial markets revolve. Its preeminence and essence in the different areas of risk management, trading, security pricing, asset allocation, portfolio optimization and monetary policy has enticed interest from investors, governments, and regulators. From this context, modeling and predicting the volatility of financial markets has been, for years, the core of extensive empirical and hypothetical investigation of both academics and practitioners. Many models have evolved and rectified to model volatility but no model has unveiled extreme superiority for predicting volatility for all financial instruments. Therefore, model reliability is associated with the foundation and distinctive feature of the asset itself.

Apart from Bitcoin, there appears to be little or no literature on modeling the volatility of crypto-currencies (Chu et al., 2017), despite the extensive literature on the predictive capacity of models in forecasting a diverse range of asset classes. Given their typical volatile behavior, coupled with their significance in the financial field and on the financial system in particular, the need to predict their volatility has become more and more imperative. Therefore, the importance of a comprehensive study encircling the behavior of cryptocurrencies with respect to fiat currencies like the Swiss Franc, Euro, and the British Pound is self-evident that may unveil unknown characteristics, amend on or improve existing findings. Thereby, this thesis contributes to the existing literature by attempting to fill this gap in current research.

The purpose of this thesis, therefore, is to inspect and demarcate the behavior and liaison of generally two types of currencies, cryptocurrencies and fiat currencies. This is addressed by monitoring and predicting their volatility, as cryptocurrencies have risen and have thrived in altering many people's exchange mechanism thereby asserting their prominence in the marketplace and on the financial system.

Formerly, most currencies were backed by physical commodities such as gold through a monetary system known as the gold standard, which was abandoned and replaced by fiat money. Investopedia (2019) defines fiat money as a currency issued by the government and declared to be legal tender whereby its value is derived from unlimited supply and demand factors and is thereby solely based on the faith and credit of the economy. Fiat money rose to eminence in the 20th century, particularly after the collapse of the Bretton Woods Agreement, which was dissolved between the late 60s and early 70s, when the United States abolished the international convertibility of the American dollar into gold, in what became known as the Nixon Shock. Ever since, a local fiat money system has been used globally, allowing for floating exchange rate regime between the major currencies. As such, fiat money is centralized and inflationary.

Unlike traditional currencies, a cryptocurrency is a digital or virtual currency and a medium of exchange that uses cryptography to secure financial transactions. A defining characteristic of most cryptocurrencies, and perhaps their most appealing allure, is that they have a confined supply and are not supported by any central authority, rendering them theoretically deflationary and decentralized thereby immune to central banking system and governmental interference providing many advantages over traditional payment methods including speed, high liquidity, lower transaction costs, and anonymity (Fantazzini et al., 2016). However, it is an emerging, retail focused, highly speculative market that lacks a legal and regulatory framework comparable to other assets. Accordingly, the unregulated and digital aspect of a cryptocurrency makes it an attractive target for hackers (Grinberg, 2011) as it is not difficult to counterfeit because of this insecurity feature. In essence, it is claimed that they could be used to hedge popular fiat currencies backed by the most powerful economies. As such, a cryptocurrency was designed to be everything fiat currency could not be. This is why it is vital to unveil the nature of the relationship between cryptocurrencies and fiat.

Bitcoin (BTC), often regarded as father of cryptocurrencies, is the most leading and prominent cryptocurrency and probably the most controversial virtual currency scheme to date that has been the main focus of interest since its release in 2009. Since 2009, the finance world has been watching the exceptional rise of Bitcoin with a combination of captivation and, in many cases, severe skepticism. The original concept was to create a digital payment protocol and peer-to-peer unit of currency that can be transferred instantly and securely between any two parties (Ametrano, 2016). The underlying technology of Bitcoin creation and transactions is called 'Blockchain', a shared public ledger on which the entire Bitcoin network relies and whereby all confirmed transactions are recorded and verified by a massive amount of computer power. As such, the blockchain technology allows rapid transactions where their history is saved in a chain (Nakamoto, 2008). However, it is a flaw when it comes to the speed of transactions. Bitcoin can only undertake a maximum of 7 transactions per second. That's why alternative cryptocurrencies or altcoins have been in development since the beginning of this decade. Their purpose is to unravel the challenges presented by Bitcoin. Nevertheless, the said wallets keep a secret piece of data called a private key or seed, which is used to sign transactions. Consequently, miners solve cryptographic puzzles to validate those transactions; through a process called mining; adding a new block to the blockchain, whereby a reward namely a fraction of a bitcoin is awarded afterwards (Brière et al., 2015). Bitcoin is limited to 21 million coins that can come into circulation. Once miners have unlocked this many, the planet's supply will be eventually tapped out and miners would get rewarded only through transactions fees unless certain protocol is adjusted to allow for a larger supply. Controversies exist around the nature of the Bitcoin, as some label it as a currency-like asset whereas others classify it as a commodity and a 'digital version of gold.' Nevertheless, Bitcoin's price has gone through cycles of appreciation and depreciation, better known as bubbles and busts. Accordingly, Bitcoin is more liable to speculative bubbles and exhibits higher volatility than traditional hard currencies. Bitcoin has therefore a central position in financial markets and in portfolio management (Dyhrberg, 2016) and examining its volatility is important.

Because it was the first, bitcoin gets all the publicity, but it competes against dozens of aspiring alternatives as currently, there are over a thousand different cryptocurrencies in circulation. Among these, one name that has garnered increasing interest is Litecoin (LTC). Launched in 2011, Litecoin is the second most popular decentralized cryptocurrency; but differs from Bitcoin with notable advantages in aspects like faster block generation rate thereby handling higher volume of transactions with a greater transactability and use of scripts as a proof of work scheme. Every 2.5 minutes, as opposed to 10 minutes for Bitcoin, the Litecoin network generates a block with the same mining procedure with minor variations. There will eventually be only 84 million litecoins in circulation at the upper limit. Nevertheless, it remains unlikely that any of Bitcoin's potentially challenging features will provoke any further substantial problems, thereby enticing people to alternatively shift towards Litecoin.

While Bitcoin has the largest market capitalization among all cryptocurrencies, many users believe that it is fundamentally flawed with its lack of privacy. Today, many cryptocurrencies exhibit certain deficiencies in terms of privacy, security, or fungibility at the protocol or fundamental code level. Launched in 2014, Monero (XMR) has successfully resolved this concern by designing a currency that operates on a secure, untraceable and private system. Monero is a decentralized open-source, privacy-oriented cryptocurrency that operates to mollify privacy concerns using concepts such as ring signatures and stealth addresses thereby preserving anonymity. Similar to bitcoin, it issues new coins and incentivize miners to secure the network and validate transactions but with a transaction speed of 2 minutes. The total cap for the ultimate supply of Monero is 18.4 million coins that are expected to be totally mined by 2022. Unlike Bitcoin however, Monero will continue generating small 0.6 XMR block rewards indefinitely after 2022, through a process known as tail emission.

Many virtual assets lack a clear purpose. They may be used to store value, acquire commodities or for personal transactions, but were not designed with a single specific application. By contrast, Ripple (XRP) is typically about the transfer of value and is designed for enterprises, rendering it one of the few digital assets with a real and clear purpose. Although Ripple also has its own cryptocurrency, it is mainly defined as a payment settlement, asset exchange, and a remittance system that works more like SWIFT, a service for international money and security transfers and the only digital asset specifically designed for banks and financial intermediaries enabling faster, cheaper and more reliable cross-border payments. No one owns Bitcoin and payments can be made without an intermediary. However, Ripple is privately owned and currently operated by Ripple Labs, Inc. rendering it theoretically, a centralized cryptocurrency and whereby payments require third party gateways. While Bitcoin transaction validations generally take 10 minutes, Ripple's transaction validations take 5 seconds. Furthermore, Ripple is not designed to be mined at all. They are pre-mined and burned as transaction fees whereby 100 billion Ripples have been pre-mined initially. This cryptocurrency is moving fast and becoming more credible each day, so much so that even the Federal Reserve are backing them and are telling banks to use Ripple XRP because of their fast and innovative technology. As of this writing, ripple ranks third on the list of top virtual currencies by market capitalization, behind Bitcoin and Ethereum.

Dogecoin is a decentralized cryptocurrency that was originally featured as a randomized reward system. The mining process is quite similar with rewards received for mining blocks of coins but the reward has not always been the same. Nonetheless, after the 600,000th block was mined, the developers emplaced an eternal reward of 10,000 Dogecoins. However, the Dogecoin system has no cap on the number of dogecoins that users can mine. Provided that miners continue on operat-ing, the Dogecoin supply will keep on increasing. The supply is set to surge by an estimated 5.2 billion dogecoins per year, endlessly. Additionally, Dogecoin differs from Bitcoin and Litecoin in several other respects. Most importantly, Dogecoin miners generally require 1 minute to verify a transaction, which significantly less than both its competitors.

Dash has gained popularity because it offers better privacy and higher transaction speed than Bitcoin. At Dash's core is an exclusive fully-incentivized peer-to-peer network. Rewards are given to miners for securing the blockchain and master nodes are rewarded for confirming, storing and providing the blockchain to users. Master nodes represent another level of network servers that operate in highly secure bundles called quorums to provide a variety of decentralized services, like fast transactions, privacy and governance, while removing the threat of low-cost network attacks. Dash is limited to 19 million coins that can come into circulation with an average block time and transaction speed of 2.5 minutes ascertaining itself as an attractive alternative.

Since the existing literature is scarce and the concept is relatively new to financial theory and the cryptocurrency itself as an asset class is still in its nascent stages, as cryptocurrencies are not widely credible, lack regulation and insurance, and are prone to illegal activities and technical flaws, the increasing circulation and abundance of cryptocurrencies is translated by some central banks' decision to regulate them. And since the stability of the market is always at a stake, the importance to look at the volatility of cryptocurrencies has gained further appeal in the recent past.

The purpose of this thesis is to evaluate and determine the best model or set of models for modeling the volatility of six of the most eminent cryptocurrencies: Bitcoin, Dash, Monero, Dogecoin, Litecoin, and Ripple against the behavior of six of the most influential currencies, namely the Euro, Japanese Yen, Swiss Franc, Canadian Dollar, Australian Dollar and the British Pound. The volatility of each cryptocurrency and hard currency is predicted for both in-sample and out-of-sample contexts. The selected models are SGARCH, IGARCH, EGARCH, GJR-GARCH, TGARCH, CGARCH, and APARCH. The literature has revealed that some models have constantly better predicted the volatility of a specific asset class. Defining the relatively best model in predicting each cryptocurrency's and fiat currency's volatility will, therefore, allow associating their behavior with that of a particular asset class and enhance existing literature with regard to their classification. Therefore, we aim to assess, through econometric analysis, financial statistics and modeling, the best-fitted volatility model by fitting GARCH-type models based on certain adopted maximum likelihood forecasting schemes, accuracy and goodness of fit measures, and assessment criteria chosen to evaluate forecasting performances and eventually conclusions are drawn about model capacity and reliability. Once the best model during the out-of-sample period for each asset is determined, we then forecast a one-step-ahead Value at Risk for each asset using the selected model. Eventually, back testing procedures are conducted to assess the accuracy of the Value at Risk results.

The thesis is structured as follows. Section 2 conducts a literature review on findings explicating the predictive capacity of the most common forecasting models and their findings on fiat and cryptocurrencies. Section 3 continues by first presenting and defining the basic structure of a volatility model and then details the specific models studied in this thesis. It then unravels the adopted procedures and methodology and the employed data while underlining the specificity of each. Their respective properties are discussed and an explicit expression of the forecasts of each model is presented. Section 4 portrays the main findings of the research where the parameters of the underlying models are estimated and the volatility for each asset is forecasted for the in sample period, and projected for the out of sample period. In addition, this section assesses the predictive ability of the selected model in accurately estimating the Value at Risk of each cryptocurrency and world currency. The empirical results are then presented, analyzed and back tested. Section 5 concludes the thesis and suggests topics for further research.

Chapter 2: Review of Literature

Modeling and forecasting the volatility of financial time series has become a field that has enticed a lot of scrutiny from empirical and theoretical practitioners. Over the past decades, volatility projection has been subject to an extensive number of studies, several of which draw a comparison between different forecasting models and assess their predictive abilities. As the future is always uncertain, the number of crashes and the size of their effects have enforced many analysts to look more prudently to the level and stationarity of volatility in time, with researchers shifting their attention towards the development and improvement of econometric models able to generate the most accurate forecasts and to optimally capture swings in returns' volatility. In the not too distant past, many theoretical models such as Merton (1969) and Black and Scholes (1973) supposed constant volatility. However, many researchers unveiled that traditional time series models that operate under the main assumption of constant variance were actually deficient in estimating asset return movements.

The traditional methods of measuring volatility (variance and standard deviation) are unconditional and cannot capture the characteristics exhibited by financial time series, such as, time varying volatility, volatility clustering, leverage effect, excess kurtosis, heavy tails, and long memory properties (Omar et al., 2017).

Today, however, it is a well-known fact that volatility of asset returns is time-varying and predictable; see Andersen and Bollerslev (1997). The original Autoregressive Conditional Heteroskedasticity (ARCH) specification, introduced by Engle (1982), was one of the first models that provided a way to model conditional heteroskedasticity in volatility and to capture these properties of financial time series data. Engle proposed the use of ARCH models that allows the conditional variance to change over time as a function of past errors leaving the unconditional variance constant by assigning equal weights to the squared residuals solely and disregarding past variances. The model was simple and intuitive but had limitations and required usually many parameters to describe adequately its volatility process.

Bollerslev (1986) amended this idea addressing the weakness found in Engle's model, which put forward the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) framework allowing a longer memory and a more adaptable lag structure. In practice, variance rates do tend to be pulled back to a long-run average level in a process known as mean reversion. GARCH incorporates mean reversion whereas ARCH does not, as GARCH integrates an additional parameter, the long term mean variance, which allows tracking the persistence of variance around the mean. Therefore, GARCH is theoretically more appealing than ARCH. Moreover, the GARCH process is generally favored by financial modeling professionals because it provides a better real-world context as it allocates more weight for recent observations and entails only three parameters allowing for an infinite number of past squared innovations to influence the conditional variance. These aspects enable GARCH to be more efficient and effective than ARCH thereby providing a better fit when the data exhibits heteroskedasticity and volatility clustering. However, when considering the forecasting accuracy of a model, the model's predictive ability should be tested both in in-sample and out of sample periods. ARCH and GARCH models were found to result in significant in sample parameters estimates and were regarded as the main tools for modeling volatility and have become widespread tools for dealing with time series heteroskedastic models as they have proven surprisingly successful in predicting conditional variances. Despite the empirical success of ARCH and GARCH models, questions still remain surrounding the true motives why volatility tends to cluster. This explains why certain models tend to outperform other models occasionally and over a specified period, rather than regularly.

Nevertheless, although the standard Gaussian GARCH has been used extensively in practice, Bollerslev evaluates his model based only on in sample forecasts. Therefore, it fails at forecasting the out of sample performance. The other problem encountered by the GARCH model is that it does not fully embrace the property of thick/heavy tails which is so much evident in the distribution of financial time series. Nelson (1991) criticizes GARCH from another perspective. He recognizes three main weaknesses in the symmetric GARCH model. First, he finds that there is an adverse correlation between assets' returns and changes in return volatility which is not captured by GARCH. Second, he asserts that the no negativity constraint of the parameters can complicate the model's forecasting process. Finally, the author expresses many concerns as to the explanation on the persistence of shocks and how it may affect the term structure of volatility whereby he claims that both ARCH and GARCH impose the assumption that the conditional volatility is affected symmetrically by positive and negative innovations. In light of the above complications, analysts were not satisfied with the existing models whereby many extended models of GARCH were proposed.

To be able to model this behavior and overcome the weaknesses in the GARCH model, Nelson (1991) proposed an extension to the GARCH model in a process known as Exponential GARCH (EGARCH). The model overcomes the previously mentioned drawbacks allowing for asymmetric effects of positive and negative asset returns by integrating both the sign and the size of shocks thereby ameliorating the estimation process. Other widely used extensions of the GARCH model and additional specifications were then developed that are known as "asymmetric power autoregressive conditional heteroskedasticity models" with each model conquering the disadvantages of the other: the Student's t-GARCH model of Bollerslev (1987), GJR-GARCH of Glosten, Jagannathan and Runkle (1993), APARCH of Ding, Granger and Engle (1993), the Threshold GARCH (TGARCH) model of Zakoian (1994), et cetera.

Virtually, a volatility model should not only aim to forecast volatility; but must integrate certain crucial elements. In their paper, Engle & Patton (2001) use the Dow Jones Industrial Index to stress on some stylized facts about volatility that should be integrated in a model. Typically, they mainly cite four essential features. First, the model should exhibit volatility clustering, meaning that periods of high (low) volatility are followed by another period of high (low) volatility. Engle and Patton displayed the daily returns on the Dow over a twelve-year period and revealed evidence that the volatility of returns varies over time. Similarly, Mandelbrot (1963), Chou (1988), and Baillie et al (1996) also reported evidence of volatility clustering. The second aspect involved is the mean reversion principle. This principle states that there is an average volatility level to which volatility shall eventually converge to. In addition, a fit model should have an asymmetric structure that permits the leverage effect, allowing negative and positive shocks to have different impacts on the volatility. Finally, a model should recognize that exogenous variables can impact volatility forecasts. The authors favor asymmetrical GARCH models to linear models; however, these models may lack the ability to take into consideration the impact of exogenous variables.

The volatility of exchange rates, specifically after the fall of the Bretton Woods agreement, has been a constant source of concern for both policymakers and academics (Héricourt & Poncet, 2015). Indeed, in the existing era of ever-escalating globalization and intensified currency volatility, fluctuations in exchange rates have a substantial impact on companies' operations and decision-making. This is why exchange rate risk has risen to be a useful measure of uncertainty as it is of crucial importance specifically in investment analysis and risk management whereby it can impact welfare, inflation, exposure in terms of transaction costs, translation exposure, international trade, etc. Over the last decade, the foreign exchange market has become one of the most volatile and liquid financial markets in the world. Therefore, volatility revolving around exchange rate risk should be effectively mitigated.

Accordingly, because of the dynamics of the foreign exchange market, it is essential to study the behavior of currencies and currency exchange. In their paper, Omari, Mwita, & Waititu (2017) applied symmetric and asymmetric GARCH models to model the exchange rates of the US Dollar with respect to the Kenyan Shilling over the period 2003-2015 in order to capture some stylized facts about exchange rate returns, such as volatility clustering and leverage effect. Daily data were extracted from the Central Bank of Kenya. The data revealed significant departure from normality combined with a negative skewness and excess kurtosis. The paper evaluates the performance of the symmetric GARCH (1,1) and GARCH-M models in addition to the asymmetric EGARCH (1,1), APARCH (1,1) and GJR-GARCH (1,1) models allowing them to follow three innovative distributions: Normal, Student's t, and Skewed Student's t distributions. The relative performance of the symmetric & asymmetric GARCH family models in estimating and forecasting the Value at Risk (VaR) is also tested. It was concluded that the asymmetric APARCH, GJR-GARCH and EGARCH models with student's t-distribution provide superior results in modeling the volatility of the US Dollar with respect to the Kenyan Shilling. The log likelihood function was applied and the models were assessed for accuracy using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Then, the authors compared the VaR estimates of each asymmetric model following the student's t distribution. It was derived that the APARCH (1,1) model is the best model for estimating the one-step ahead value at risk. Similarly, Thorlie et al. (2012) inspected the volatility of the SLL/USD exchange rate over the period 2004-2013. Their results showed that the forecasting performance of GJR-GARCH model using the skewed Student t-distribution is most successful in measuring the conditional variance as the Asymmetric GARCH and GARCH models were found to better fit under the non-normal distribution than the normal distribution.

Olowe (2009) modeled the volatility of Naira / US Dollar exchange rates on a sample of monthly data from 1970 to 2007. Six different GARCH models were tested. His findings were quite analogous as the best fitted models were the Asymmetric Power ARCH and the Threshold Symmetric GARCH. As such, many studies have shown that countries from Western, Eastern, and Central Africa tend to perform relatively better under asymmetric GARCH. However, a limitation of those studies is that each empirical research focused solely on a single exchange rate and therefore their findings cannot be generalized to other exchange rates in the market. Furthermore, the study itself might have become obsolete with the ever alternating dynamics of the foreign exchange market as the findings may have become outdated and thereby invalid.

Nevertheless, although there have been extensive empirical studies accentuating on modeling and estimating exchange rate volatility in developing, emerging and developed countries by applying different specifications, little attention has been devoted to Arab countries despite the importance of the Gulf States in the region. Abdalla (2012) is the first to investigate this concern by modeling the exchange rate return volatility in a panel of nineteen Arabian countries using daily observations covering the year span 2000 - 2011. The main currencies examined are the Bahraini Dinar (BHD), Kuwaiti Dinar (KWD), Omani Rial (OMR), and etc. all against the US Dollar. The daily returns for all currencies display either a positive or negative skewness or excess kurtosis indicating departure from normality. In addition, a highly leptokurtic distribution is observed for all series. The ARCH-LM test results provided strong evidence of heteroskedasticity and presence of ARCH effects in the residuals for all currencies except for the Iraqi Dinar and Libyan Dinar. In order to investigate further the existence of leverage effect, he conducts sign and size bias tests for asymmetry. The empirical results suggest that returns volatility exhibit asymmetric behavior, suggesting that the asymmetric volatility models are better suited for capturing dynamics of the volatility process in the data series. Under GARCH (1, 1), the conditional variance is an explosive process for ten of the nineteen currencies, while it was found to be quite persistent for seven currencies. This revealed that shocks to volatility are very high and will remain forever as the variance is not stationary under GARCH (1, 1). The asymmetrical EGARCH found evidence of leverage effects for all currencies except for the Jordanian Dinar inferring that negative shocks imply a higher next period conditional variance than positive shocks. The author concludes by asserting that exchange rates volatility can be adequately modeled by the different classes of GARCH models. However, a stronger and a more accurate insight about exchange rate behavior would have been attained if further models were integrated.

Yet, Granger and Poon (2005) compare 93 studies that carried out tests on the predictive ability of volatility forecast models. They classify the models as historical volatility models, ARCH models, stochastic volatility models and implied volatility models. Then they present pair wise comparisons of the models in order to determine which models are the most frequent winners. It is found that implied volatility models outperform other models. One possible explanation could be that options prices incorporate all market information and characteristics. Historical volatility models and GARCH rank next, while stochastic volatility models are the worst performers. However, the authors report that within the ARCH class, asymmetric GARCH models do better than the symmetric GARCH (1, 1).

Many authors have also shown interest in comparing predictive models beyond the ARCH/GARCH class. Pilbeam and Langeland (2014) for instance draw a comparison between GARCH models and the implied volatility model in estimating volatilities of foreign exchange markets. They choose to evaluate the traditional GARCH (1,1) model, two asymmetric GARCH models- EGARCH and GJR-GARCH- and the implied volatility model from call and put options prices. In addition to comparing the predictive ability of the models, the authors examine the pricing efficiency of currency options and observe whether market characteristics are well captured. The models are tested over the period of 2002–2012, and in-sample forecasts are generated. It is found that the implied volatility model outperforms symmetric and asymmetric GARCH models, as corroborated by Granger and Poon (2005). However, the authors highlight an interesting point that was missed by other authors and which shows that during periods of high volatility, the accuracy of all models lessens and models' outputs are further away from realized volatilities. The limitation of this study is that the dataset does not include out of sample forecasting.

The literature depicting the predictive capacity of models does not only revolve around exchange rates but almost all asset classes. Since 2009, controversies have scaled with the rise of Bitcoin as it was unclear whether to classify it as a currency or a commodity, and many inquiries were raised regarding its essence and implications in the world of finance. Baek and Elbeck (2015) address two vital issues concerning Bitcoin: how unstable is it and what is 'electrifying' its volatility. First, the authors compare the volatility of Bitcoin with respect to the S&P 500. The comparison, which is an indication of the intrinsic risk of the Bitcoin, shows that the Bitcoin market is extremely risky and speculative. Then, in order to identify the drivers of such volatility, the authors conduct a regression analysis to study the impact of many market variables on the Bitcoin's return. It is found that external economic factors do not influence the cryptocurrency's market returns, and the volatility is internally driven by forces of demand and supply. On January 5 2017, Bitcoin has marked one of its sharpest rallies and falloffs over just 24 hours of trading as Bitcoin's value plunged from a high $1,151 down to a low $874. The crash was so swift that at one point Bitcoin lost more than $3 Billion in market value in just 40 minutes, "slashing" its market capitalization. Bitcoin has also revealed its upside volatility particularly in December. Between November and December 2017, Bitcoin's price has increased by around $12,000 as the CBOE started its futures trading on bitcoin that has reached a record high of $19,783 (Worah, 2018). Shortly after in 2018, Bitcoin kept on trading below $10,000 declining by more than 80% from its record-high. Therefore, within a period of 12 months, Bitcoin rose by 2,373% marking its presence, thus far, as one of the highest volatile financial instruments.

Accordingly, volatility has been heavily accentuated recently specifically after the subprime mortgage crisis that has seen the financial system dissolve. Compared with traditional currencies, Bitcoin and cryptocurrencies, in general, have been relentlessly scrutinized as they have proven to be less stable, easier influenced by speculative factors and revealed intensified levels volatility. And with many forecasting models at hand, considering the massive fluctuation in cryptocurrencies and its associated percussions on the financial system, it is getting more and more urgent to predict their volatility.

At this stage, the amount of research that has been done on Bitcoin and other cryptocurrencies is still in short supply. Naimy & Hayek (2018) were the first to contrast and assess the predictive abilities of GARCH (1, 1), EWMA, and EGARCH in forecasting the volatility of the Bitcoin/USD exchange rate. 1093 daily observations were extracted covering the sampled period April 1st 2013 until March 31st 2016. Upon plotting the return series, Bitcoin seemed to exhibit persistence and volatility clustering implying that the volatility can be forecasted. The authors used the student's t distribution and generalized error distribution along with the normal distribution but the latter was found to be the most precise probability density function. Parameters were calibrated and calculated using maximum likelihood estimates from in-sample returns in order to compute in-sample volatility. Out of sample volatility is calculated afterwards. Estimated volatilities are then compared to realized volatilities relying on error statistics, namely the mean absolute error (MAE), the root mean square error (RMSE) and the mean absolute percentage error (MAPE) that pointed out the relative superiority of EGARCH (1,1) in both in sample and out of sample contexts with increased accuracy in out of sample period. Their findings are, therefore, consistent with the conclusions reached by Engle & Patton (2001) with the authors concluding that the predictive abilities of the models are worsened when volatility exhibits extreme movements and improves when volatility is relatively low. The authors note that the implied volatility model is not tested since the Bitcoin options' market is very recent and immature as data on historical option prices is not yet available. Finally, the authors assert that the Bitcoin's behavior is not similar to the behavior of currencies. A limitation of the study is that the research focused entirely on Bitcoin which was still in its nascent stages with short history whereby past behaviors might not be good indicators to reflect on its future performance.

Similarly, Krogt (2018) investigates the behavior of Bitcoin and whether it can be classified as a currency or security. In his paper, he analyses the volatility of Bitcoin and compares its volatility process with that of a security (S&P-500) and a currency (EUR/USD exchange rate). The one-day ahead volatility is examined for the GARCH (p,q), TGARCH (1,1), EGARCH (1,1) and APARCH (1,1) models. The goodness-of-fit and forecasting abilities are evaluated with the use of Akaike/Bayesian information criteria and the Mincer-Zarnowitz regression, respectively. The analysis shows there are similarities between Bitcoin and the S&P-500 from a volatility process point of view. The EGARCH (1, 1) model fitted the volatility best for the Bitcoin and the S&P-500 whereby APARCH (1, 1) seemed to have fairly accurate forecasting power on the EUR/USD. Nevertheless, the author demands for the use of a more detailed analysis on choosing the ex-post volatility proxy for Bitcoin in the Mincer-Zarnowitz regression model. He also emphasizes the highly volatile distinctive feature of Bitcoin that is 35 times as high as the S&P-500 and 200 times as high as the EUR/USD exchange rate. Briere et al. (2015) and Cheah and Fry (2015) confirm that the Bitcoin market is highly speculative and more susceptible to speculative bubbles than other currencies. The European Central Bank has a valid motivation, therefore, which is consistent with the findings of this paper for not labeling Bitcoin as a true currency due to its high volatility (ECB,

2015) as Bitcoin behaves similar to a security rather than a currency. As such, the asymmetry in the Bitcoin market is still significant suggesting that the Bitcoin market is still far from being mature.

The recent spike in the Bitcoin prices has triggered an increased interest in exploring the dynamics of the cryptocurrency markets. In a turbulent time with the main-stream financial system reeling from the aftershocks of the 2008 crisis and the European Sovereign debt-crisis, cryptocurrencies started gaining traction with the cryptocurrency market expanding rapidly. As such, Kumar & Anandarao (2019) investigate the dynamics of volatility spillover across four major cryptocurrency returns namely Bitcoin, Ethereum, Ripple and Litecoin and implement a comprehensive methodology in order to capture the cryptocurrency market dynamics. The period selected is August 2015 till January 2018 because it is characterized by extreme fluctuations in the cryptocurrency markets and therefore providing better insights about the nature of volatility spillover across cryptocurrency markets. It is seen that there is statistically significant volatility spillover from Bitcoin to Ethereum and Litecoin during that period with increased spillover after 2017. Overall, the results indicate the possibility of herding behavior and turbulence in the cryptocurrency markets. Noting that the cryptocurrencies' market capitalization lost at least 243 billion US dollars in the first quarter of 2018 and by September 2018, cryptocurrencies collapsed by 80% from their peak marking this crash as the worst in the history of cryptocurrencies. From here, the cryptocurrency market still remains a potential source of financial instability and uncertainty. For that reason, predicting their volatility has become more evident than ever.

Chu et al. (2017) provided the first GARCH modeling of the seven most popular cryptocurrencies ranked by market capitalization as of May 2017. They fitted SGARCH (1, 1), EGARCH (1, 1), GJRGARCH (1, 1), APARCH (1, 1), IGARCH (1, 1), CSGARCH (1, 1), GARCH (1, 1), TGARCH (1, 1), AVGARCH (1, 1), NGARCH (1, 1), NAGARCH (1, 1) and ALL GARCH (1, 1) models to the log returns of the exchange rates of each of the following cryptocurrencies: Bitcoin, Dash, Dogecoin, Litecoin, Maidsafecoin, Monero and Ripple. The distribution of the innovation process were taken to be one of normal distribution, skew normal distribution, Student's t distribution, skew Student's t distribution, skew generalized error distribution, normal inverse Gaussian distribution, generalized hyperbolic distribution or Johnson's SU distribution. The goodness of fit measures were evaluated based on the values of the Akaike Information Criterion (AIC), the corrected Akaike Information Criterion (AICc), the Bayesian Information Criterion (BIC), the Hannan-Quinn criterion (HQC) and the Consistent Akaike Information Criterion (CAC). The normal distribution was found to give the most accurate measures for each cryptocurrency and among the GARCH models, IGARCH (1,1) provided the best fit for Bitcoin, Dash, Litecoin, Maidsafecoin and Monero; while the GJRGARCH (1,1) model gave the best fit for Dogecoin; and the GARCH (1,1) was found to be the optimal fit for Ripple. Upon having a clear insight about the best suited model for each cryptocurrency, the authors then move on to perform the unconditional and conditional coverage value at risk exceedance test. All p-values were found to be significant at the five percent confidence level. Hence, it was derived that the best fitting models can be used to provide acceptable estimates of the Value at Risk. Eventually, the results show that cryptocurrencies exhibit extreme volatility with a future work depicting their joint behavior using a multivariate GARCH deemed necessary.

The study of Holtappels (2018) inspects this issue by quantifying the manner in which the variance of cryptocurrencies behaves compared to this same effect for fiat currencies and indices. For cryptocurrencies, he constructs six individual portfolios (Bitcoin, Ethereum, Ripple, Litecoin, EOS, NEO) and an altcoin package consisting of 15 other cryptocurrencies (Dogecoin, Byetcoin, Verge, Lisk, Waves, Dash, Bitshares, NEM, Steem, Ethereum Classic, Monero, Stellar, Siacoin, DigixDAO, and Stratis). The fiat currencies examined are the Euro, Japanese Yen, British pound, Canadian dollar, Australian dollar, and the Swiss franc whereas the Indices data considered where the USA Dow Jones Industrial Average, Japanese Nikkei 225 Stock Average, French CAC 40, Chinese SSE Composite, German DAX 30 and the British FTSE 100. The author uses the MGARCH model for the joint modelling of several time series since the volatility of one time series is influenced by both its own past values and the past values of other time series in regression. With the influence of Bitcoin on altcoins confirmed in many studies and since the MGARCH model also captures the volatility spillover between variables, it is of high interest now to detect what influences cryptocurrencies' volatility. The study revealed that the lagged values of the variance have a relatively strong impact on the current variance in terms of strength and persistence as compared to fiat currencies and indices. Additionally, the correlation among cryptocurrencies was found to be substantially larger than those between fiat and indices. Finally, the results show that the variance of fiat currencies and indices seems to revert to a long run average level whereby cryptocurrencies tend to exhibit an unstable and explosive variance forecast. A limitation of this study, however, is quite similar to that of Naimy & Hayek (2018) whereby the author emphasizes that the cryptocurrency market is relatively new compared to that of equities and fiat currencies whereby their behavior might change in the future as the cryptocurrency market matures. Therefore, the findings of this research may become irrelevant in the distant future in terms of using them to explain the behavior of variances. Over a certain amount of years, a more comprehensive research can be executed, as results can be compared within multiple periods of economic prosperity and economic downturn. Also, this research is limited to the number of cryptocurrencies used, whereby an increase in the dataset provides a possibility to run further divergent regressions and hence, examine a more detailed relationship among several asset classes.

In light of the above and with the ever increasing interest in cryptocurrencies and their importance in the financial world, there is need for a comprehensive analysis to study volatility dynamics and out-of-sample forecasting behavior of cryptocurrencies. Moreover, the complex dynamics underlying the evolution of the cryptocurrencies' volatility is yet to be "fully" explored. However, despite the growing interest, acceptance and integration of cryptocurrencies to the global financial markets, the majority of recent studies have focused entirely on Bitcoin's behavior or a few other cryptocurrencies and specifically on the in-sample modelling framework (Trucios, 2019). Nevertheless, the most out-of-sample comparisons available in the existing literature focusing on the volatility dynamics of the cryptocurrency market are restrictive since they only consider few models leaving out several GARCH-type models and several innovations distributions.

In their paper, Omari et al. (2019) unfold those issues. Their study contributes and extends on the existing literature on modeling cryptocurrencies volatility dynamics by employing a wider range of GARCH-type models (namely: SGARCH, IGARCH, EGARCH, GJR-GARCH, TGARCH, APARCH, CSGARCH, AVGARCH, NGARCH, NAGARCH, FGARCH, and FIGARCH), and assuming nine different innovations term distributions and a longer time period to try and fill a gap in the literature. The eight most popular cryptocurrencies considered (by market capitalization as of August 2018) are: Bitcoin, Ethereum, Litecoin, Ripple, Monero, Dash, Stellar and NEM. For each cryptocurrency, the parameters for all GARCH models were estimated using the QMLE method and the optimal model was selected based on the AIC, BIC, and HQIC. Subsequently, the out-of-sample forecast results were used to determine the GARCH-type specification that has a better VaR forecasting performance. The accuracy tests of VaR forecast of each model are evaluated next by means of back testing procedures using the conditional and unconditional coverage tests. The results demonstrate that the asymmetric GARCH models mostly have better VaR forecasting performance for all cryptocurrencies especially at the 99% level of significance. Moreover, the skewed versions of student-t, GED, and hyperbolic distributions confirm their predominance over the alternatives in terms of better predictive ability. Finally, concerning the accuracy tests, the VaR forecasting performance results differ among cryptocurrencies. The authors conclude by recommending asymmetric GARCH models with a long memory property, skewed and heavy tailed distributions to optimally forecast the value of risk of cryptocurrencies.

As the cryptocurrency market has relatively now conciliated, it is now apparent that a study analyzing the volatility of the cryptocurrency market with respect to fiat currencies is inevitable with questions being raised about whether cryptocurrencies have a stand to be a viable alternative to fiat currencies. Section 3 continues by first presenting and defining the basic structure and assumptions of each volatility model. It then unravels the adopted procedures and methodology and eventually analyzes the employed data while underlining the required specificities to model financial time series.

Chapter 3: Methodology & Procedure

The review of the available literature in the previous section reveals a complete absence of research on comparing the behavior of cryptocurrencies with that of exchange rates, and more specifically on assessing their relative performance upon identifying the best volatility process around which each one revolves. Unsurprisingly, the main reasons behind the inconsistencies found in research studies are due to the time frames involved and volatility models employed, as the returns of cryptocurrencies are highly volatile with regard to other asset classes. Despite the inconsistency and inconclusiveness found in research studies, GARCH models are identified among the most prevalent superior ones. In this section, we first present the GARCH-type models that will be used to model time-varying volatility in cryptocurrencies and exchange rates return series. Their parameters are estimated and the volatility for each asset under each of the selected models is then computed for the in-sample and out-of-sample periods. To assess the accuracy of the tested models, the estimated volatility is compared to the realized volatility. The selection criteria that will be exploited to reveal the most pertinent model with its relative distribution for each asset class are then described. Accordingly, the optimal GARCH-type models selected eventually are used to forecast the one-day ahead conditional variance for all cryptocurrencies and exchange rates. Next, formulas are provided for estimating the value at risk based on the volatility updated historical simulation method. Afterwards, back testing tests are employed. Finally, the data is presented and statistical analysis is conducted for qualification purposes to determine whether the inspected models can be implemented.

3.1. Applied Models

Let Pt denote by the daily observations of the respective cryptocurrencies and exchange rate data series at time t for $t = 1, \ldots, n$. In risk management, daily volatility is defined as the standard deviation of the proportional change in the variable during a day whereby the simple return is used instead of the continuously compounded return (Hull, 2012). Hence, daily prices are converted into daily returns as per the following equation:

$$u_t = \frac{P_t - P_{t-1}}{P_{t-1}} \qquad \textbf{(1)}$$

where u_t represents the return at day t, *Pt* and *Pt-1* are the corresponding cryptocurrencies and fiat currencies prices against the US Dollar at the end of day *t* and at the end of the preceding day *t-1* respectively. Then, GARCH models can be specified as:

$$X_t = \mu_t + \sigma Z_t \quad \textbf{(2)}$$

Where μ_t denotes the conditional mean and σ_t denotes the volatility process. For brevity, all models are restricted to a maximum order of one where we consider only the first order lags (p=q=1), since empirical evidence has proven them to be more flexible, efficient and significant with higher order models rarely performing better than lower order models in the out-of-sample analysis (Hansen & Lunde, 2005).

Essentially, we employ seven GARCH-type specifications to model the volatility behavior of cryptocurrencies and exchange rates, namely: SGARCH, IGARCH, EGARCH, GJR-GARCH, APARCH, TGARCH and CGARCH models. All of the GARCH models implemented follow the above specification in (2); however, in each case, the volatility process "*σt*" is different. A brief description on the applied models is illustrated in the following subsection.

3.1.1. Estimated Volatility using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Models

Bollerslev (1986) provided an extension to the basic ARCH model, allowing the conditional variance to vary as a function of its lagged returns and conditional variance. As such, the conditional variance for the **Standard GARCH (1, 1)** process introduced by Bollerslev is given by:

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 \qquad \textbf{(3)}$$

$$\omega = \gamma V_L \qquad \textbf{(4)}$$

Where σ_t^2 is the estimate of the variance for day t, u_{t-1}^2 and σ_{t-1}^2 represent the associated return and the variance on the previous day with α and β being their respective weights. The long run variance "V_L" is an average level towards which variances revert to through a principle called mean reversion, with γ being the weight assigned to V_L. The model is considered stable when the weights γ, α and β sum-up to 1. The main feature of this model is that it captures volatility clustering and persistence in the data through the parameters α+β with restrictions $\omega \geq 0$, $\alpha \geq 0$, $\beta \geq 0$ and $\alpha+\beta < 1$ to ensure a uniquely stationary process and positivity of the conditional variance. However, if the sum of the parameters α and β equals 1, the GARCH model converges to the Integrated GARCH model where the long term volatility bears an explosive process.

Engle and Bollerslev (1986) later unveiled the **Integrated GARCH** model. It is a restricted version of the standard GARCH (1, 1) model where the parameters α and β sum up to 1 and typically imports a unit root under the GARCH process. Thus, the IGARCH (1, 1) can be expressed as follows, given that β is now set equal to (1 – α) with restrictions $\omega \geq 0$, $\alpha \geq 0$ and $1 - \alpha \geq 0$:

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + (1-\alpha)\sigma_{t-1}^2 \qquad \textbf{(5)}$$

In the SGARCH and IGARCH models, the impact of positive and negative news on the conditional variance are symmetrical. Although the Standard GARCH model has a greater applicability due to its easy computation, the GARCH model cannot explain the negative correlation between return and volatility. Moreover, the GARCH model restraints all coefficients to be greater than zero, which complicates the model's application. For this reason, Black (1976) discovered that current return and future volatility have negative correlation and that the impact of positive and negative shocks on the conditional variance is rather asymmetrical. This came to be known as the "leverage effect" after which more advanced models were developed to incorporate its effect.

In 1991, a few years later, Nelson unfolded a more prevalent model known as **Exponential GARCH** model, denoted by EGARCH (p, q), to incorporate for the asymmetric impact of positive and negative shocks on volatility whereby the latter is believed to produce greater levels of volatility, despite having the same magnitude. A difference from the SGARCH (1, 1) model is that it considers log returns, which suggests that the parameters are unrestricted, and are thereby allowed to take negative values while ensuring a positive conditional variance. In addition, the conditional variance is written as a function of past standardized innovations, instead of past innovations. Formally, an EGARCH (1, 1) can be written as:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \quad \textbf{(6)}$$

Where β represents the persistence parameter and α and γ capture the size and the sign (leverage) effect, respectively. The above specification exhibits an asymmetric effect when $\gamma \neq 0$. More specifically, if the leverage parameter "γ" is negative, this means that negative news affect volatility more than positive news. Conversely, if returns and volatility are positively correlated, γ will be positive thereby positive shocks will have a higher impact on volatility than negative shocks, which is irregularly the case.

The **Glosten-Jagannathan-Runkle GARCH** (GJR-GARCH) model by Glosten et al. (1993) is similar to EGARCH (1, 1) in that they both incorporate the asymmetric impact of positive and negative shocks. However, GJRGARCH is given by:

$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})u_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (\textbf{7})$$

Where It-1 = 1 if u_{t-1} < 0 and It-1 = 0 if $u_{t-1} \geq 0$. A defining feature of this model is that a positive shock will increase volatility by α_t, whereas a negative shock will increase volatility by α_t+ γ_t at a specified time t. However, in contrast to the EGARCH model, the leverage effects exists when γ > 0, indicating that past "bad news" have stronger impact on current volatility than past "good news”. If γ < 0, then past positive returns increase current volatility more than past negative returns. The persistence in this model relies on $\alpha, \beta,$ and γk with k representing the average value of standardized errors. Parameters restrictions are similar to the Standard GARCH whereby ω ≥ 0, α ≥ 0, and β ≥ 0.

The **Asymmetric Power ARCH** model by Ding et al. (1993) models for both the leverage and the effect that the sample autocorrelation of absolute returns are usually larger than that of squared returns through its “power parameter”; allowing for more flexibility where σ_t^2 is replaced by σ_t^δ and it is given by:

$$\sigma_t^\delta = \omega + \alpha(|u_{t-1}| - \gamma u_{t-1})^\delta + \beta\sigma_{t-1}^\delta \qquad (\mathbf{8})$$

For δ, α, β and ω being ≥ 0 and -1 ≤ γ ≤ 1, where δ is the Taylor (power effect) parameter after Taylor (1986) for the Box-Cox Transformation [1], γ is the leverage parameter and the persistence parameter is given by $\beta + \alpha k$. Signs analysis for the leverage parameter are similar to the GJR-GARCH model, where a leverage effect exists once γ > 0. Noting that APARCH (1, 1) converges to the GJR-GARCH (1, 1) model when δ = 2 and to the Standard GARCH (1, 1) model with restrictions: δ = 2 and γ = 0.

The **Threshold GARCH** model due to Zakoian (1994) is similar to the GJR-GARCH model and is a particular case of APARCH (1,1) with δ = 1, which models for the conditional standard deviation instead of the conditional variance with the restraint -1 ≤ γ ≤ 1. TGARCH (1, 1) is typically expressed as follows:

$$\sigma_t = \omega + \alpha(|u_{t-1}| - \gamma u_{t-1}) + \beta\sigma_{t-1} \qquad (\mathbf{9})$$

[1] The Box-Cox Transformation is a statistical technique used to transform non-normal dependent variables into a normal shape.

By contrast to the standard GARCH (1,1) model that shows mean reversion to a constant term "ω", the **Component GARCH** model by Engle and Lee (1999) allows mean reversion to a varying level "q_t", known as the time varying long run volatility. Bauwens & Storti (2009) emphasized that the volatility is modeled as a convex combination of unobserved GARCH components where the weights are time varying as a function of appropriately chosen state variables. As such, The CGARCH (1, 1) splits the conditional variance into its transient (eq.10) and permanent components (eq.11) to examine short and long-term effects on volatility, as presented below:

$$\sigma_t^2 = q_t + \alpha(u_{t-1}^2 - q_{t-1}) + \beta(\sigma_{t-1}^2 - q_{t-1}) + \gamma(u_{t-1}^2 - q_{t-1})I_{t-1} \quad \mathbf{(10)}$$

$$q_t = \omega + \rho(q_{t-1} - \omega) + \phi(u_{t-1}^2 - \sigma_{t-1}^2) \quad \mathbf{(11)}$$

Similar to GJR-GARCH model, The CGARCH specification in eq. (10) captures asymmetric responses to shocks by introducing the slope dummy variable "I_{t-1}" to the leverage parameter that takes the value of "1" for $u_{t-1} < 0$, and $I_{t-1} =$ "0" if otherwise. A positive gamma "γ" indicates the presence of transitory leverage effect in the conditional variance. Stationarity of the CGARCH model and non-negativity of the conditional variance are ensured once the following inequality constraints are satisfied: $\omega \geq 0$, $\alpha \geq 0$, $\phi \geq 0$, $\beta \geq 0$, $\beta \geq \phi$ and $\alpha+\beta \leq \rho \leq 1$.

3.1.2. Realized Volatility using Merton's Model

In order to gauge the accuracy of the tested models, estimated volatility should be compared to the realized volatility (Naimy & Hayek, 2018). In 1980, Robert Merton proposed a model to calculate the realized volatility based simply on the asset's returns. His model suggested that when the sampled variable consists of many observations, the sum squared returns is an accurate estimation of volatility. As such, the formula that will be adopted for the annual realized volatility "σ_t" is expressed as follows:

$$\sigma_t = \sqrt{\frac{252}{22}\sum_{t-22}^{t-1} u_i^2} \quad \mathbf{(12)}$$

Where t represents the day of observation and u_i is the return on day i such that $t-22 < i < t-1$. In other words, this indicates that the monthly realized volatility "σ_t" is calculated based on the most recent 22 daily returns. Results are then annualized by multiplying monthly volatilities by 252/22.

3.2. Maximum Likelihood Methodology for Parameters Estimation

After setting $\omega = \gamma V_L$, the parameters ω, α and β in equation (3) can be estimated using the Maximum Likelihood Method where:

$$\gamma = 1 - \alpha - \beta \qquad \textbf{(13)}$$

Therefore, the Long-run Variance V_L will be set equal to:

$$V_L = \frac{\omega}{\gamma} \qquad \textbf{(14)}$$

The Maximum Likelihood Estimation (MLE) is a method for estimating the parameters of a distribution by maximizing a certain likelihood function so that under the assumed statistical model the observed data is most probable. In finance, however, the Likelihood Function is often replaced by the Log Likelihood Function (LLF). It is the adopted approach to estimate the parameters of the underlying models in this study. This method is generally preferred since it is consistent, intuitive, efficient and provides asymptotic standard errors that are valid under non-normality. The LLF is given as per the below equation:

$$\ln L^* = \prod_{t-1}^{T} \ln f(y_{t-1}, y_{t-2}, \dots , y_1, \theta_1, \theta_2, \theta_k), \qquad \textbf{(15)}$$

Where f is the conditional probability density function, y_t and θ_t are the respective values of the time series and the model parameters at time t. Throughout this thesis, however, the following log likelihood function is applied:

$$\textbf{(16)} - \ln(\sigma_n) - u_n^2 \,/\, 2\sigma_n^2$$

Where σ_n and σ_n^2 represent the daily conditional volatilities and variances over each observation and u_n^2 being the respective square of returns.

3.3. Distribution & Model Selection

For each model, the innovation process Zt is allowed to follow one of three distributions. In addition to the Normal Distribution, the Student's t Distribution and the Generalized Error Distribution (GED) are used as skewed and heavy-tailed distributions have recently proven to yield better results such as in the papers of Naimy & Hayek (2018) and Omari et al. (2019).

The selection of the best distribution curve is based on three information criteria: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and the Hannan-Quinn Information Criterion (HQC).

- ✓ The Akaike Information Criterion due to Akaike (1974) is defined by:

$$AIC = 2k - 2lnL(\hat{\Theta}) \quad \textbf{(17)}$$

Where k denotes the number of unknown parameters, Θ the vector of unknown parameters and $\hat{\Theta}$ their maximum likelihood estimates.

- ✓ The Bayesian Information Criterion due to Schwarz (1978) is defined by:

$$BIC = k\ ln\ n - 2lnL(\hat{\Theta}) \quad \textbf{(18)}$$

Where n denotes the number of observations

- ✓ The Hannan-Quinn Criterion due to Hannan and Quinn (1979) is given by:

$$HQC = -2lnL(\hat{\Theta}) + 2k \ln\ln n \quad \textbf{(19)}$$

In addition, other test statistics are used to assess the forecasting accuracy of the models: the Mean Absolute Error (MAE), the Root Mean Square Error (RMSE) and the Mean Absolute Percentage Error (MAPE). Generally, the error term is expressed as follows:

$$e_t = \hat{y}_t - y_t \quad \textbf{(20)}$$

With $\hat{y}_t$ and y_t representing the predicted and actual values respectively.

- ✓ The Mean Absolute Error is simply an average of the absolute errors and is given by:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|e_t| \quad (\mathbf{21})$$

- ✓ The Root Mean Squared Error, however, is the square root of the average squared errors:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2} \quad (\mathbf{22})$$

- ✓ The Mean Absolute Percentage Error equation below shows how each residual is scaled against the actual value, as follows:

$$MAPE = \frac{100\%}{n}\sum_{t=1}^{n}\frac{|e_t|}{|y_t|} \quad (\mathbf{23})$$

A lower measure for these test statistics indicate a better performance. Ranks from the alternative evaluation methods are then presented for the in sample and out of sample periods. The purpose of selecting the optimal out-of-sample GARCH model for each currency and cryptocurrency is to forecast the one-day ahead volatility that will be successively used to make VaR projections.

3.4. <u>Value at Risk & Estimation</u>

Nowadays, the importance and the adoption of an efficient risk management framework have increased dramatically. It is important for risk managers and financial institutions to continuously update and monitor the volatilities of market variables compromising their portfolios on which their values depend. Often a financial institution's portfolio depends on hundreds, or even thousands, of market variables. While very useful to traders, some risk measures do not provide senior management, fund managers and financial institutions with an indication of the total risk to which a financial institution is exposed (Hull, 2012).

Value-at-Risk (VaR) has become the main tool of reporting to the bank regulators the risk that financial institutions face (Angelidis & Degiannakis, 2008). It is a standard risk measure that is commonly used in risk management which measures and quantifies the level of downside risk into a single value. It is defined as the maximum loss expected for a given portfolio/position over a set time horizon *"T"* and a confidence level *"X"* percent. Put alternatively, it is the loss at the X^{th} percentile of the losses distribution over the next T days (Hull, 2012). Generally, risk is mainly categorized in five areas: market, liquidity, business, credit and operational risk. The focus of this thesis is on market risk, as VaR is widely adopted for measuring it. It is worth noting that Market Risk exposure is usually computed using a 99% confidence level and a 1-day time horizon.

The VaR forecast for the GARCH-type models relies on the one-day ahead conditional mean, μ_{t+1} and the conditional variance forecast, σ^2_{t+1} of the volatility model. Under each of the innovations term distribution assumptions, the one-day-ahead VaR forecast is calculated as:

$$VaR_{t+1}(\alpha) = \mu_{t+1} + F^{-1}(\alpha)\,\sigma_{t+1} \tag{24}$$

Where $F^{-1}(\alpha)$ is the α-quantile of the cumulative distribution function of the innovation distribution. Once the optimal out-of-sample GARCH model has been computed for each cryptocurrency and exchange rate, their corresponding Value at Risk can be forecasted.

3.4.1. The Historical Simulation Method & Volatility Updating Procedure

First, the "rolling returns" of each cryptocurrency and fiat currency are calculated from a 400 day rolling window procedure on their prices, simulated over 250 times. Then, the out-of-sample optimal models of each cryptocurrency and fiat currency are applied to compute the conditional variances and volatilities accordingly over each of the 250 sub-samples, with each sub-sample having 400 observations. Note that the number of sub-sample periods is chosen in accord with the number of days in the out of sample period.

The original approach integrates the optimal model selected for each cryptocurrency and fiat currency and involves using *"n"* day-to-day changes in the values of cryptocurrencies and exchange rates that have been observed in the past in a direct way to estimate the probability distribution of the change in the value of these assets between today and tomorrow thereby providing *"n -1"* alternative scenarios of what could be the value of those assets over the succeeding day using the following expression:

$$\text{Value under } i\text{th scenario} = v_n \frac{v_i}{v_{i-1}} \qquad \mathbf{(25)}$$

Where v_i and v_{i-1} are the respective values of the cryptocurrencies or exchange rates on day *i* and day *i-1* and v_n being a fixed value representing the asset's price on the most recent day of the selected sampled time frame.

However, since the volatility of market variables may vary over time, Hull and White (1998) suggested an extension to the Basic Historical Simulation approach to reflect on those variations in the market. They proposed a further adjustment by integrating "volatility updating" to the original procedure. In general, when this approach is used, the expression in equation (25) for the value of each cryptocurrency or currency under the *ith* scenario becomes:

$$\text{Value under } i\text{th scenario} = v_n \frac{v_{i-1} + (v_i - v_{i-1})\sigma_{n+1}/\sigma_i}{v_{i-1}} \qquad \mathbf{(26)}$$

The main modifications to the original approach to be raised from the above expression are the volatility parameters "σ_i" and "σ_{n+1}" denoting the estimated volatility at day *i* and the most recent estimate of volatility. For this reason, this approach typically accounts for volatility changes in a natural and intuitive way and produces VaR estimates that incorporate fresher information.

Subsequently, using the rolling window procedure, a viable list of return scenarios is computed under each simulation trial thereby generating a list of "possible percentage gains or losses", as per the below equation:

$$\text{Return under } i\text{th scenario} = \frac{(v_{ith\ scenario} - v_n)}{v_n} \tag{27}$$

With $v_{ith\ scenario}$ representing one of the possible future values for the selected cryptocurrency or fiat currency, previously calculated from equation (26). However, v_n represents the value of the selected cryptocurrency or fiat currency on the most recent out of sample date of the rolling window trial. Once all simulation trials have been generated, the VaR can be now calculated as the appropriate percentile of the probability distribution of this change.

The above procedure is undergone repeatedly for the 250 sub-sample periods in order to compute the VaR for the 250 days (extending from December 4th 2018 till November 18th 2019) at the 1%, 2.5%, 5% and 10% levels of significance.

3.5. Back testing Value at Risk

Whatever the method used for calculating VaR, an important reality check is back testing. Back testing is a technique for simulating a model on past data to gauge the accuracy and effectiveness of the Value at Risk calculations. In other words, it is a test of how well the current procedure for calculating VaR would have worked if it was performed in the past. Therefore, if a VaR model truly provides the level of coverage defined by its confidence level, then the failure rate (also known as the hit ratio) over the full sample will equal α for the $(1-\alpha)^{th}$ percentile VaR.

The accuracy of the estimated VaR in forecasting returns is assessed by comparing the out-of-sample VaR forecasts to the actual realized returns during the same time period and this is summarized in terms of a violation ratio. If violations happen on α *%* of the days (say on about 1% of the days for a 99% VaR), we can feel reasonably comfortable with the current methodology for calculating VaR. However, if the actual observed loss over the forecast period exceeds the VaR forecast, then the VaR limit is said to have been violated and an exception is recorded. If the number of exceptions surpass expectations, then the reported VaR measure systematically understates the asset's actual level of risk. Similarly, if the number of exceptions fall behind expectations, then this alternatively signals an overly conservative VaR measure.

Therefore, if the number of violations differ considerably from α *x 100%* of the sampled series, then the accuracy of the underlying model is called into question. Progressively, we perform the most prominent test widely known as Kupiec's Unconditional Coverage Test to conduct the back testing process and to determine whether the incorporated VaR model should be accepted.

3.5.1. Unconditional Coverage Test

Kupiec (1995) proposed a likelihood ratio test, known as the "Unconditional Coverage Test" that gauges the level of accuracy in back testing VaR. Specifically, the test is employed to verify whether the sample point estimate is statistically consistent with the model's prescribed confidence level. In other words, the Kupiec test will reject the model if it overstates/understates the true VaR. The null hypothesis states that the observed failure rate $\hat{p}$ is equal to the failure rate suggested by the VaR's confidence interval which is expressed as:

$$H_0 : p = \hat{p} = X / T$$

Where:

p – The specified model probability (in accordance to the VaR confidence level)

$\hat{p}$ – The observed failure rate

X – Number of exceptions/violations

T – Number of trials

Effectively, the likelihood ratio, denoted by "LR_K", after Paul Kupiec (1995) is given by:

$$LR_K = -2\ln\frac{[p^x (1-p)^{T-X}]}{\left[\left(\frac{X}{T}\right)^X \left(1-\frac{X}{T}\right)^{T-X}\right]} \sim X^2 \qquad \mathbf{(28)}$$

For the purpose of making a valid conclusion about the model's accuracy, the critical value from the Chi-Squared Distribution is used. If the likelihood ratio is greater (lower) than the associated critical value of '3.84', the test statistic reveals that the model should be rejected (accepted) at the level of confidence. Noting that the above expression can be greater than the said value of '3.84' for either a high or a low number of violations, implying that the model would be rejected on both occasions. As mentioned earlier, the number of violations, denoted by *"X"*, is computed by recording how regularly the actual loss exceeds VaR, which is the amount of exceptions noted once all days are account for. All cryptocurrencies and fiat currencies will have equal number of trials. Therefore, the number of trials denoted by "T"*,* is 250 at all times. We proceed with the below subsection to present and analyze the data related to the selected cryptocurrencies and fiat currencies.

3.6. Data and Descriptive Statistics

The data employed in this study are the global historical daily prices extracted from "FinanceYahoo" for cryptocurrencies, whereas the data for exchange rates was extracted from "Investing.com". Estimates from the softwares Microsoft Excel & Spider Financial's Numerical Analysis for Excel (NumXL) will be integrated as a base for our evaluation.

For our analysis, we extracted the daily closing prices spanning from 10th October 2015 until 18th November 2019 yielding a total of 1,501 daily observations for cryptocurrencies and 1,071 daily observations for exchange rates. Note this difference is because data for fiat currencies can only be obtained for weekdays.

To avoid any disparity that may result from the relative quotings obtained on the respective days between cryptocurrencies and exchange rates, the data is filtered and adjusted for the gaps on weekends to conserve reliability and consistency in our estimations. Accordingly, this adjustment provided us with 1,071 compatible observations ranging from 12th October 2015 until 18th November 2019 (excluding weekends.)

As a result, the in-sample period for each cryptocurrency and exchange rate was chosen to extend from October 13th 2015 till December 3rd 2018 yielding a total of 820 returns whereas the out-of-sample period ranges from December 4th 2018 till November 18th 2019 generating a total of 250 returns.

The six cryptocurrencies chosen to be a part of our analysis are: Bitcoin (BTC), Dash (DASH), Monero (XMR), Dogecoin (DOGE), Litecoin (LTC) and Ripple (XRP) on one hand, and the currencies: Euro (EUR), Japanese Yen (JPY), British Pound (GBP), Swiss Franc (CHF), the Canadian Dollar (CAD), and the Australian Dollar (AUD) on the other hand; all against the US dollar. Noting that the latter was excluded given that all currencies are priced with respect to the US Dollar. Furthermore, an influential cryptocurrency, Ethereum, was also excluded as its price was relatively stable until early 2017, after which we started noticing considerable movements in its volatility. Moreover, due to data deficiency, several other cryptocurrencies were omitted from our study such as Bitcoin Cash, EOS, Cardano, Neo, etc. Nevertheless, the selected currencies were purposely chosen as recent research such as Chu et al. (2017) and Holtappels (2018) have suggested them to be among the most deliberated and traded currencies among investors and practitioners.

It is worth noting, however, that Bitcoin represented 70% of the overall cryptocurrency market capitalization as of July 2019 with the next largest share corresponding to Ethereum (a cryptocurrency excluded in this study due to the volume of available data; given that Ethereum started witnessing movements beginning Q2/2017) that relatively holds only 10% (Cap, 2019). Intrinsically, the remaining cryptocurrencies hold only a combined share of 8% and a presentable combined total share of 79%. However, due to the volatility of cryptocurrencies, the rankings of the respective cryptocurrencies changes continuously. Nevertheless, a summary of our cryptocurrencies' relative share is summarized below.

Cryptocurrency	Market Cap. ($B)	Percentage (%)
BTC	215.47	70.50
XRP	15.37	5.03
LTC	6.76	2.21
XMR	1.62	0.53
DASH	1.35	0.44
DOGE	0.40	0.13
Portfolio	**$ 240.97B**	**78.84%**
Cryptocurrency Market	**$ 305.63B**	**100.00%**

Table 1: The Relative Dollar and Percentage Share of the Selected Cryptocurrencies (Bitcoin, Ripple, Litecoin, Monero, Dash & Dogecoin) with respect to the entire Cryptocurrency Market as of July 10th 2019 (Cap, 2019).

Therefore, taking into consideration the dominance that Bitcoin imposes on the cryptocurrency market, it is more reasonable to conduct a separate analysis on each cryptocurrency independently to avoid any biased inferences.

Table 2 presents the summary statistics for the returns of the six cryptocurrencies and currencies. All cryptocurrencies have positive average returns and a significant positive skewness. The excess kurtosis for all series implies that they follow a heavy tailed leptokurtic distribution, coupled with high probability of extreme outlier values and a peakness significantly greater than that of a normal distribution, specifically for Ripple and Dogecoin. To verify, the results of the Jarque-Bera test, after Jarque & Bera (1987), reject the null hypothesis of normality for all series since the calculated test statistics are greater than the critical values at their respective significance levels. Table 2 displays the results of the Jarque-Bera test for all cryptocurrencies and fiat currencies.

Conversely, due to the lower volatile nature of fiat money, the selected currencies revealed an average return of 0% and a relatively smaller standard deviation and a slighter kurtosis. With the exception of the British Pound, all currencies display an approximately symmetrical distribution that, however, exhibit a leptokurtic distribution and demonstrate deviation from normality.

	BTC	**XRP**	**LTC**	**XMR**	**DASH**	**DOGE**	**EUR**	**GBP**	**CAD**	**AUD**	**CHF**	**JPY**
Nbr of Obser	1070	1070	1070	1070	1070	1070	1070	1070	1070	1070	1070	1070
Mean	0.0043	0.0068	0.0050	0.0080	0.0052	0.0061	0.0000	-0.0001	0.0000	-0.0001	0.0000	0.0001
Standard Err	0.0014	0.0027	0.0022	0.0026	0.0020	0.0027	0.0001	0.0002	0.0001	0.0002	0.0001	0.0002
Median	0.0031	-0.0039	-0.0009	-0.0003	-0.0012	-0.0004	-0.0001	-0.0001	-0.0003	0.0001	-0.0001	0.0000
Standard Dev	0.0461	0.0875	0.0707	0.0859	0.0664	0.0876	0.0046	0.0062	0.0046	0.0056	0.0044	0.0058
Variance	0.0021	0.0077	0.0050	0.0074	0.0044	0.0077	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Kurtosis	7.5221	42.3348	24.8775	31.3553	8.9870	55.9986	6.1380	30.2116	4.2499	3.9739	4.6042	7.1852
Skewness	0.4127	4.4201	2.8073	3.1523	1.1189	4.8158	0.2084	-1.8542	0.1970	-0.2030	0.2550	0.5342
Range	0.4649	1.4155	1.0421	1.2866	0.7106	1.6322	0.0545	0.1109	0.0388	0.0436	0.0411	0.0639
Minimum	-0.2124	-0.2967	-0.3263	-0.2541	-0.2308	-0.3891	-0.0238	-0.0806	-0.0190	-0.0237	-0.0158	-0.0306
Maximum	0.2525	1.1188	0.7157	1.0325	0.4798	1.2431	0.0307	0.0303	0.0198	0.0198	0.0253	0.0333
Sum	4.6519	7.2994	5.3236	8.5071	5.6026	6.5082	-0.0139	-0.1485	-0.0045	-0.0608	-0.0178	0.1220
Jarque-Bera	942.05	72455	22740	37613.1	1820.66	129353	446.72	33626	76.439	49.541	126.44	831.85
P-Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2: Summary Statistics of the daily returns of the cryptocurrencies (Bitcoin, Ripple, Litecoin, Monero, Dash and Dogecoin) and currencies (Euro, British Pound, Canadian Dollar, Australian Dollar, Swiss Franc and Japanese Yen) over the sampled period 10th October 2015 – 18th November 2019.

Figure 1 presents the times series plot of the six cryptocurrencies and hard currencies. A defining feature for all cryptocurrencies, as per presented figures, is that their prices increased abruptly as they recorded “exceptional” highs near the end of 2017, as prices started to plunge successively during 2018. The figure also highlights the main aspect of hard currencies regarding their relative stability.

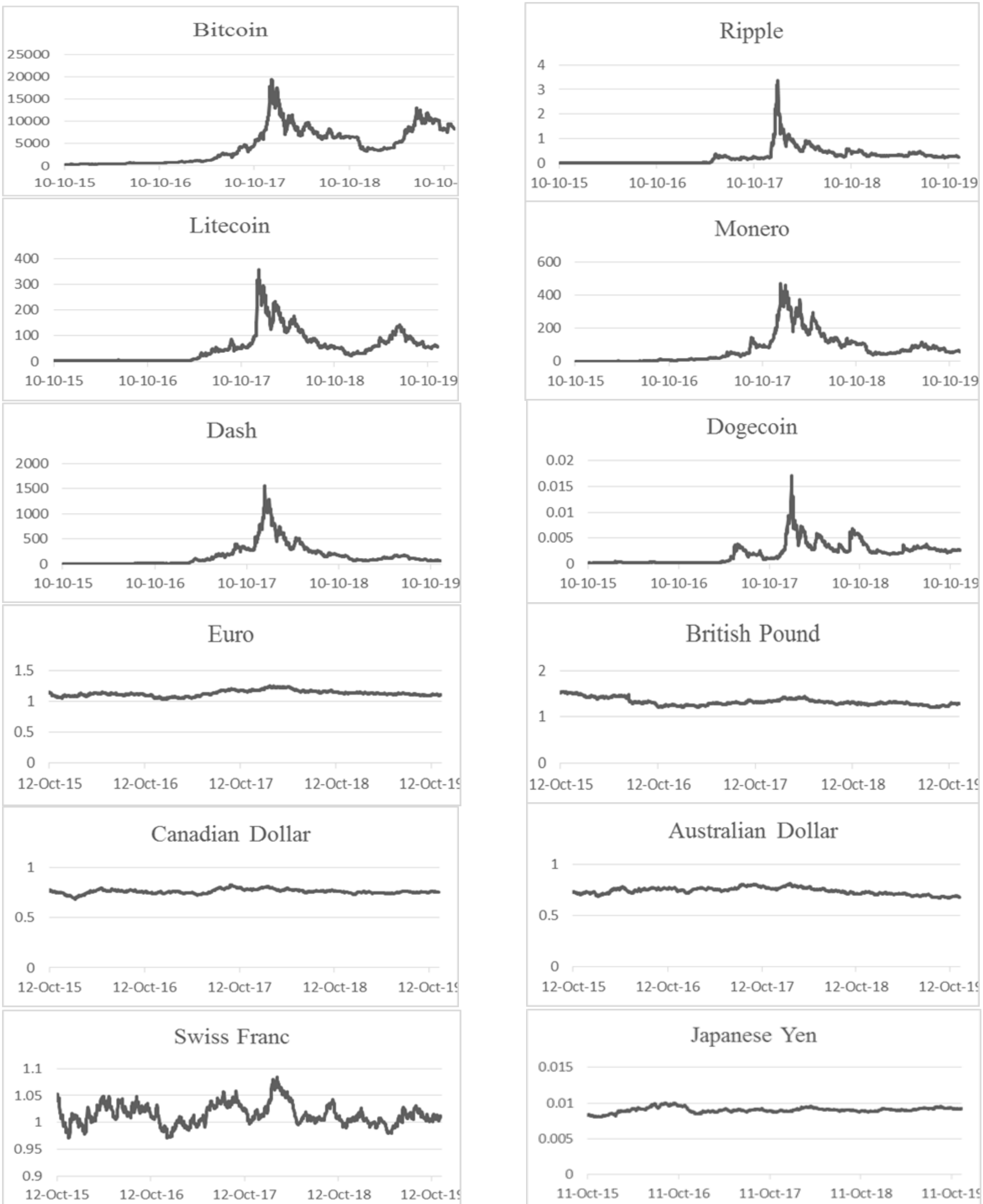


Figure 1: Time series plot of the daily prices of the six cryptocurrencies and exchange rates between October 10, 2015 and November 18, 2019

When analyzing their historical returns, however, Figure 2 validates a stylized and distinctive feature of leptokurtosis in cryptocurrencies that arises from the pattern of time-varying volatility clustering in the market where periods of high (low) volatility are followed by periods of high (low) volatility underlining, undeniably, the high probability of extreme returns in cryptocurrencies mentioned earlier. As a result, from the plot of return series below, persistence and volatility clustering are visible, which implies that volatility can be forecasted.

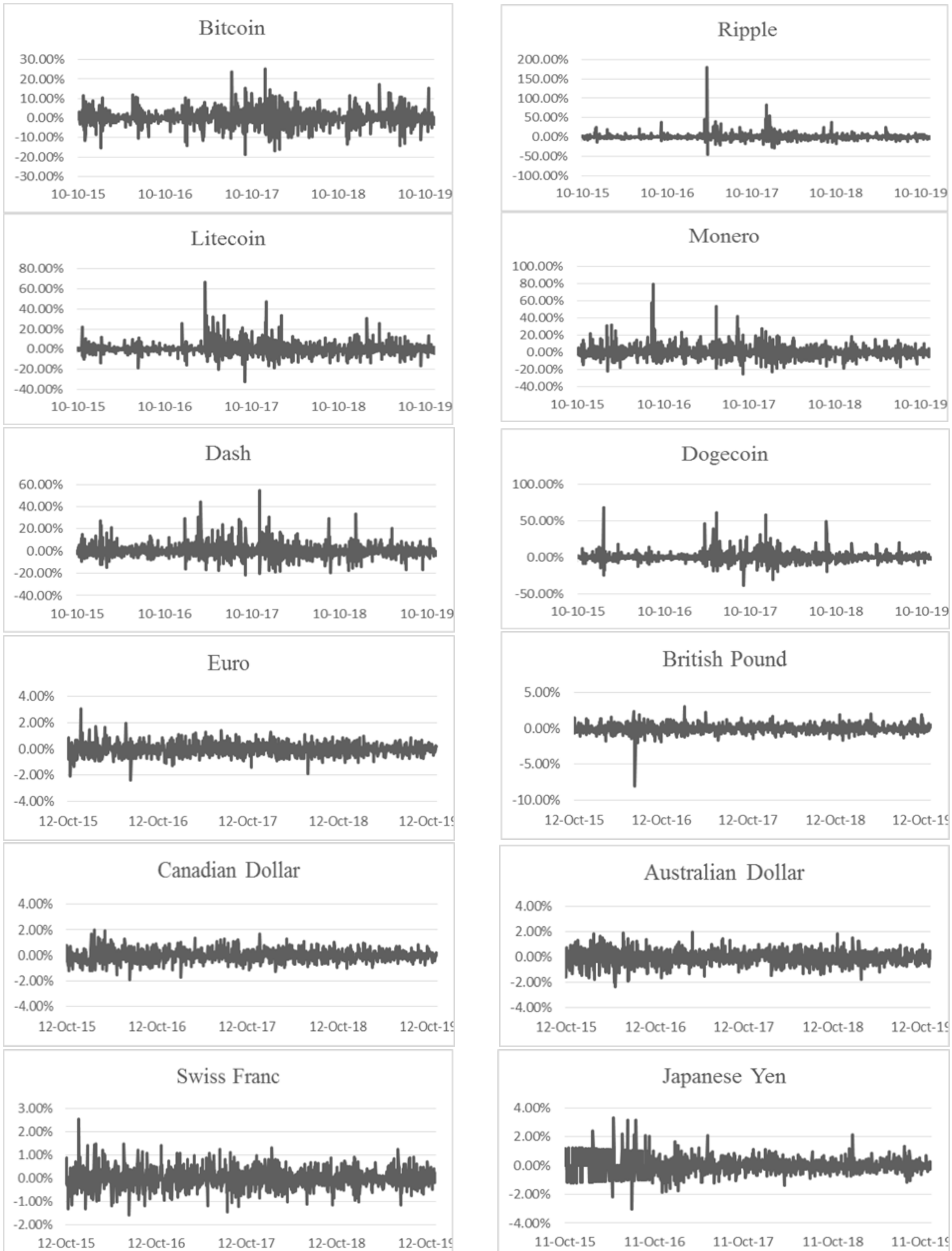


Figure 2: Time series plot of the daily simple returns of the six cryptocurrencies and exchange rates between October 10, 2015 and November 18, 2019.

Subsequently, the Durbin-Watson test, after Durbin & Watson (1950, 1951) verified that no serial correlation was present in the residuals for all series. Therefore, no autocorrelation is detected and no remediation is required. This means that there is neither an AR, nor a MA, nor an ARMA [2] process. Hence, our data is most likely to exhibit an ARCH effect. Moving forward, the below table depicts the Augmented Dickey-Fuller (ADF) test statistics, after Dickey & Fuller (1979), and their corresponding P-values for each data set. Results, inevitably, rejected the original hypothesis of non-stationary and asserted that in sample returns are strongly stationary for all series, suggesting that no transformation in the return series is required.

Augmented Dickey-Fuller Test Statistics												
	BTC	**XRP**	**LTC**	**XMR**	**DASH**	**DOGE**	**EUR**	**GBP**	**CAD**	**AUD**	**CHF**	**JPY**
Statistic	-31.581	-18.388	-28.999	-11.729	-32.443	-7.599	-33.826	-32.551	-32.332	-34.948	-32.262	-35.415
P-Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 3: ADF Test verifying the stationarity of the cryptocurrencies and exchange rates data series over the sampled period.

In essence, the above section has thoroughly described the employed models and the relative evaluation techniques having also integrated value at risk into the framework. Following certain statistical analysis, this section has also ensured that the data is qualified. Before proceeding with the advised approach and prior to implementing any forecasts, heteroskedasticity test for ARCH effect was conducted for the squared residuals. Given that the probability of the Chi Squared turned out to be less than 5% for all series, the null hypothesis of no ARCH effect has been rejected for all series. Therefore, an ARCH effect exists. Accordingly, full justification is gained to run GARCH volatility models.

[2] In statistical analysis of time series, the Auto-Regressive Moving Average (ARMA) models provide parsimonious description of a weakly stationary stochastic process in terms of two polynomials; where the AR part involves regressing the variable against its own lagged/prior values and the MA part involves regressing the variable against the current and previous white noise error terms.

Chapter 4: Findings

After theoretically describing the adopted methodology approach, this section presents the detailed findings under each of the volatility models for each of the selected cryptocurrencies and fiat currencies.

4.1. In-Sample Modeling

Upon extracting the historical daily prices for each cryptocurrency and fiat currency and computing their respective daily returns, the daily conditional variance for each observation is calculated from equations (3), (5), (6), (7), (8), (9), (10 & 11) for each of the GARCH (1,1), IGARCH (1,1), EGARCH (1,1), GJR-GARCH (1,1), APARCH (1,1), TGARCH (1,1) and CGARCH (1,1) models respectively. Results are then annualized assuming 250 trading days per year and the annualized volatility is deduced from the annualized variance by taking the square root of the latter.

4.1.1. Parameters Estimation & Volatility Modeling

First, parameters estimation is performed for the in-sample period extending from October 13th 2015 through December 3rd 2018, which will be subsequently used to forecast volatility for both the in-sample and out-of-sample periods.

For each model and under each of the selected cryptocurrencies and fiat currencies, the sum of the log likelihood estimates on each observation for the in-sample data set is maximized using the “Solver” add-in function in Excel, subject to the conditions and constraints defined in section 3.1.1. The resulting parameters obtained once the LLF is maximized are those used to estimate the conditional volatilities for the in-sample period. Below is a detailed table presenting the computed parameters under each model for each of the selected cryptocurrencies and fiat currencies.

BTC							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00003798	0.00003693	-0.2911965	0.00003265	0.00003798	0.00190668	0.03162807
α	0.121	0.1233	0.2843	0.1428	0.1231	0.1409	0.0482
β	0.8769	0.8767	0.9479	0.8839	0.8769	0.8635	0.1180
α + β	0.9979	1.0000	1.2322	1.0267	1.0000	1.0044	0.1662
ϒ	-	-	0.0153	-0.0508	0.0085	0.0033	-0.0548
VL	213.71 %	-	96.64 %	-	-	-	-
δ	-	-	-	-	2	-	-
ρ	-	-	-	-	-	-	0.99890
Ø	-	-	-	-	-	-	0.11797
LLF	2193.47	2193.45	2195.9	2195.71	2193.47	2191.83	2193.94
XRP							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00053230	0.00054250	-0.9315335	0.00346279	0.00053230	0.01043814	0.11880610
α	0.2971	0.3735	0.4166	0.5240	0.1943	0.1997	0.4077
β	0.6536	0.6265	0.8152	0.6529	0.6536	0.6643	0.5423
α + β	0.9507	1.0000	1.2317	1.1769	0.8479	0.8640	0.9500
ϒ	-	-	0.1916	-0.9601	-0.2365	-0.4523	0.0454
VL	164.241 %	-	127.204 %	-	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.99936
Ø	-	-	-	-	-	-	0.00000
LLF	1823.69	1822.37	1819.15	1755.14	1823.69	1806.71	1845.22
LTC							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00022466	0.00016634	-0.3066892	0.00020611	0.00022463	0.00384824	0.00662378
α	0.0979	0.1325	0.1577	0.1398	0.0456	0.0607	0.1581
β	0.8583	0.8675	0.9401	0.8724	0.8583	0.8708	0.0636
α + β	0.9561	1.0000	1.0978	1.0122	0.9038	0.9315	0.2216
ϒ	-	-	0.1129	-0.1158	-0.4657	-0.8174	0.2292
VL	113.150 %	-	122.227 %	-	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.98976
Ø	-	-	-	-	-	-	0.04070
LLF	1884.26	1877.83	1901.16	1894.21	1884.26	1882.15	1893.43
XMR							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00073569	0.00034368	-8.4872648	0.00076781	0.00073572	0.00701040	0.00752625
α	0.0911	0.1564	0.1047	0.1316	0.0308	0.0552	0.0000
β	0.8103	0.8436	-0.7175	0.8141	0.8103	0.8444	0.0898
α + β	0.9014	1.0000	-0.6128	0.9458	0.8411	0.8996	0.0898
ϒ	-	-	0.0568	-0.1151	-0.7204	-0.9407	0.0295
VL	136.573 %	-	133.623 %	131.036 %	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.90329
Ø	-	-	-	-	-	-	0.08976
LLF	1646.19	1634.46	1617.29	1654.09	1646.19	1637.21	1646.28

DASH							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00019596	0.00015124	-0.2836452	0.00017579	0.00019595	0.00346804	0.00814057
α	0.1530	0.1776	0.2750	0.1727	0.1078	0.1340	0.0000
β	0.8217	0.8224	0.9441	0.8366	0.8217	0.8359	0.1587
α + β	0.9747	1.0000	1.2191	1.0093	0.9295	0.9700	0.1587
ϒ	-	-	0.0495	-0.0687	-0.1913	-0.1913	-0.0871
VL	139.095 %	-	124.864 %	-	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.97624
Ø	-	-	-	-	-	-	0.15870
LLF	1854.74	1853.83	1860.34	1857.20	1854.74	1854.82	1855.68
DOGE							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00016896	0.00016896	-0.2071793	0.00015342	0.00012582	0.00204339	1.02507108
α	0.2286	0.2286	0.3702	0.4085	0.1542	0.1865	0.1234
β	0.7714	0.7714	0.9521	0.7545	0.7598	0.8138	0.8195
α + β	1.0000	1.0000	1.3223	1.1630	0.9140	1.0003	0.9429
ϒ	-	-	0.1137	-0.2167	-0.4093	-0.3580	0.0689
VL	N/A	-	181.860 %	-	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.99995
Ø	-	-	-	-	-	-	0.11627
LLF	1824.99	1824.99	1840.91	1838.22	1829.76	1824.76	1833.07
EUR							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00000090	0.00000000	-5.7318176	0.00000006	0.00000106	0.00760915	0.00002167
α	0.0000	0.0091	-0.0630	0.0051	0.0000	-0.0383	0.0000
β	0.9614	0.9909	0.4607	0.9935	0.9520	-0.5048	0.7594
α + β	0.9614	1.0000	0.3977	0.9985	0.9520	-0.5431	0.7594
ϒ	-	-	0.1266	-0.0024	0.1018	0.0158	-0.0661
VL	7.635 %	-	7.784 %	7.406 %	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.99767
Ø	-	-	-	-	-	-	0.05089
LLF	3936.08	3937.54	3940.18	3938.33	3934.83	3936.76	3927.40
GBP							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00000382	0.00000072	-0.7482446	0.00000445	0.00000378	0.00041999	0.00003529
α	0.1635	0.0872	0.2727	0.2500	0.1123	0.1202	0.1821
β	0.7557	0.9128	0.9256	0.7349	0.7524	0.8133	0.7473
α + β	0.9192	1.0000	1.1983	0.9849	0.8646	0.9335	0.9294
ϒ	-	-	0.0934	-0.1690	-0.2355	-0.3651	-0.1129
VL	10.872 %	-	10.375 %	10.572 %	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.99446
Ø	-	-	-	-	-	-	0.00488
LLF	3777.10	3765.28	3782.59	3784.01	3777.13	3776.15	3780.18

CAD							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00002253	0.00000000	-16.922744	0.00000001	0.00000000	0.00007177	0.00005638
α	0.0923	0.0171	0.2088	0.0053	0.0037	0.0239	0.0267
β	0.9014	0.9829	-0.5943	1.0021	0.9902	0.9647	0.9683
$\alpha + \beta$	0.9937	1.0000	-0.3854	1.0074	0.9939	0.9886	0.9949
Υ	-	-	0.0323	-0.0155	-0.5712	-0.1151	-0.0144
VL	7.877 %	-	7.834 %	-	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	1.00000
$\emptyset$	-	-	-	-	-	-	0.00000
LLF	3936.71	3942.93	3939.12	3925.18	3942.55	3943.82	3944.81
AUD							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00000022	0.00000000	-0.0748181	0.00000020	0.00000016	0.00002450	0.00003206
α	0.0060	0.0186	0.0466	0.0000	0.0123	0.0134	0.0000
β	0.9867	0.9814	0.9927	0.9889	0.9782	0.9794	0.0168
$\alpha + \beta$	0.9927	1.0000	1.0393	0.9889	0.9905	0.9928	0.0168
Υ	-	-	0.0167	0.00650	-0.1757	-0.5494	0.0121
VL	8.692 %	-	9.463 %	8.01 %	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.99521
$\emptyset$	-	-	-	-	-	-	0.01683
LLF	3806.22	3809.20	3811.48	3801.02	3810.42	3811.10	3810.44
CHF							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00000166	0.00000000	-4.6933254	0.00000650	0.00000155	0.0000114	0.00004540
α	0.0001	0.0124	-0.0526	0.0650	0.0028	0.0016	0.1643
β	0.9204	0.9876	0.5651	0.6825	0.9257	0.9998	0.0043
$\alpha + \beta$	0.9204	1.0000	0.5125	0.7474	0.9229	1.0014	0.1686
Υ	-	-	0.1571	-0.1132	-0.2330	-1.0000	-0.2319
VL	7.232 %	-	7.171 %	7.250 %	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	1.00000
$\emptyset$	-	-	-	-	-	-	0.00434
LLF	3999.46	4005.80	4007.80	4004.35	3997.44	4007.09	4002.84
JPY							
	GARCH	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
ω	0.00000000	0.00000000	0.00417207	0.00000003	0.00000000	0.0000149	0.00000000
α	0.0195	0.0260	0.0005	0.0224	0.0087	0.0298	0.0000
β	0.9793	0.9740	1.0006	0.9894	0.9850	0.9811	0.0230
$\alpha + \beta$	0.9989	1.0000	1.0011	1.0118	0.9937	1.0109	0.0230
Υ	-	-	0.0512	-0.0309	-0.2400	0.0033	-0.0054
VL	-	-	55.739 %	-	-	-	-
δ	-	-	-	-	2.000	-	-
ρ	-	-	-	-	-	-	0.99895
$\emptyset$	-	-	-	-	-	-	0.02301
LLF	3828.77	3828.67	3822.82	3828.45	3826.89	3805.11	3828.97

Table 4: Summary of the Estimated Parameters for all GARCH models under each Cryptocurrency and Fiat Currency.

4.1.1.1. GARCH (1,1)

		ERROR DISTRIBUTION											
		Normal Distribution				Student's t - Distribution				Generalized Error Distribution			
		AIC	BIC	HQC	LLF	AIC	BIC	HQC	LLF	AIC	BIC	HQC	LLF
Cryptocurrency	BTC	-3.4566	-3.4426	-3.4513	1852.26	**-3.7812**	**-3.7626**	**-3.7742**	**2026.94**	-3.7797	-3.7611	-3.7726	2026.12
	XRP	-2.6956	-2.6817	-2.6903	1445.16	**-3.1375**	**-3.1189**	**-3.1305**	**1682.59**	-3.1181	-3.0995	-3.1110	1672.17
	LTC	-2.6430	-2.6291	-2.6378	1417.03	**-3.2896**	**-3.2710**	**-3.2825**	**1763.92**	-3.2482	-3.2296	-3.2411	1741.76
	XMR	-2.2290	-2.2150	-2.2237	1195.49	**-2.5882**	**-2.5696**	**-2.5812**	**1388.71**	-2.5637	-2.5451	-2.5567	1375.60
	DASH	-2.7436	-2.7296	-2.7383	1470.81	-2.9452	-2.9266	-2.9382	1579.68	**-2.9474**	**-2.9288**	**-2.9403**	**1580.84**
	DOGE	-2.7026	-2.6887	-2.6973	1448.90	**-3.1679**	**-3.1493**	**-3.1609**	**1698.84**	-3.1425	-3.1239	-3.1354	1685.23
Fiat Currency	EUR	-8.0013	-7.9873	-7.9960	4283.68	**-8.0137**	**-7.9951**	**-8.0067**	**4291.35**	-8.0132	-7.9946	-8.0062	4291.07
	GBP	-7.4616	-7.4477	-7.4564	3994.98	**-7.5599**	**-7.5413**	**-7.5529**	**4048.57**	-7.5384	-7.5198	-7.5314	4037.06
	CAD	-7.9941	-7.9801	-7.9888	4279.82	-8.0014	-7.9828	-7.9943	4284.73	**-8.0050**	**-7.9864**	**-7.9980**	**4286.68**
	AUD	-7.5843	-7.5703	-7.5790	4060.58	**-7.5922**	**-7.5736**	**-7.5851**	**4065.81**	-7.5898	-7.5712	-7.5828	4064.55
	CHF	-8.0640	-8.0501	-8.0588	4317.26	-8.0776	-8.0590	-8.0706	4325.52	**-8.0860**	**-8.0674**	**-8.0789**	**4329.99**
	JPY	-7.6656	-7.6516	-7.6603	4104.08	**-7.7152**	**-7.6966**	**-7.7082**	**4131.63**	-7.6779	-7.6621	-7.6835	4129.45

Table 5: The Goodness-of-Fit of the GARCH (1,1) Model assuming Three Innovation Term Distributions and covering the entire sampled period for each Cryptocurrency & Fiat Currency.

First, an important component in fitting a GARCH process is the distribution of the innovations term. In order to select the most appropriate distribution for the innovations term for each crypto-currency and fiat currency, the GARCH (1,1) model is utilized covering the entire sampled period, from October 13th 2015 until November 18th 2019, and assuming three distributions for the innovations term: Normal, Student's t and the Generalized Error distributions. Based on the empirical results from table 5, we note that the use of skewed and heavy-tailed innovations distributions is justified, as they give better results based on the goodness-of-fit measures. The Student's t distribution demonstrates superiority having the highest log-likelihood value, as well as the lowest AIC, BIC and HQC values among the three innovation term distributions for 9 out of the 12 modeled assets. In particular, the Student's t distribution is selected for Bitcoin, Ripple, Litecoin, Monero, Dogecoin, Euro, the British Pound, Australian Dollar and the Japanese Yen while the Generalized Error Distribution is selected for Dash, the Canadian Dollar and the Swiss Franc. Progressively, a thorough analysis surrounding the achieved results is conducted, as depicted below:

GARCH (1,1)					
	ω	α	β	VL	LLF
BTC	0.00003798	0.1210	0.8769	213.708 %	2193.47
XRP	0.00053230	0.2971	0.6536	164.241 %	1823.69
LTC	0.00022466	0.0979	0.8583	113.150 %	1884.26
XMR	0.00073569	0.0911	0.8103	136.573 %	1646.19
DASH	0.00019596	0.1530	0.8217	139.095 %	1854.74
DOGE	0.00016896	0.2286	0.7714	-	1824.99
EUR	0.00000090	0.0000	0.9614	7.635 %	3936.08
GBP	0.00000382	0.1635	0.7557	10.872 %	3777.10
CAD	0.00002253	0.0923	0.9014	7.877 %	3936.71
AUD	0.00000022	0.0060	0.9867	8.692 %	3806.22
CHF	0.00000166	0.0001	0.9204	7.232 %	3999.46
JPY	0.00000000	0.0195	0.9793	-	3828.77

Table 6: *GARCH (1,1) Estimated Parameters for each Cryptocurrency and Fiat Currency*

The ARCH component "α", known also as the GARCH reaction parameter, ranges between 9% and 30% for the cryptocurrencies and between 0% and 9% for the fiat currencies, except for the British Pound having an α of 16%. This parameter determines the impact of market shocks on volatility. Predictably, the relatively high disturbance realized in the British Pound compared to the remaining hard currencies is due to the Brexit turmoil following the UK-wide referendum in June 2016 and effectively, its associated repercussions. Intrinsically, the ARCH components for the Euro, Australian Dollar, and Swiss Franc are 0%, implying that market shocks have no effect on their volatilities, unlike the remaining fiat currencies. Predictably, all cryptocurrencies are however, sensitive to disturbances in the market, specifically Ripple and Dogecoin.

The GARCH component "β" explains the relative significance of today's returns in determining the variance for periods ahead. With the exception of the British Pound, all fiat currencies exhibit a relatively larger beta compared to cryptocurrencies suggesting that exchange rates are more explicable and less "spiky" as illustrated in figures 1 & 2 in section 3.6.

Noticeably, the "ω" term for fiat currencies is relentlessly insignificant and close to zero. A high persistence and a low omega often suggests that volatility might be nonstationary which points towards conversion of the Standard GARCH model to the Integrated GARCH (Zivot, 2008) in the case of fiat currencies. Also, it is important to note that this is exceptionally true in the case of Dogecoin, where the sum of parameters α+β equals 1, thereby indicating that the conditional variance is strictly stationary with an unattainable long term variance. As for Bitcoin, Ripple, Litecoin, Monero and Dash, the series are stationary and mean reverting with long term volatilities surpassing the 100% mark. Specifically, Bitcoin and Ripple reported the highest long term volatilities with respective values of 214% and 164%, which further underlines cryptocurrencies' "intensifying" levels of volatility.

4.1.1.2. IGARCH (1,1)

IGARCH (1,1)				
	ω	α	β	LLF
BTC	0.00003693	0.1233	0.8767	2193.45
XRP	0.00054250	0.3735	0.6265	1822.37
LTC	0.00016634	0.1325	0.8675	1877.83
XMR	0.00034368	0.1564	0.8436	1634.46
DASH	0.00015124	0.1776	0.8224	1853.83
DOGE	0.00016896	0.2286	0.7714	1824.99
EUR	0.00000000	0.0091	0.9909	3937.54
GBP	0.00000072	0.0872	0.9128	3765.28
CAD	0.00000000	0.0171	0.9829	3942.93
AUD	0.00000000	0.0186	0.9814	3809.20
CHF	0.00000000	0.0124	0.9876	4005.80
JPY	0.00000000	0.0260	0.9740	3828.67

Table 7: IGARCH (1,1) Estimated Parameters for each Cryptocurrency and Fiat Currency

The Integrated GARCH model validates the presumptions drawn from the GARCH model and provides further clarification concerning the British Pound. Remarkably, the GARCH component "β" estimates for all cryptocurrencies and fiat currencies were reasonably analogous in both models once compared to the British Pound, where the latter's beta has increased drastically from 76% in GARCH(1,1) to 91% in IGARCH(1,1). This suggests that if it wasn't of the UK-referendum and Brexit's effect on UK's economy, the Pound would have behaved "smoother" to market shocks, thereby providing a clearer insight with regard to predicting its volatility.

In contrast to the Standard GARCH model, however, the unconditional variance in the Integrated GARCH model is not finite and therefore the model does not exhibit volatility mean reversion (Zivot, 2008). Hence, the omega term "ω" now takes the form of a constant.

Nevertheless, it is worth highlighting again the "ω" term for fiat currencies where each of the Euro, Pound, Canadian Dollar, Australian Dollar, Swiss Franc and Japanese Yen have a ω of 0. This provides further verification that the IGARCH model provides a very good fit for the fiat currencies, while at the same time, drawing attention towards advanced GARCH models as they may provide better explanation to cryptocurrencies' volatility.

4.1.1.3. EGARCH (1,1)

EGARCH (1,1)						
	ω	α	β	ϒ	VL	LLF
BTC	-0.291197	0.2843	0.9479	0.0153	96.636 %	2195.90
XRP	-0.931534	0.4166	0.8152	0.1916	127.204 %	1819.15
LTC	-0.306689	0.1577	0.9401	0.1129	122.227 %	1901.16
XMR	-8.487265	0.1047	-0.7175	0.0568	133.623 %	1617.29
DASH	-0.283645	0.2750	0.9441	0.0495	124.864 %	1860.34
DOGE	-0.207179	0.3702	0.9521	0.1137	181.860 %	1840.91
EUR	-5.731818	-0.0630	0.4607	0.1266	7.784 %	3940.18
GBP	-0.748245	0.2727	0.9256	0.0934	10.375 %	3782.59
CAD	-16.922744	0.2088	-0.5943	0.0323	7.834 %	3939.12
AUD	-0.074818	0.0466	0.9927	0.0167	9.463 %	3811.48
CHF	-4.693325	-0.0526	0.5651	0.1571	7.171 %	4007.80
JPY	0.004172	0.0005	1.0006	0.0512	55.739 %	3822.82

Table 8: *EGARCH (1,1) Estimated Parameters for each Cryptocurrency and Fiat Currency*

What distinguishes this model from the Standard GARCH model is the specification of its conditional variance equation allowing to incorporate the impact of asymmetries on volatility and thereby the variance to react differently depending on the sign and size of the shock it receives. For clarity, bear in mind that the ARCH term "α" represents the extent towards which the magnitude (size) of shocks to the variance affects future volatility in the returns. The GARCH term "β" gives insight into persistence of past volatility and how it helps predict future volatility. The key coefficient to look at, however, is the leverage effect term "γ". This parameter describes how the sign of the shock influences the future volatility of returns.

The leverage coefficient "γ" ranges between 1% and 19% and carries a positive value for all cryptocurrencies and fiat currencies. This implies that none of the cryptocurrencies and fiat currencies exhibit a leverage effect and positive shocks have a greater impact on their volatility than negative shocks, particularly for Ripple (19%) and the Swiss Franc (16%). However, the asymmetry effect on Bitcoin, Australian Dollar and Canadian Dollar is relatively insignificant (≤ 3%).

The ARCH term "α" is positive for all cryptocurrencies and fiat currencies except for the Euro and Swiss Franc, with the highest values displayed by Ripple (42%) and Dogecoin (37%). This shows that cryptocurrencies and fiat currencies generally exhibit a positive relationship between their past variances and current variances in absolute value, which means that the bigger the magnitude of shocks to their variance, the higher their volatility.

The GARCH term "β" is quite significant for all cryptocurrencies and fiat currencies except for the Euro, Canadian Dollar and the Swiss Franc, revealing that one distinctive feature in cryptocurrencies is persistence in their volatility.

The long term volatility "VL" of fiat currencies ranges between 7% and 10%, and 56% exceptionally for the Japanese Yen. Nevertheless, cryptocurrencies' long term volatility ranges between 97% and 182%. For instance, Ripple's long term volatility is around 18 times larger than the Swiss Franc's long term volatility, further emphasizing the increased volatility in cryptocurrencies with respect to fiat currencies.

4.1.1.4. GJR-GARCH (1,1)

GJR-GARCH (1,1)						
	ω	**α**	**β**	**ϒ**	**VL**	**LLF**
BTC	0.000033	0.1428	0.8839	-0.0508	-	2195.71
XRP	0.003463	0.5240	0.6529	-0.9601	-	1755.14
LTC	0.000206	0.1398	0.8724	-0.1158	-	1894.21
XMR	0.000768	0.1316	0.8141	-0.1151	131.036 %	1654.09
DASH	0.000176	0.1727	0.8366	-0.0687	-	1857.20
DOGE	0.000153	0.4085	0.7545	-0.2167	-	1838.22
EUR	0.000000	0.0051	0.9935	-0.0024	7.406 %	3938.33
GBP	0.000004	0.2500	0.7349	-0.1690	10.572 %	3784.01
CAD	0.000000	0.0053	1.0021	-0.0155	-	3925.18
AUD	0.000000	0.0000	0.9889	0.0065	8.010 %	3801.02
CHF	0.000007	0.0650	0.6825	-0.1132	7.250 %	4004.35
JPY	0.000000	0.0224	0.9894	-0.0309	-	3828.45

Table 9: GJR-GARCH (1,1) Estimated Parameters for each Cryptocurrency and Fiat Currency

The GJR-GARCH model is similar to the EGARCH model in that it models asymmetries effects on volatility. However, this model incorporates the non-negativity constraint on the three parameters: ω, α, and β, while only the leverage term can be negative. As described in chapter 3, this model differs from the Standard GARCH model by introducing a dummy variable "I_{t-1}" allowing the conditional variance to increase in response to bad news more than good news.

Typically, the generated figures display similar results to the Standard GARCH model in terms of mean reverting (ω), volatility clustering (α) and volatility persistence (β). Firstly, all cryptocurrencies display higher long term volatilities with a value for "ω" different from 0, whereas for fiat currencies the constant term is set to be 0. Secondly, the volatility of cryptocurrencies tend to cluster in response to market shocks, unlike fiat currencies. Thirdly, the larger beta in the case of most fiat currencies evidences that they are relatively more explicable and are subject to less 'spikes' than cryptocurrencies, in general.

The leverage coefficient "γ" ranges between 0% and -96%. Results show consistency with the EGARCH model, where a negative leverage coefficient implies the absence of leverage effect for all cryptocurrencies and fiat currencies. This confirms that positive shocks have a higher impact on volatility than negative shocks, except for the Euro and Australian Dollar whose gamma are 0, thereby pose no asymmetry effects. Noticeably, the gamma coefficient is relatively significant for all cryptocurrencies compared to fiat currencies, specifically Ripple (-96%). However, the low values for the leverage coefficient attained for most of the fiat currencies (namely: Australian Dollar, Canadian Dollar, Euro, and Japanese Yen) in conjunction with the absence of the constant term "ω" proves yet again that the IGARCH model provides the best fit for fiat currencies.

4.1.1.5. APARCH (1,1)

APARCH (1,1)						
	ω	α	β	ϒ	δ	LLF
BTC	0.000038	0.1231	0.8769	0.0085	2.000	2193.47
XRP	0.000532	0.1943	0.6536	-0.2365	2.000	1823.69
LTC	0.000225	0.0456	0.8583	-0.4657	2.000	1884.26
XMR	0.000736	0.0308	0.8103	-0.7204	2.000	1646.19
DASH	0.000196	0.1078	0.8217	-0.1913	2.000	1854.74
DOGE	0.000126	0.1542	0.7598	-0.4093	2.000	1829.76
EUR	0.000001	0.0000	0.9520	0.1018	2.000	3934.83
GBP	0.000004	0.1123	0.7524	-0.2355	2.000	3777.13
CAD	0.000000	0.0037	0.9902	-0.5712	2.000	3942.55
AUD	0.000000	0.0123	0.9782	-0.1757	2.000	3810.42
CHF	0.000002	0.0000	0.9257	-0.2329	2.000	3999.45
JPY	0.000000	0.0087	0.9850	-0.2400	2.000	3826.89

Table 10: APARCH (1,1) Estimated Parameters for each Cryptocurrency and Fiat Currency

The Asymmetric Power ARCH, as the GJR-GARCH and EGARCH models, additionally captures asymmetry in return volatility. The computed values of the alpha, beta and omega parameters show consistency in the behavior of cryptocurrencies and exchange rates. A defining and distinctive aspect of this model is that the conditional volatility originally assumes a varying power of "δ" However, since the maximum likelihood estimates of the power parameter for all cryptocurrencies and fiat currencies ranged between "1.8" and "2.1", the modeled version in this thesis assumes a fixed value of "2.00" for the delta parameter to preserve consistency among the assets and to conform with the remaining models. Consequently, the APARCH model exposes similar properties to the GJR-GARCH model.

Estimates for the leverage parameter "γ" for each cryptocurrency and hard currency have changed significantly compared to the GJR-GARCH model, with gamma ranging between 0% and -72%. Curiously, the lowest percentage (in absolute value) for the leverage coefficient was for Bitcoin (0%) which reveals that its volatility is affected symmetrically to positive and negative shocks. Nonetheless, all remaining cryptocurrencies and fiat currencies have a significant negative value for gamma verifying that positive shocks have a higher impact on cryptocurrencies' and fiat currencies' volatility than negative shocks. In particular, the leverage parameter appears to be largest (in absolute value) for Monero (72%) and the Candadian Dollar (57%) with only the Euro having a positive leverage parameter (10%), which is still significant. These results are inconsistent with the previous models where some fiat currencies showed insignificant asymmetry effects.

4.1.1.6. TGARCH (1,1)

TGARCH (1,1)					
	ω	α	β	ϒ	LLF
BTC	0.001907	0.1409	0.8635	0.0033	2191.83
XRP	0.010438	0.1997	0.6643	-0.4523	1806.71
LTC	0.003848	0.0607	0.8708	-0.8174	1882.15
XMR	0.007010	0.0552	0.8444	-0.9407	1637.21
DASH	0.003468	0.1340	0.8359	-0.1913	1854.82
DOGE	0.002043	0.1865	0.8138	-0.3580	1824.76
EUR	0.007609	-0.0383	-0.5048	0.0158	3936.76
GBP	0.000420	0.1202	0.8133	-0.3651	3776.15
CAD	0.000072	0.0239	0.9647	-0.1151	3943.82
AUD	0.000025	0.0134	0.9794	-0.5494	3811.10
CHF	0.000011	0.0016	0.9998	-1.0000	4007.09
JPY	0.000015	0.0298	0.9811	0.0033	3805.11

Table 11: TGARCH (1,1) Estimated Parameters for each Cryptocurrency and Fiat Currency

The TGARCH model is a particular case of the APARCH model once the power effect, “δ”, takes the value of 1. However, in contrast to the remaining models, the TGARCH distinctively models for the standard deviation instead of the conditional variance.

As shown in the above table, the Bitcoin and Japanese Yen have a positive gamma of 0.33%, which is insignificant and relatively close to 0. This implies that the impact of returns on their volatility is symmetrical and thereby, they do not exhibit an asymmetric effect. The Euro appears to be the only asset with a positive leverage parameter of 1.58%, indicating that a leverage effect is present. However, this effect is neglected since its non-representative, once the remaining cryptocurrencies and hard currencies are considered. All of the remaining 9 cryptocurrencies and fiat currencies have a significant negative leverage parameter, further emphasizing the results attained earlier.

4.1.1.7. CGARCH (1,1)

CGARCH (1,1)								
	ω	α	β	α + β	ϒ	ρ	Ø	LLF
BTC	0.031628	0.0482	0.1180	0.1662	-0.0548	0.99890	0.1180	2193.94
XRP	0.118806	0.4077	0.5423	0.9500	0.0454	0.99936	0.0000	1845.22
LTC	0.006624	0.1581	0.0636	0.2216	0.2292	0.98976	0.0407	1893.43
XMR	0.007526	0.0000	0.0898	0.0898	0.0295	0.90329	0.0898	1646.28
DASH	0.008141	0.0000	0.1587	0.1587	-0.0871	0.97624	0.1587	1855.68
DOGE	1.025071	0.1234	0.8195	0.9429	0.0689	0.99995	0.1163	1833.07
EUR	0.000022	0.0000	0.7594	0.7594	-0.0661	0.99767	0.0509	3927.40
GBP	0.000035	0.1821	0.7473	0.9294	-0.1129	0.99446	0.0049	3780.18
CAD	0.000056	0.0267	0.9683	0.9949	-0.0144	1.00000	0.0000	3944.81
AUD	0.000032	0.0000	0.0168	0.0168	0.0121	0.99521	0.0168	3810.44
CHF	0.000045	0.1643	0.0043	0.1686	-0.2319	1.00000	0.0043	4002.84
JPY	0.000000	0.0000	0.0230	0.0230	-0.0054	0.99895	0.0230	3828.97

Table 12: CGARCH (1,1) Estimated Parameters for each Cryptocurrency and Fiat Currency

Perhaps the most complex among all selected models is the Component GARCH model, as it allows the conditional variance to revert to a varying level “q_t” instead of “ω”. In other words, the intercept parameter is now a time-varying first order autoregressive process. As a result, the conditional variance is highly dependent its permanent component “q_t”.

Curiously, the high value attained for the trend intercept “ω” in the case of Ripple and Dogecoin points towards the relative significance of their permanent component, suggesting that the CGARCH model may provide a good fit for these two cryptocurrencies.

This is further supported by the fact that Ripple and Dogecoin are the only cryptocurrencies among all cryptocurrencies that present shocks of transitory nature (sum of alpha and beta coefficients “α+β” are close to “ρ”). Noting that the British Pound and the Canadian Dollar also reveal that their volatilities are highly prone to short term effects.

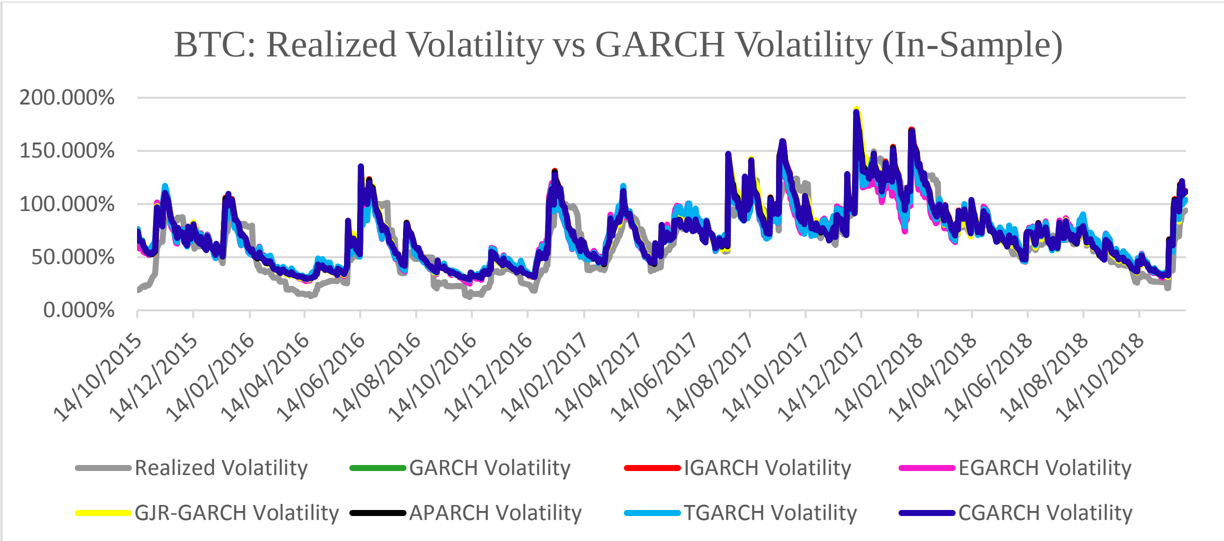
The AR coefficient of the permanent volatility “ρ” is highly significant (almost 1) for all cryptocurrencies and fiat currencies and its size exceeds the coefficients of the transitory component in all cases implying that the CGARCH model is quite stable for all cryptocurrencies and fiat currencies.

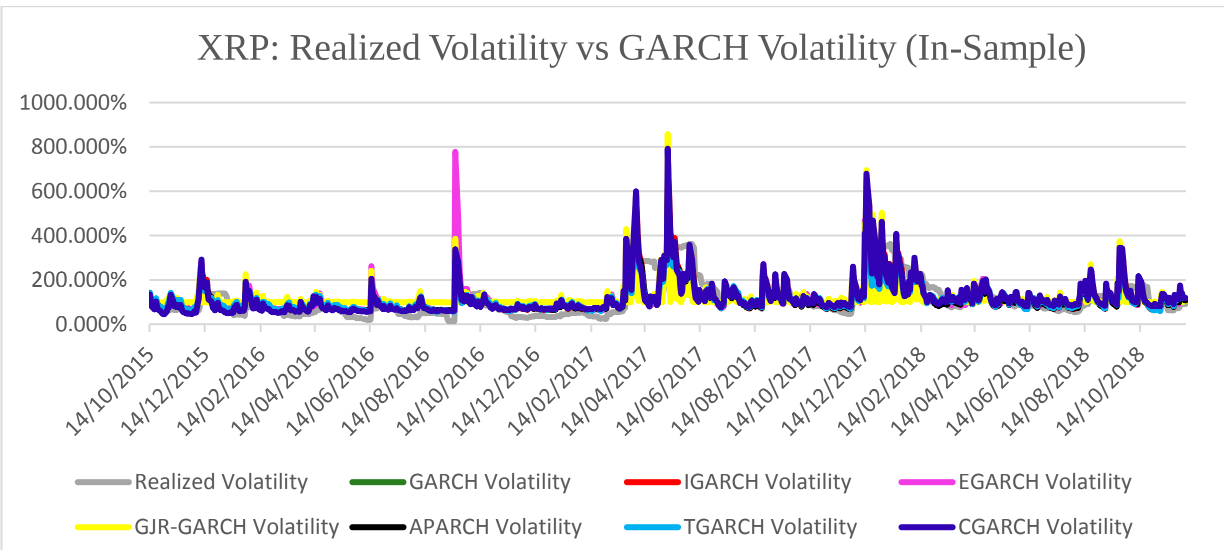
The forecasting error term "$\emptyset$" is positive but insignificant for most cryptocurrencies and fiat currencies which implies that actual volatilities are close to estimated volatilities.

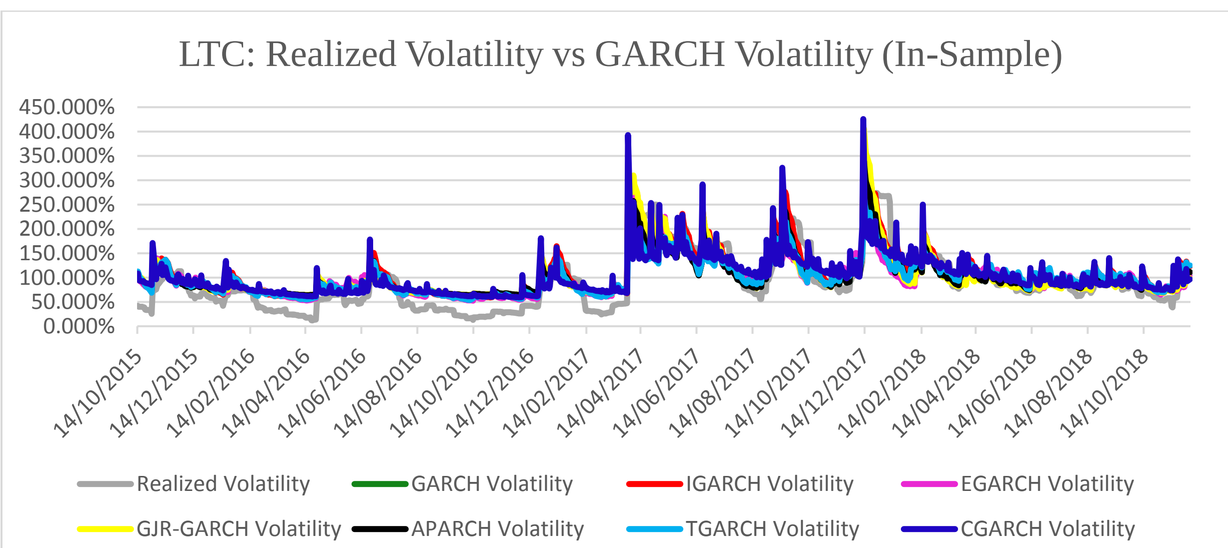
However, in contrast to all remaining models, the CGARCH is the only model that reports the presence of leverage effect in most cryptocurrencies, particularly for Litecoin (γ = 23%). This implies that negative shocks have generally a higher impact on cryptocurrencies' volatility than positive shocks. However, this isn't the case for fiat currencies where positive shocks still have a higher influence than negative shocks, specifically in the case of Swiss Franc (γ = -23%).

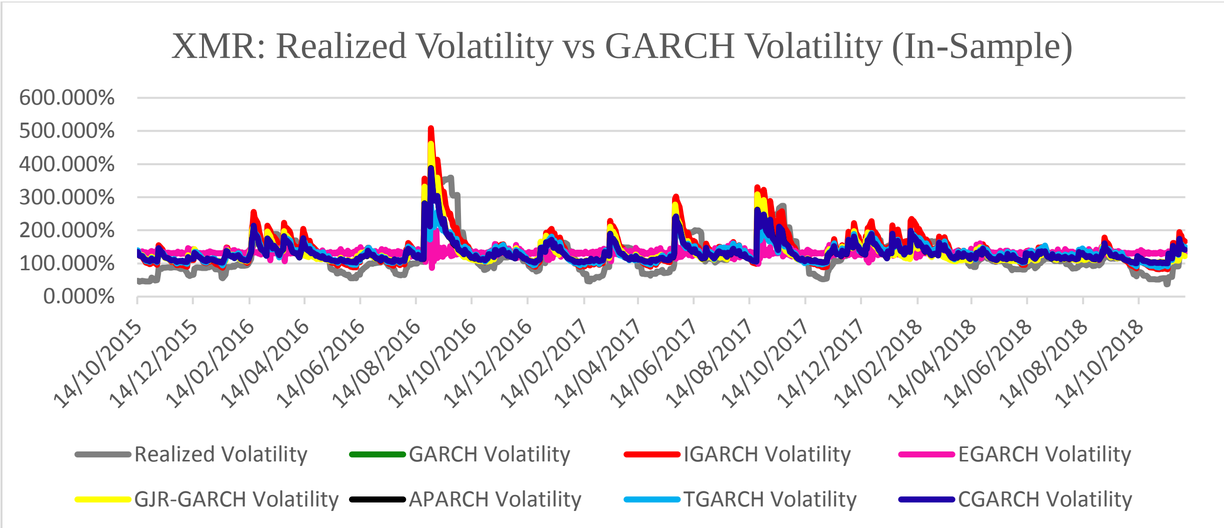
4.1.2. Realized Volatility & Volatility Comparison

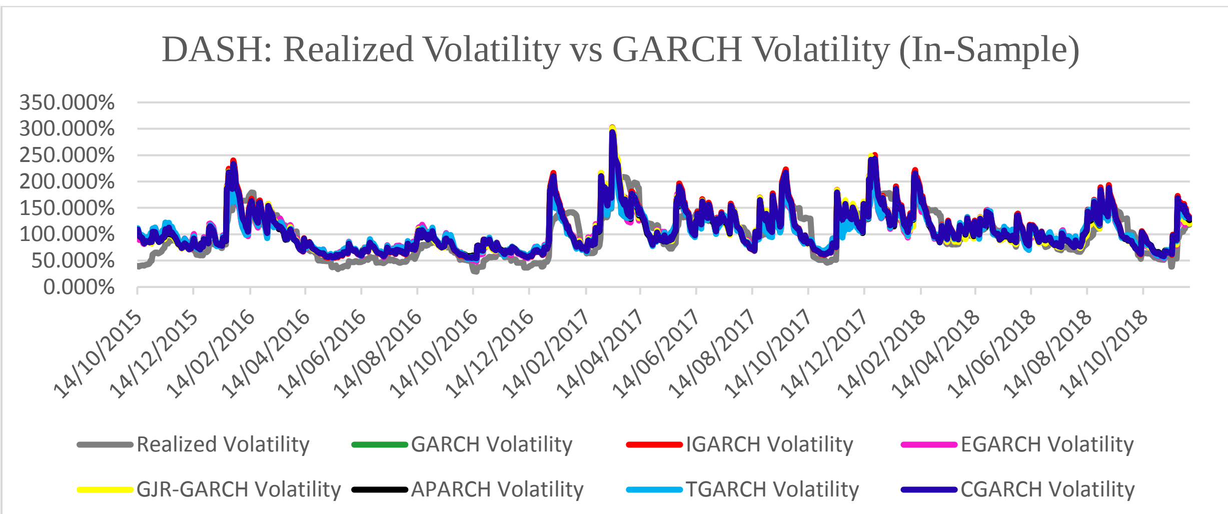
To proceed, realized volatilities are computed using equation (12), described in section 3.1.2, as many research such as Andersen & Benzoni (2008) and Lanne & Ahoniemi (2010) have evinced that the square root of sum of squared returns provides a viable approximation of realized volatility. As mentioned earlier, annualized volatilities were derived from monthly volatilities by multiplying the latter by the amount of "trading months per trading year," that is 252/22. The below figure plots the realized volatility against GARCH volatilities for each cryptocurrency and fiat currency over the in-sample period.

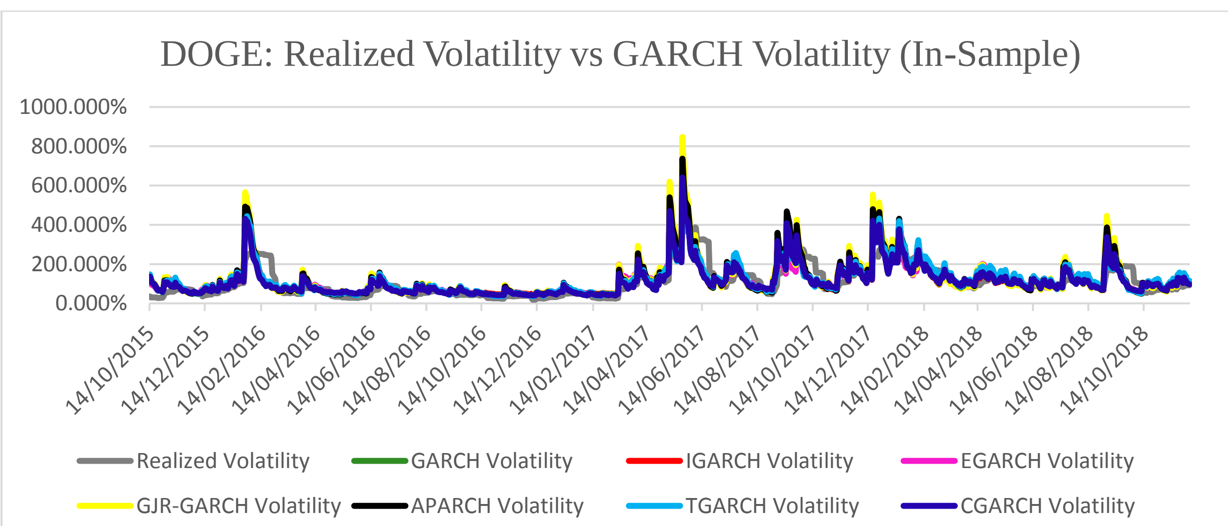

BTC: Realized Volatility vs GARCH Volatility (In-Sample)
200.000%
150.000%
100.000%
50.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

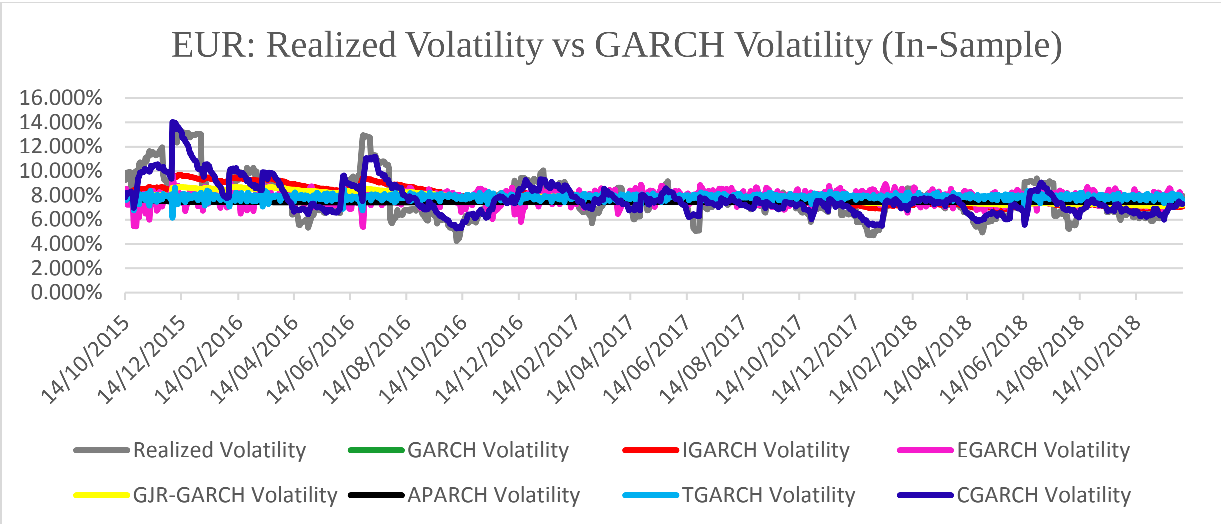

XRP: Realized Volatility vs GARCH Volatility (In-Sample)
1000.000%
800.000%
600.000%
400.000%
200.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

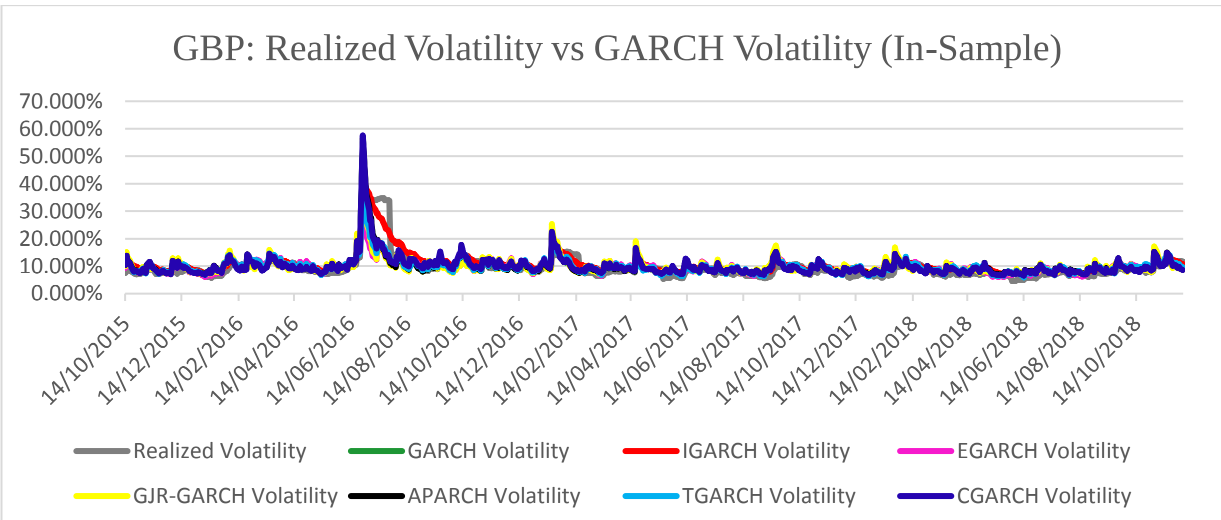

LTC: Realized Volatility vs GARCH Volatility (In-Sample)
450.000%
400.000%
350.000%
300.000%
250.000%
200.000%
150.000%
100.000%
50.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

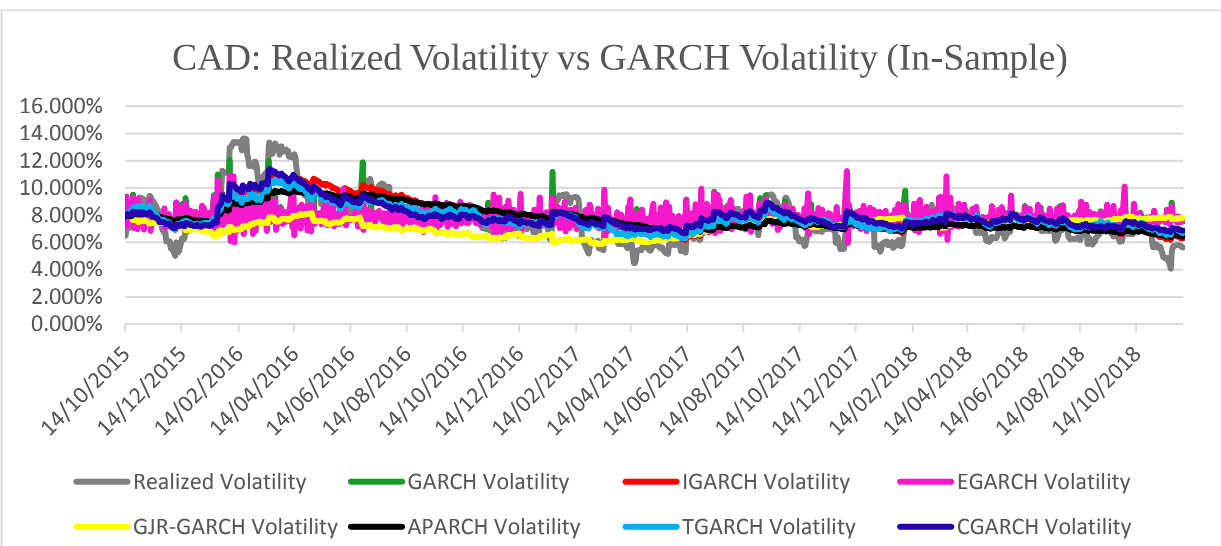

XMR: Realized Volatility vs GARCH Volatility (In-Sample)
600.000%
500.000%
400.000%
300.000%
200.000%
100.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

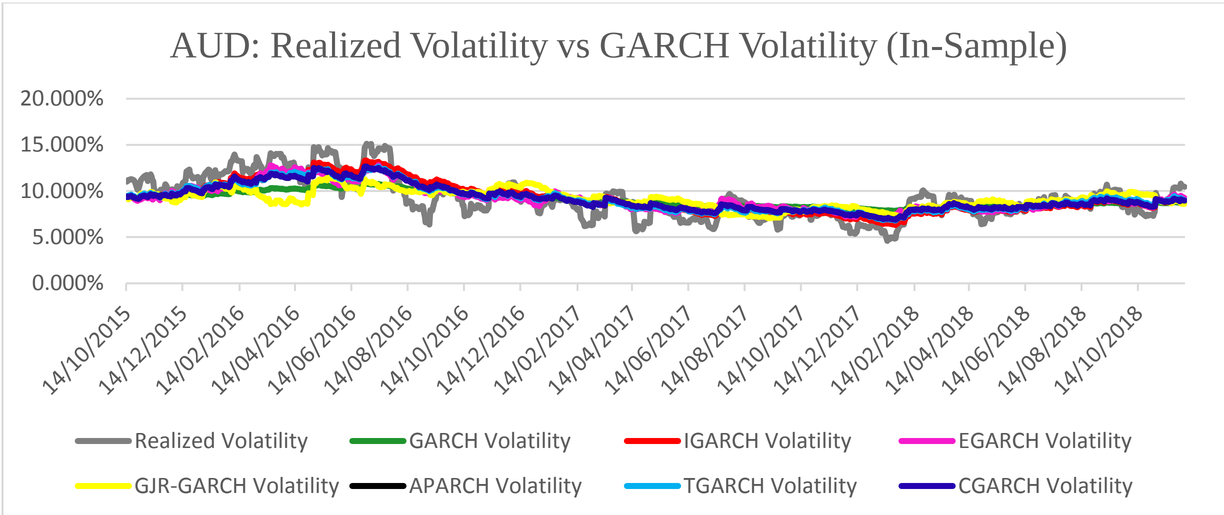

DASH: Realized Volatility vs GARCH Volatility (In-Sample)
350.000%
300.000%
250.000%
200.000%
150.000%
100.000%
50.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

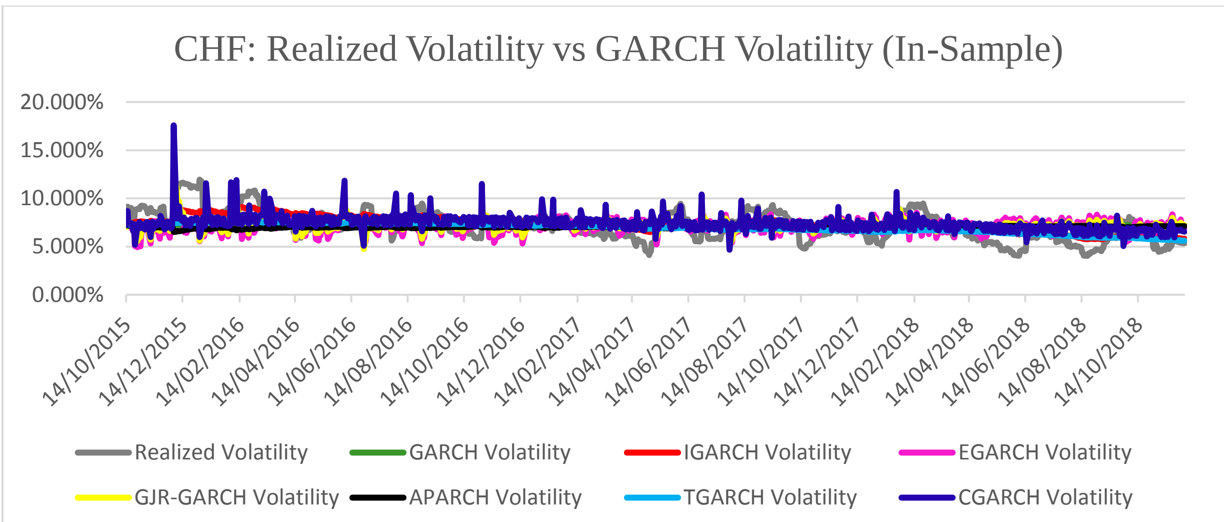

DOGE: Realized Volatility vs GARCH Volatility (In-Sample)
1000.000%
800.000%
600.000%
400.000%
200.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility


EUR: Realized Volatility vs GARCH Volatility (In-Sample)
16.000%
14.000%
12.000%
10.000%
8.000%
6.000%
4.000%
2.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility


GBP: Realized Volatility vs GARCH Volatility (In-Sample)
70.000%
60.000%
50.000%
40.000%
30.000%
20.000%
10.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility


CAD: Realized Volatility vs GARCH Volatility (In-Sample)
16.000%
14.000%
12.000%
10.000%
8.000%
6.000%
4.000%
2.000%
0.000%
14/10/2015
14/12/2015
14/02/2016
14/04/2016
14/06/2016
14/08/2016
14/10/2016
14/12/2016
14/02/2017
14/04/2017
14/06/2017
14/08/2017
14/10/2017
14/12/2017
14/02/2018
14/04/2018
14/06/2018
14/08/2018
14/10/2018
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility





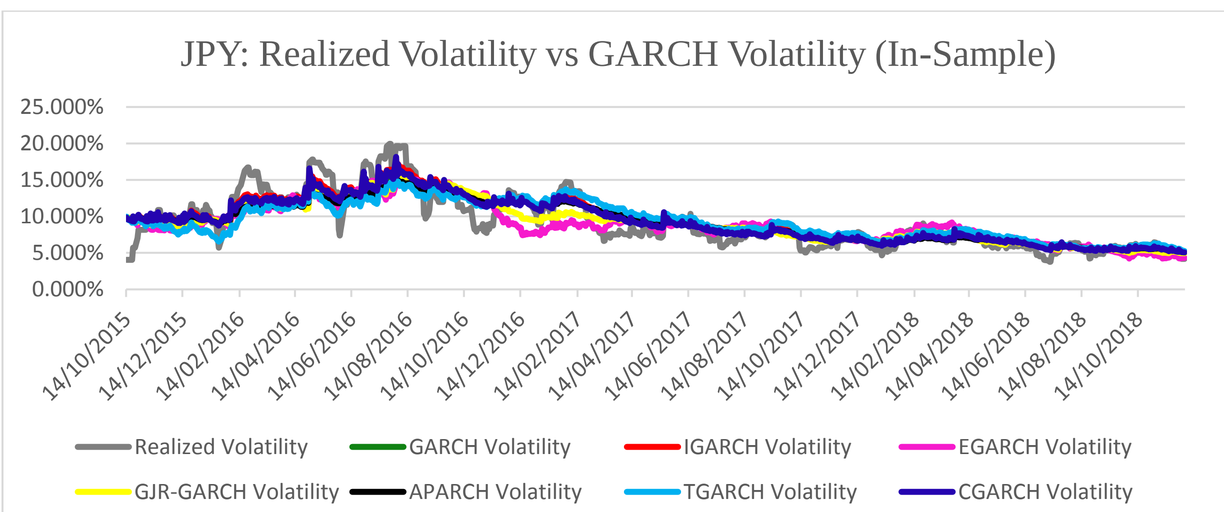


Figure 3: Realized Volatility versus GARCH Models Volatilities for each Cryptocurrency and Fiat Currency covering the In-Sample period.

Once computed, the realized volatilities related to the six cryptocurrencies and six fiat currencies are compared to their calculated in-sample volatilities under each model in order to determine the most accurate model for predicting their volatilities. This comparison is addressed using the three error metrics, as defined in section 3.3: The Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE). These error metrics determine the optimal model by subtracting the calculated volatility from the realized volatility under each model for each cryptocurrency and fiat currency. The model with the least error difference is considered the most accurate. The following table details the in-sample error statistics values along with their rankings for each cryptocurrency and fiat currency under each of the selected models:

		GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	TGARCH	CGARCH
Bitcoin	**RMSE**	0.1438834	0.1452684	0.1626541	0.1420283	0.1438827	0.1543428	0.1446455
	Rank	3	5	7	**1**	2	6	4
	MAPE	0.2597495	0.2600098	0.2872678	0.2513765	0.2597456	0.2917471	0.2570306
	Rank	4	5	6	**1**	3	7	2
	MAE	0.1150521	0.1160461	0.1361096	0.1118629	0.1150513	0.1295820	0.1143421
	Rank	4	5	7	**1**	3	6	2
Ripple		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.5606818	0.6118490	0.6899646	0.7824047	0.5606813	0.5310634	0.6502455
	Rank	3	4	6	7	2	**1**	5
	MAPE	0.4470383	0.4684544	0.4947561	0.7136717	0.4470379	0.4704342	0.4792861
	Rank	2	3	6	7	**1**	4	5
	MAE	0.3831406	0.4076085	0.4384743	0.5492027	0.3831403	0.3870977	0.4342990
	Rank	2	4	6	7	**1**	3	5
Litecoin		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.3142638	0.3159657	0.3365539	0.3258989	0.3142511	0.3240884	0.3882587
	Rank	2	3	6	5	**1**	4	7
	MAPE	0.4986707	0.4956276	0.4674167	0.4915881	0.4986553	0.4883770	0.5264090
	Rank	6	4	**1**	3	5	2	7
	MAE	0.2501523	0.2647562	0.2612368	0.2493461	0.2501420	0.2643273	0.2985987
	Rank	3	6	4	**1**	2	5	7
Monero		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.3797907	0.3752803	0.5668703	0.4026492	0.3797942	0.3895298	0.3798416
	Rank	2	**1**	7	6	3	5	4
	MAPE	0.2982211	0.2959970	0.4417629	0.3070350	0.2982247	0.3041831	0.2992323
	Rank	2	**1**	7	6	3	5	4
	MAE	0.2921630	0.2997151	0.4386953	0.3084665	0.2921662	0.3048044	0.2929214
	Rank	**1**	4	7	6	2	5	3

		GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	TGARCH	CGARCH
Dash	**RMSE**	0.2534699	0.2697845	0.2521968	0.2533920	0.2534681	0.2489615	0.2576457
	Rank	5	7	2	3	4	**1**	6
	MAPE	0.2470614	0.2486341	0.2445397	0.2395079	0.2470599	0.2554356	0.2465609
	Rank	5	6	2	**1**	4	7	3
	MAE	0.2032780	0.2114108	0.2014295	0.1985643	0.2032765	0.2040712	0.2053800
	Rank	4	7	2	**1**	3	5	6
Dogecoin		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.5064305	0.5064303	0.5490583	0.6602732	0.5920399	0.4847540	0.5473900
	Rank	3	2	5	7	6	**1**	4
	MAPE	0.3729656	0.3729680	0.3851809	0.4153554	0.3994000	0.3855438	0.3777930
	Rank	**1**	2	4	7	6	5	3
	MAE	0.3393688	0.3393693	0.3612364	0.4055001	0.3836772	0.3454698	0.3614334
	Rank	**1**	2	4	7	6	3	5
Euro		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0167027	0.0137294	0.0182184	0.0149769	0.0168460	0.0172814	0.0081503
	Rank	4	2	7	3	5	6	**1**
	MAPE	0.1592232	0.1413479	0.1819595	0.1489521	0.1521989	0.1753247	0.0770562
	Rank	5	2	7	3	4	6	**1**
	MAE	0.0123338	0.0104168	0.0138541	0.0112251	0.0121169	0.0132459	0.0059228
	Rank	5	2	7	3	4	6	**1**
British Pound		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0317732	0.0199425	0.0340562	0.0344916	0.0321520	0.0297481	0.0317099
	Rank	4	**1**	6	7	5	2	3
	MAPE	0.1866371	0.1744161	0.1920843	0.2026332	0.1892557	0.1804646	0.1777932
	Rank	4	**1**	6	7	5	3	2
	MAE	0.0183853	0.0153797	0.0192712	0.0200396	0.0186582	0.0176004	0.0177472
	Rank	4	**1**	6	7	5	2	3
Canadian Dollar		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0170002	0.0109265	0.0179887	0.0184561	0.0129582	0.0107604	0.0103680
	Rank	5	3	6	7	4	2	**1**
	MAPE	0.1721560	0.1169429	0.1832628	0.1627358	0.1330819	0.1085776	0.1139570
	Rank	6	3	7	5	4	**1**	2
	MAE	0.0129722	0.0087688	0.0138477	0.0133409	0.0101546	0.0081901	0.0082551
	Rank	5	3	7	6	4	**1**	2

		GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	TGARCH	CGARCH
Australian Dollar	**RMSE**	0.0167435	0.0120629	0.0125309	0.0178898	0.0123189	0.0122403	0.0123128
	Rank	6	**1**	5	7	4	2	3
	MAPE	0.1487503	0.1112955	0.1160254	0.1596206	0.1153037	0.1146732	0.1150881
	Rank	6	**1**	5	7	4	2	3
	MAE	0.0132555	0.0097394	0.0100715	0.0142299	0.0100603	0.0100071	0.0100386
	Rank	6	**1**	5	7	4	2	3
Swiss Franc		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0152728	0.0108196	0.0173273	0.0157562	0.0161374	0.0130508	0.0146707
	Rank	4	**1**	7	5	6	2	3
	MAPE	0.1816106	0.1316578	0.2056852	0.1879071	0.1845078	0.1393637	0.1742015
	Rank	4	**1**	7	6	5	2	3
	MAE	0.0121120	0.0088518	0.0137593	0.0124809	0.0126762	0.0099685	0.0116783
	Rank	4	**1**	7	5	6	2	3
Japanese Yen		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0163927	0.0149071	0.0220859	0.0191509	0.0186790	0.0207713	0.0153348
	Rank	3	**1**	7	5	4	6	2
	MAPE	0.1389362	0.1298187	0.1827249	0.1563098	0.1551345	0.1755943	0.1300348
	Rank	3	**1**	7	5	4	6	2
	MAE	0.0120576	0.0109139	0.0166857	0.0142432	0.0137705	0.0155573	0.0112381
	Rank	3	**1**	7	5	4	6	2

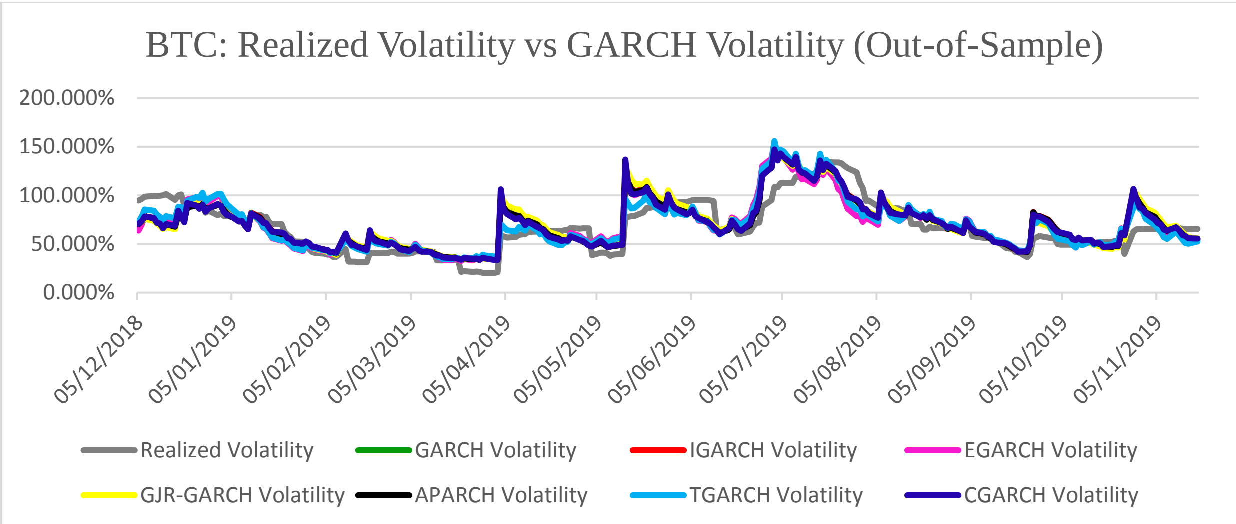
Table 13: Error Statistics by Rankings under each Volatility Model for each Cryptocurrency and Fiat Currency covering the In-Sample Period.

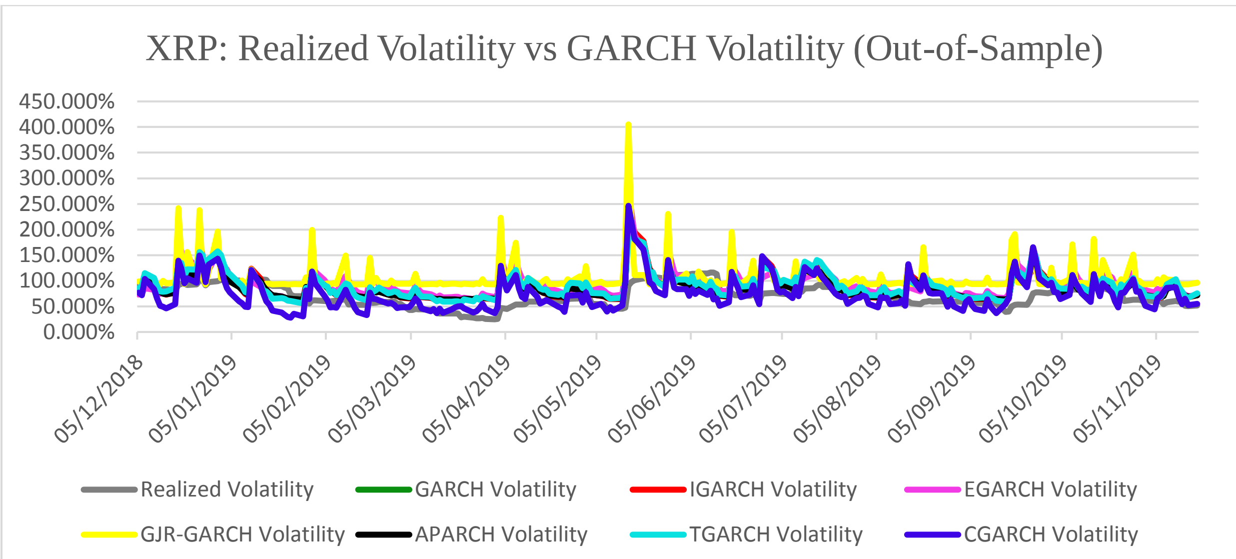
4.2. <u>Out-of-Sample Modeling</u>

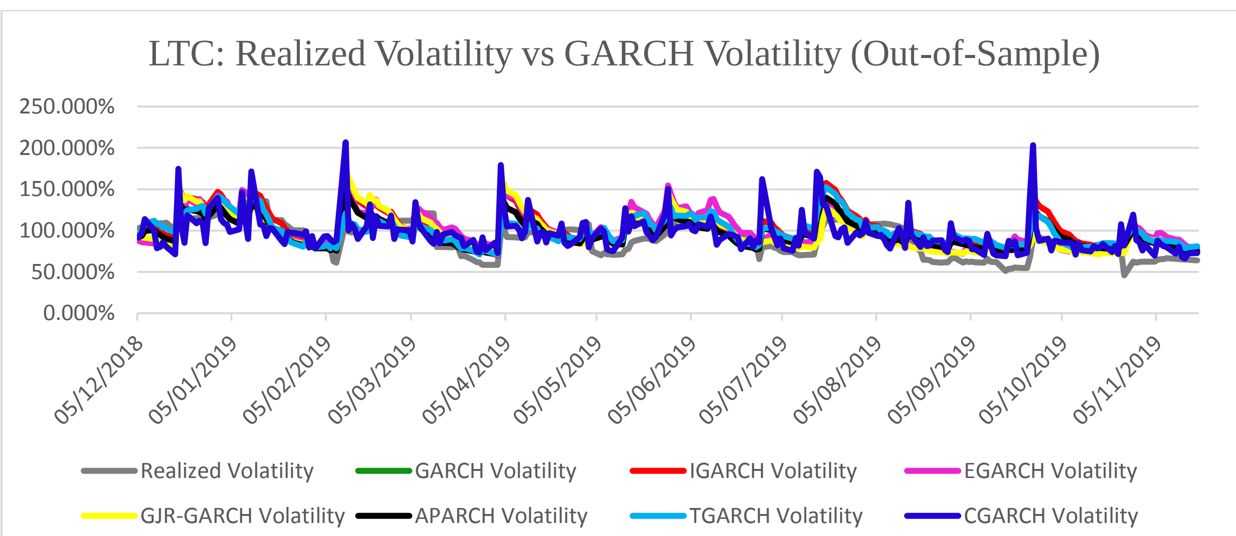
Exact calculations are carried forward for the out-of-sample period extending from December 4th 2018 till November 18th 2019, where the only modification is that the resulting parameters obtained from the in-sample period (check Table 4) are those used to forecast the conditional volatilities for the out-of-sample period. Therefore, the parameters are now plugged into formulas rather than being re-calculated. This is done in order to determine whether there are any consistencies among the selected models upon shifts in time.

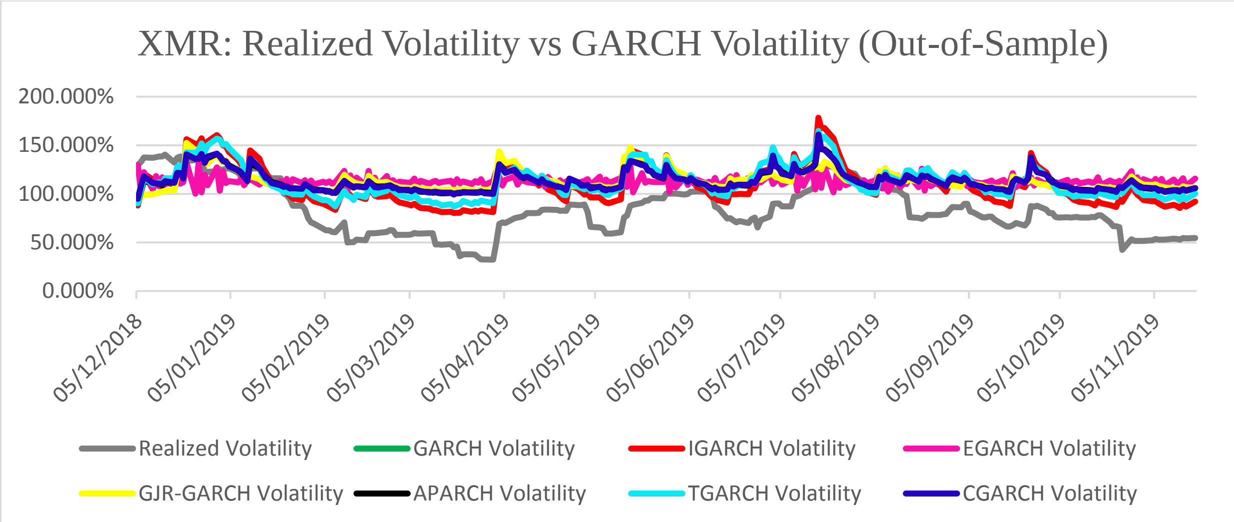
4.2.1. <u>Realized Volatility & Volatility Comparison</u>

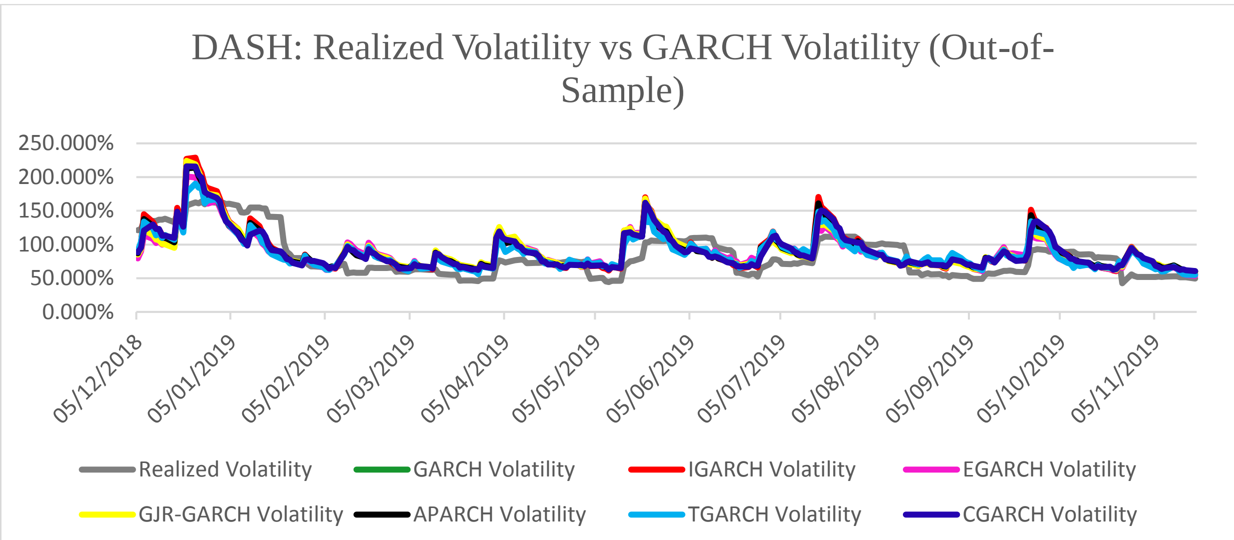
To proceed, realized volatilities are computed as in section 4.1.2 and are compared to calculated volatilities. The below figure plots the realized volatility against the modeled GARCH volatilities for each cryptocurrency and fiat currency covering the out-of-sample period.

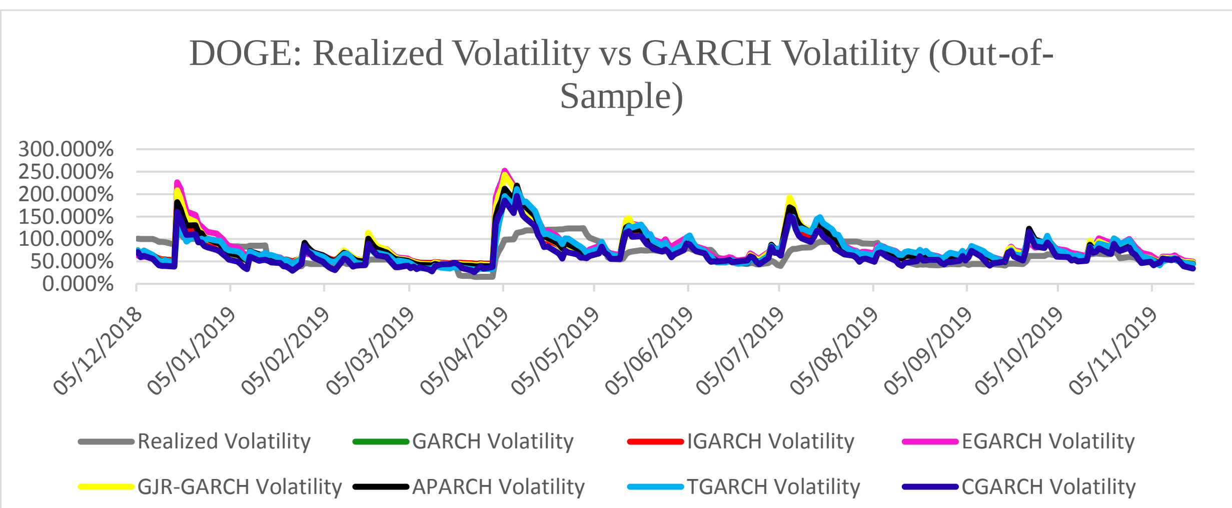

BTC: Realized Volatility vs GARCH Volatility (Out-of-Sample)
200.000%
150.000%
100.000%
50.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

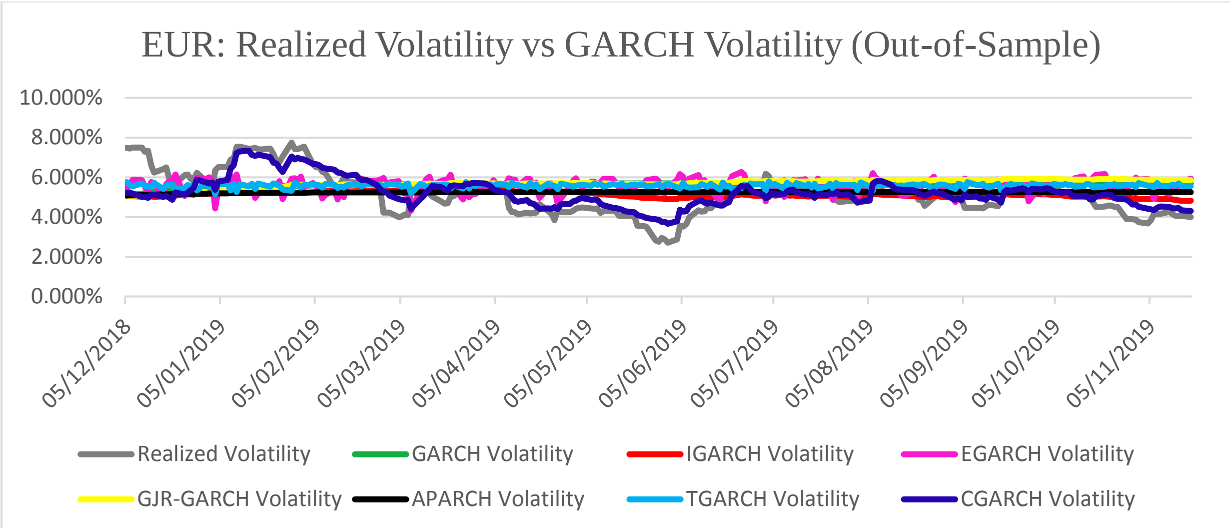

XRP: Realized Volatility vs GARCH Volatility (Out-of-Sample)
450.000%
400.000%
350.000%
300.000%
250.000%
200.000%
150.000%
100.000%
50.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

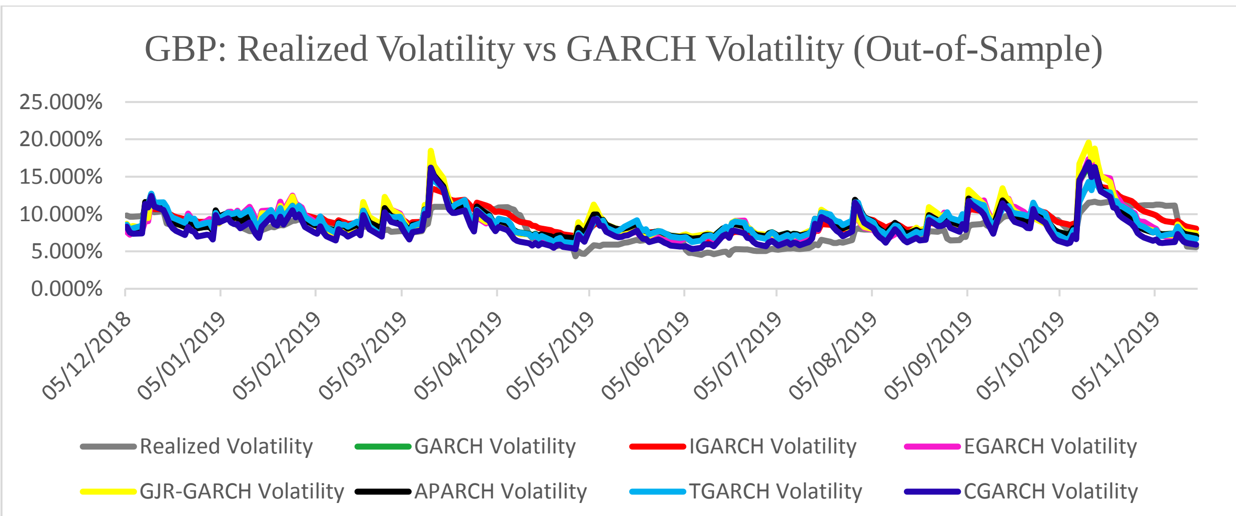

LTC: Realized Volatility vs GARCH Volatility (Out-of-Sample)
250.000%
200.000%
150.000%
100.000%
50.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

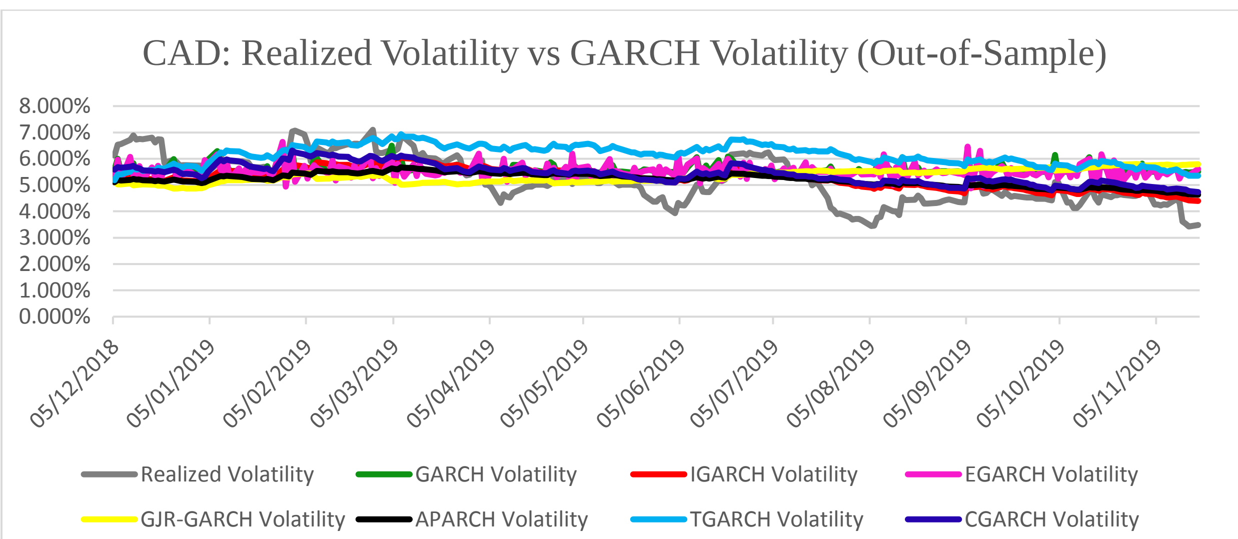

XMR: Realized Volatility vs GARCH Volatility (Out-of-Sample)
200.000%
150.000%
100.000%
50.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

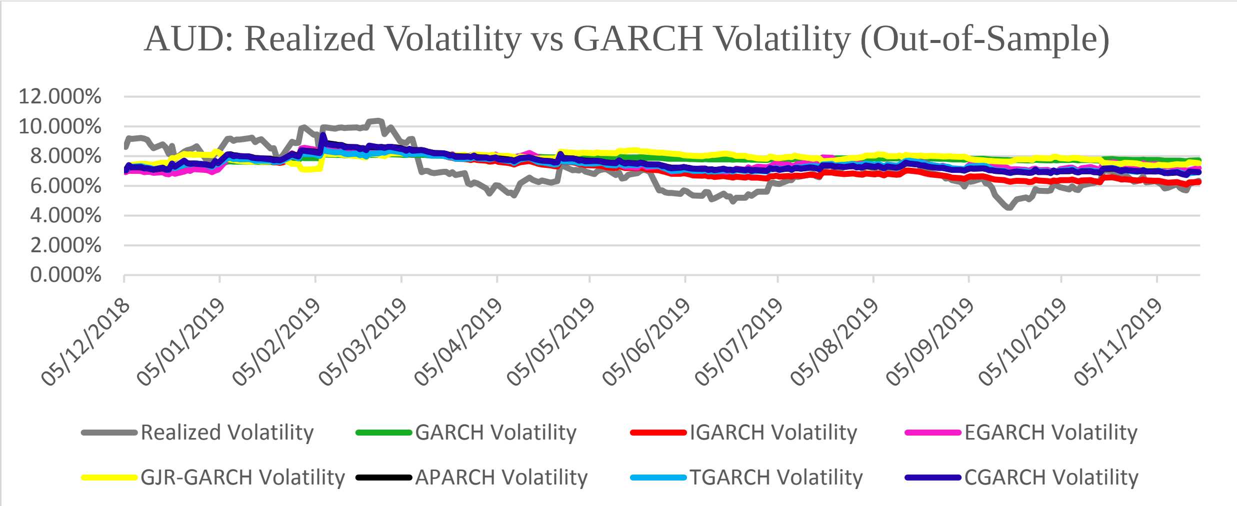

DASH: Realized Volatility vs GARCH Volatility (Out-of-Sample)
250.000%
200.000%
150.000%
100.000%
50.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility

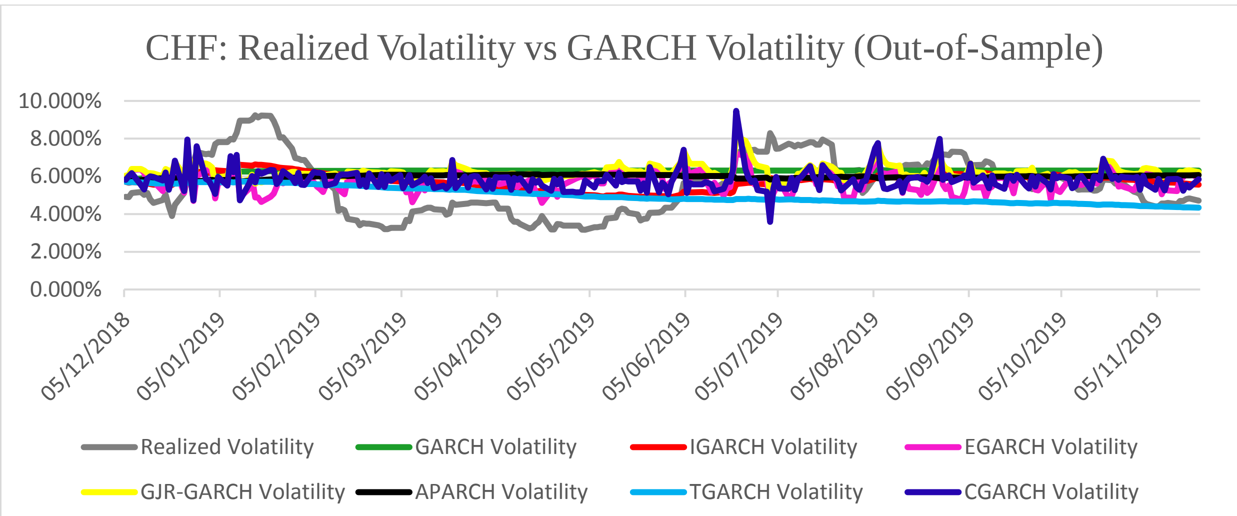

DOGE: Realized Volatility vs GARCH Volatility (Out-of-Sample)
300.000%
250.000%
200.000%
150.000%
100.000%
50.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility


EUR: Realized Volatility vs GARCH Volatility (Out-of-Sample)
10.000%
8.000%
6.000%
4.000%
2.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility


GBP: Realized Volatility vs GARCH Volatility (Out-of-Sample)
25.000%
20.000%
15.000%
10.000%
5.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility


CAD: Realized Volatility vs GARCH Volatility (Out-of-Sample)
8.000%
7.000%
6.000%
5.000%
4.000%
3.000%
2.000%
1.000%
0.000%
05/12/2018
05/01/2019
05/02/2019
05/03/2019
05/04/2019
05/05/2019
05/06/2019
05/07/2019
05/08/2019
05/09/2019
05/10/2019
05/11/2019
Realized Volatility
GARCH Volatility
IGARCH Volatility
EGARCH Volatility
GJR-GARCH Volatility
APARCH Volatility
TGARCH Volatility
CGARCH Volatility





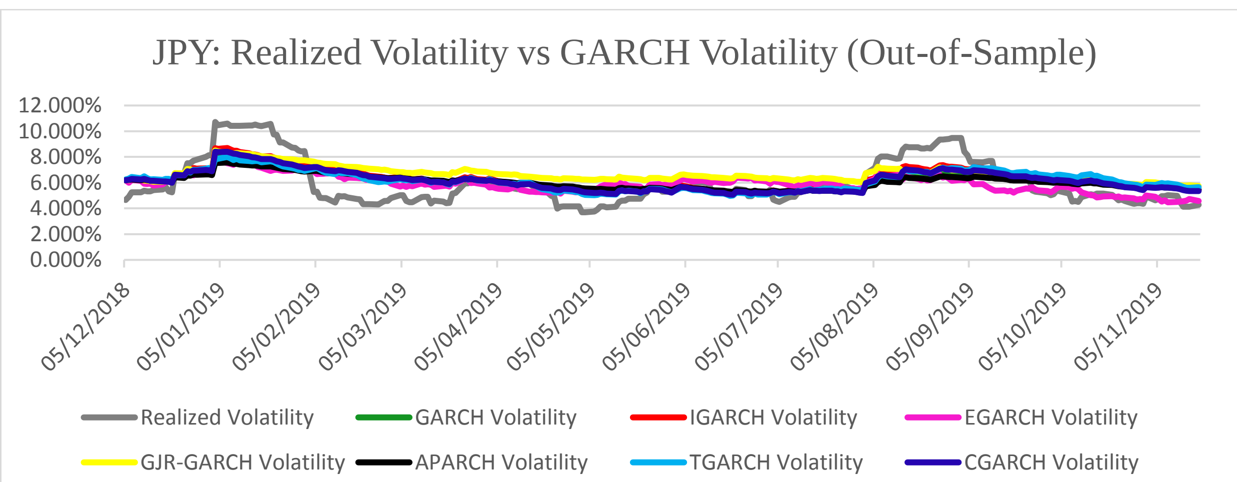


Figure 4: Realized Volatility versus GARCH Models Volatilities for each Cryptocurrency and Fiat Currency covering the Out-of-Sample period.

Subsequently, the realized volatilities are compared to the calculated out-of-sample volatilities to determine the optimal model for predicting each cryptocurrency's and fiat currency's volatility for the out-of-sample period. This comparison is also addressed using the error statistics: RMSE, MAPE and MAE. The following table details the out-of-sample error statistics values along with their rankings for each cryptocurrency and fiat currency under each of the selected models.

		GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	TGARCH	CGARCH
Bitcoin	**RMSE**	0.1408560	0.1422242	0.1576708	0.1514731	0.1408557	0.1349229	0.1426873
	Rank	3	4	7	6	2	**1**	5
	MAPE	0.1746768	0.1754714	0.1920261	0.1842919	0.1746752	0.1718826	0.1743728
	Rank	4	5	7	6	3	**1**	2
	MAE	0.1065257	0.1071604	0.1205260	0.1127929	0.1065251	0.1049094	0.1063803
	Rank	4	5	7	6	3	**1**	2
Ripple		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.2836429	0.3157890	0.3587448	0.5122478	0.2836421	0.3072511	0.2988604
	Rank	2	5	6	7	**1**	4	3
	MAPE	0.3751473	0.3982823	0.4826442	0.6771766	0.3751465	0.4158141	0.3139430
	Rank	3	4	6	7	2	5	**1**
	MAE	0.2241280	0.2430162	0.2883991	0.3849528	0.2241274	0.2531108	0.2147268
	Rank	3	4	6	7	2	5	**1**
Litecoin		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.1646110	0.2140478	0.2057390	0.1920021	0.1646023	0.1765855	0.2361351
	Rank	2	6	5	4	**1**	3	7
	MAPE	0.1705084	0.2090270	0.2099013	0.1811546	0.1705011	0.1843552	0.2037097
	Rank	2	6	7	3	**1**	4	5
	MAE	0.1390615	0.1720250	0.1716690	0.1523151	0.1390551	0.1438349	0.1708536
	Rank	2	7	6	4	**1**	3	5
Monero		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.3642975	0.3154638	0.5537820	0.3856993	0.3643024	0.3472789	0.3645876
	Rank	3	**1**	7	6	4	2	5
	MAPE	0.4929623	0.4053284	0.7601671	0.5192801	0.4929694	0.4613187	0.4934940
	Rank	3	**1**	7	6	4	2	5
	MAE	0.3210113	0.2796983	0.4965147	0.3388234	0.3210159	0.3126987	0.3213154
	Rank	3	**1**	7	6	4	2	5
Dash		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.2268232	0.2389969	0.2330716	0.2345549	0.2268218	0.2147497	0.2283810
	Rank	3	7	5	6	2	**1**	4
	MAPE	0.2497570	0.2504086	0.2582297	0.2573711	0.2497556	0.2391720	0.2500200
	Rank	3	5	7	6	2	**1**	4
	MAE	0.1905541	0.1945864	0.1967049	0.1967433	0.1905530	0.1814664	0.1918504
	Rank	3	5	6	**7**	2	**1**	4

		GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	TGARCH	CGARCH
Dogecoin	**RMSE**	0.2579243	0.2579248	0.3538872	0.3413393	0.3031388	0.2714862	0.2684947
	Rank	**1**	2	7	6	5	4	3
	MAPE	0.3333432	0.3333463	0.3971993	0.3788513	0.3485463	0.3323091	0.2913433
	Rank	3	4	7	6	5	2	**1**
	MAE	0.1972936	0.1972946	0.2521531	0.2382838	0.2190991	0.2068837	0.1938899
	Rank	2	3	7	6	5	4	**1**
Euro		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0263039	0.0096160	0.0298588	0.0127606	0.0246930	0.0293008	0.0061158
	Rank	5	2	7	3	4	6	**1**
	MAPE	0.5244953	0.1541455	0.6007713	0.2306426	0.4924492	0.5936001	0.0909191
	Rank	5	2	7	3	4	6	**1**
	MAE	0.0238637	0.0075435	0.0275541	0.0107483	0.0223726	0.0272811	0.0044955
	Rank	5	2	7	3	4	6	**1**
British Pound		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0191869	0.0147745	0.0200180	0.0226044	0.0195144	0.0185480	0.0182524
	Rank	4	**1**	6	7	5	3	2
	MAPE	0.2152740	0.1865596	0.2125813	0.2370252	0.2170239	0.2109993	0.1744509
	Rank	5	2	4	7	6	3	**1**
	MAE	0.0154997	0.0131359	0.0160869	0.0175669	0.0157099	0.0152953	0.0139454
	Rank	4	**1**	6	7	5	3	2
Canadian Dollar		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0259045	0.0066675	0.0265882	0.0105759	0.0075323	0.0122491	0.0136170
	Rank	6	**1**	7	3	2	4	5
	MAPE	0.5102805	0.1091050	0.5181345	0.1863255	0.1248719	0.2282569	0.2635371
	Rank	6	**1**	7	3	2	4	5
	MAE	0.0244134	0.0054879	0.0248008	0.0091914	0.0063423	0.0106642	0.0125132
	Rank	6	**1**	7	3	2	4	5
Australian Dollar		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0161378	0.0101321	0.0132442	0.0167421	0.0119262	0.0122043	0.0119056
	Rank	6	**1**	5	7	3	4	2
	MAPE	0.2272484	0.1228304	0.1794958	0.2335342	0.1587185	0.1611424	0.1583059
	Rank	6	**1**	5	7	3	4	2
	MAE	0.0148469	0.0084156	0.0118765	0.0151698	0.0104083	0.0107356	0.0103784
	Rank	6	**1**	5	7	3	4	2

		GARCH	IGARCH	EGARCH	GJR-GARCH	APARCH	TGARCH	CGARCH
Swiss Franc	**RMSE**	0.0224845	0.0130925	0.0241501	0.0229456	0.0219850	0.0168339	0.0157209
	Rank	5	**1**	7	6	4	3	2
	MAPE	0.4328319	0.2270885	0.4617816	0.4446551	0.4218030	0.2694923	0.2690886
	Rank	5	**1**	7	6	4	3	2
	MAE	0.0189396	0.0110173	0.0204856	0.0197631	0.0185408	0.0146837	0.0131818
	Rank	5	**1**	7	6	4	3	2
Japanese Yen		**GARCH**	**IGARCH**	**EGARCH**	**GJR-GARCH**	**APARCH**	**TGARCH**	**CGARCH**
	RMSE	0.0131185	0.0121331	0.0137439	0.0152725	0.0143299	0.0130453	0.0124106
	Rank	4	**1**	5	7	6	3	2
	MAPE	0.1830458	0.1765844	0.1734588	0.2487440	0.1923998	0.1832938	0.1745765
	Rank	4	3	**1**	7	6	5	2
	MAE	0.0107012	0.0100164	0.0106849	0.0135592	0.0115151	0.0107230	0.0101379
	Rank	4	**1**	3	7	6	5	2

Table 14: Error Statistics by Rankings under each Volatility Model for each Cryptocurrency and Fiat Currency covering the Out-of-Sample Period.

4.3. Model Optimization Results

		In-Sample *(A)*	Out-of-Sample (*B*)
Cryptocurrencies	**BTC**	GJR-GARCH (1,1)	TGARCH (1,1)
	XRP	APARCH (1,1)	CGARCH (1,1)
	LTC	GJR-GARCH (1,1)	APARCH(1,1)
	XMR	IGARCH (1,1)	IGARCH (1,1)
	DASH	GJR-GARCH (1,1)	TGARCH (1,1)
	DOGE	GARCH (1,1)	CGARCH (1,1)
Fiat Currencies	**EUR**	CGARCH (1,1)	CGARCH (1,1)
	GBP	IGARCH (1,1)	IGARCH (1,1)
	CAD	TGARCH (1,1)	IGARCH (1,1)
	AUD	IGARCH (1,1)	IGARCH (1,1)
	CHF	IGARCH (1,1)	IGARCH (1,1)
	JPY	IGARCH (1,1)	IGARCH (1,1)

Table 15: Optimal Models for each Cryptocurrency and Fiat Currency under the (A) In-Sample & (B) Out-of-Sample Periods.

As illustrated in the above table, the results show consistency among fiat currencies, whereby the Integrated GARCH has proven to perform best for most of the fiat currencies, particularly the British Pound, Australian Dollar, Swiss Franc and the Japanese Yen. The IGARCH model was also found to be the most accurate model for the Canadian Dollar, but only for the out-of-sample period given that the Threshold GARCH performed better during the in-sample period. However and quite surprisingly, the Component GARCH modeled the Euro almost "impeccably". This may be contributed to the distinctive characteristics of the CGARCH model which divides the conditional variance into its transitory and permanent components, whereby the long-run component is allowed to be continuously updated rather than held uniform, thereby better capturing and reflecting on volatility clusters and persistence in Euro's returns. It is important to note that when the omega term "ω" takes the value of zero, the IGARCH model becomes nothing different from the Exponentially Weighted Moving Average (EWMA) Model, which is the case of all fiat currencies. Therefore, the IGARCH has proven to be the prevailing model when modeling foreign exchange markets. This may be contributed to their low volatile nature, their typical symmetrical behavior to shocks, and 'persistent variance' in which current information remains important when forecasting volatility.

Exceptionally and among all cryptocurrencies, the Integrated GARCH was also the best performing model for Monero, for both sampled periods. This might be due to the fact that the absence of a long-run average variance in the IGARCH model entails that any disturbance in the market brings an everlasting change in Monero's volatility structure, which explains the overstated volatility estimates obtained under the IGARCH model (Figures 3 & 4).

As for the remaining cryptocurrencies, the GJR-GARCH model proved to be superior during the in-sample period while the CGARCH and TGARCH models proved to be best performers during the out-of-sample period which validates the assumption that advanced GARCH models better model asymmetries in cryptocurrencies' volatilities. Specifically, for the in-sample period, the GJR-GARCH model is selected for Bitcoin, Litecoin and Dash, APARCH is selected for Ripple, and GARCH is selected for Dogecoin. For the out-of-sample period, TGARCH performed best for Bitcoin and Dash while CGARCH is selected for Ripple and Dogecoin and APARCH is selected for Litecoin. Apparently, it is natural to observe some discrepancies among cryptocurrencies due their relatively highly volatile feature. But remarkably, however, the EGARCH model which was considered superior in many papers such as Krogt (2018) and Abdalla (2012), was one of the worst performing models among all fiat and virtual currencies.

Nevertheless, the rankings obtained are consistent with what can be visually observed in the time series of volatilities (Figures 3 & 4) where the volatilities predicted by the optimal models seem to graphically best fit the realized volatilities. Consequently, the optimal models inferred from the out-of-sample period and implied from table (15/B) will be used in the next section to calculate Value at Risk.

4.4. Value at Risk & Back Testing Results

This section details how the out-of-sample results are integrated as part of our adopted procedure for estimating the Value at Risk for each cryptocurrency and fiat currency at different significance levels (90%, 95%, 97.5% and 99% CLs) and in performing the corresponding Kupiec Likelihood Ratio Test, which is ultimately used to measure the model's efficiency by back testing the obtained VaR results.

4.4.1. The Rolling Window Procedure for Variance Estimation

Given that the number of simulated VaR is 250 days and since the rolling window procedure is conducted for every time interval of 400 days to each VaR, a further 400 days of additional data on the daily closing prices of each cryptocurrency and fiat currency is required from the in-sample period. As a result, the rolling window framework spans from May 23rd 2017 till November 18th 2019 yielding to a total of 650 daily closing prices for each cryptocurrency and fiat currency.

Subsequently, the chosen sample of 650 days will be divided into 250 sub-sample periods with each sub-sample consisting of 400 daily prices/observations. As mentioned in section 3.4.1, the rolling window procedure is conducted and simulated over 250 times, in accordance with the chosen period for VaR and number of days in the out-of-sample period.

Then, the "rolling returns" of each cryptocurrency and fiat currency are calculated from the previously "rolled prices" generating a list of 399 returns for each sub-sample period.

Accordingly, the daily variances are computed 399 times for each sub-sample period using the "rolled returns" and the out-of-sample parameters (from tables 4 & 15/B) for each cryptocurrency and fiat currency. Therefore, the out-of-sample optimal models of each cryptocurrency and fiat currency are integrated into the rolling window procedure to compute the variance 99,750 times (399 x 250) for each asset. Particularly, the parameters from the CGARCH (1,1) model are used for Ripple, Dogecoin and the Euro, while those of TGARCH (1,1) are used for Bitcoin and Dash, and the parameters of the APARCH (1,1) are used for Litecoin. As for Monero, the British Pound, Canadian Dollar, Australian Dollar, Swiss Franc and the Japanese Yen, the IGARCH (1,1) model and its relevant parameters are used to compute each of their variances. Note that the parameters have not been re-estimated in this section and thereby the out of sample optimal model and their previously estimated parameters for each cryptocurrency and fiat currency have been utilized to calculate the variances, as the simulated trial period from which variances are calculated exactly mirrors the out-of-sample period extending from December 4th 2018 through November 18th 2019. The resulting volatilities, however, are deduced by simply taking the square root of the corresponding variances.

4.4.2. <u>Adjusting for the Values of Cryptocurrencies & Fiat Currencies</u>

The "rolled" values of each cryptocurrency and fiat currency from the previous subsection are adjusted using equation (26) from section 3.4.1 to update for variations in volatility taking place in the market, as proposed by Hull and White (1998). This leads to 399 different possible scenarios for the value of each cryptocurrency and fiat currency on each of the days between December 4th 2018 and November 18th 2019.

Subsequently, once the 399 scenarios corresponding to the values of these assets are generated on each of the days between December 4th 2018 and November 18th 2019, a viable return scenario is estimated accordingly from each value computed thereby generating a combined total of 99,750 returns (399 x 250) for all the days between December 4th 2018 and November 18th 2019. For instance, the return scenarios, which represent the "possible percentage gains or losses" after adjustment, for the day "December 4 2018" is calculated as per equation (27) as follows:

$$\text{Return on December 4th 2018 under } i\text{th scenario } = \frac{(v_{Ai} - v_{4/12/2018})}{v_{4/12/2018}}$$

Where "v_{Ai}" represents 1 of the 399 adjusted values for the selected cryptocurrency or fiat currency, previously calculated from equation (26), in correspondence with the scenario number "i" where 1 < i < 399. The original value of the cryptocurrency or fiat currency on December 4th 2018, denoted by "$v_{4/12/2018}$", is deducted from each of the 399 adjusted values and the results are divided by the original amount "$v_{4/12/2018}$" again in order to obtain the returns, whether the relative percentage changes are interpreted as gains or losses.

4.4.3. Value at Risk Calculations & Comparison with Actual Returns

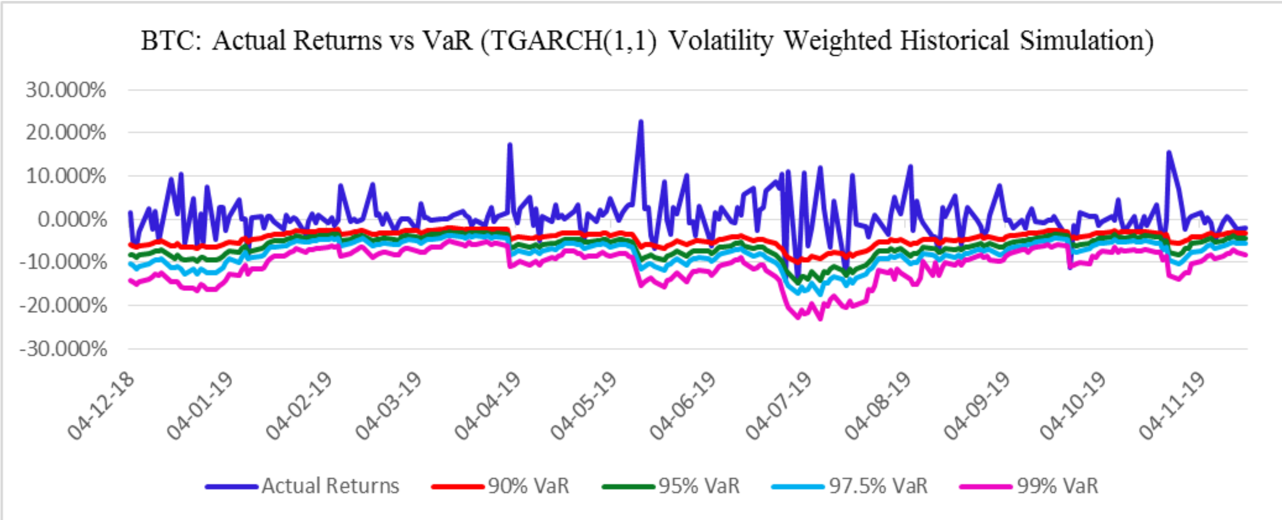
Upon computing the return scenarios for every cryptocurrency and fiat currency on each day extending from December 4th 2018 till November 18th 2019, the 90th, 95th, 97.5th and 99th percentiles of the loss distribution are computed resulting in 250 Value at Risk estimates obtained for each confidence level and for each of the selected cryptocurrencies and fiat currencies. The below table presents a partial illustration of the daily actual returns, the VaR estimates and number of exceptions at each confidence level in the case of Bitcoin. A further illustration on the computed results for the remaining cryptocurrencies and fiat currencies is available in the Appendix (table 18), noting that similar calculations are carried for all cryptocurrencies and fiat currencies.

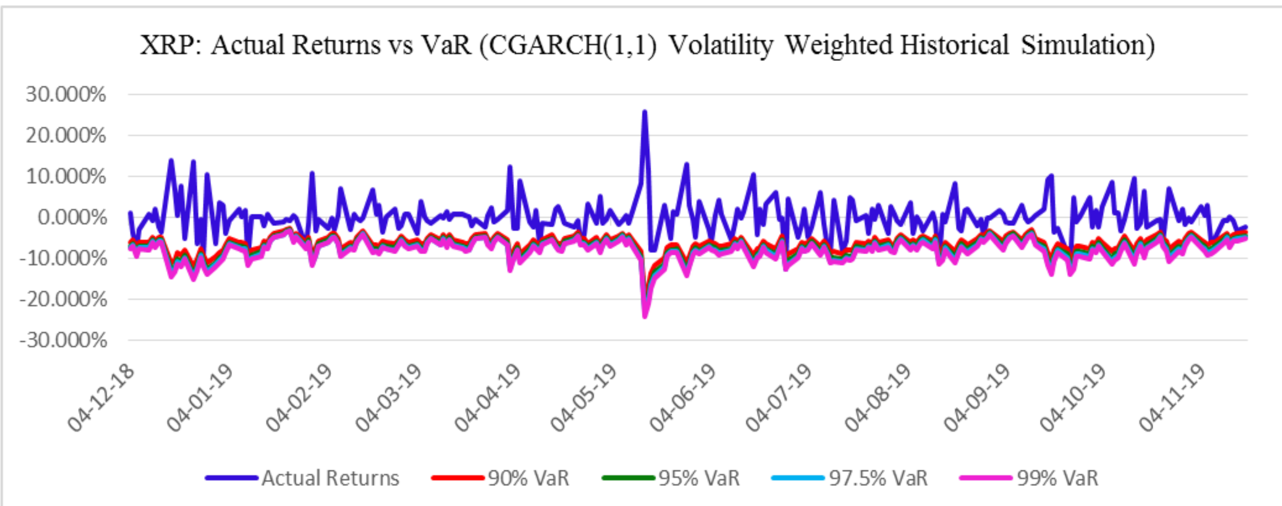
Bitcoin									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-5.834%	-6.144%	-6.626%	-6.234%	**…**	-3.124%	-2.987%	-3.113%	-3.168%
Exceptions	-	-	-	-	**15**	-	-	-	-
VaR at 95% CL	-8.208%	-8.393%	-8.770%	-8.425%	**…**	-4.464%	-3.976%	-4.434%	-4.512%
Exceptions	-	-	-	-	**6**	-	-	-	-
VaR at 97.5% CL	-10.408%	-10.760%	-11.469%	-10.916%	**…**	-5.475%	-5.280%	-5.509%	-5.639%
Exceptions	-	-	-	-	**2**	-	-	-	-
VaR at 99% CL	-14.279%	-14.601%	-15.257%	-14.656%	**…**	-8.131%	-7.242%	-7.626%	-8.375%
Exceptions	-	-	-	-	**1**	-	-	-	-
Actual Returns	1.612%	-5.128%	-6.204%	-2.873%	**…**	-0.084%	-1.137%	-2.482%	-2.152%

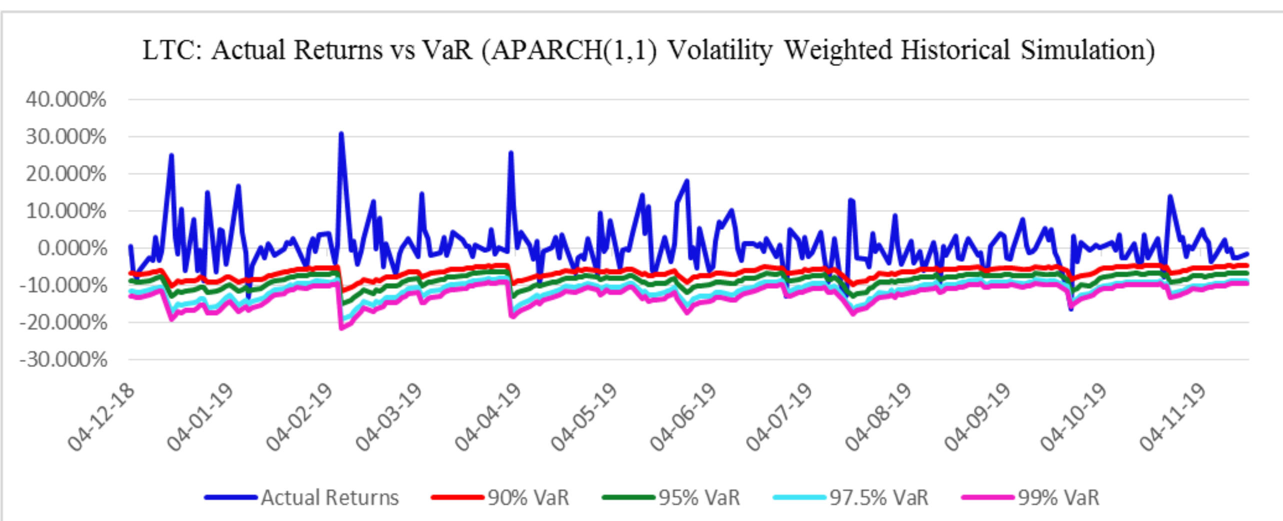
Table 16: Partial Illustration of the Actual Returns, Value at Risk, and Number of Exceptions Estimates for Bitcoin at the Different Levels of Significance between December 4th 2018 and November 18th 2019.

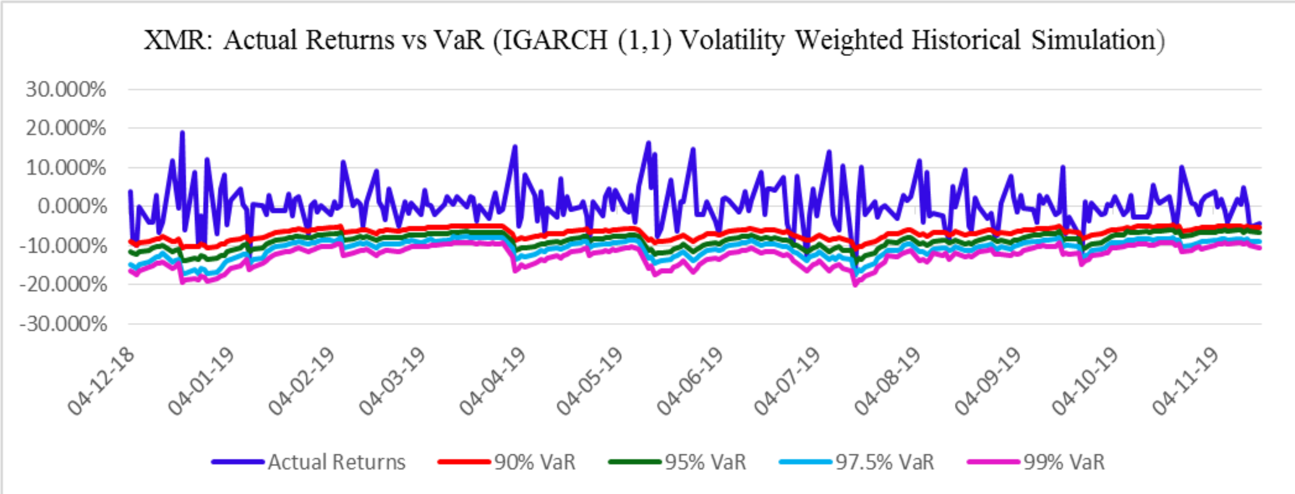
Notice how on each date the VaR increases evidently with a corresponding increase in confidence level. It is worth mentioning that the missing dates between columns in the above table pertain to weekend days that have been previously adjusted for, as per section 3.6. The above VaR results are computed using the function =PERCENTILE.EXC(ReturnScenariosArray, α) where "α" takes the values of 10%, 5%, 2.5% and 1% corresponding to the 90th, 95th, 97.5th ad 99th percentiles respectively. The above function is computed 250 times for each confidence interval to determine the VaR on each of the 250 days extending from December 4th 2018 till November 18th 2019. As a result, each cryptocurrency and fiat currency has a combined total of 1,000 (250 x 4) VaR estimates.

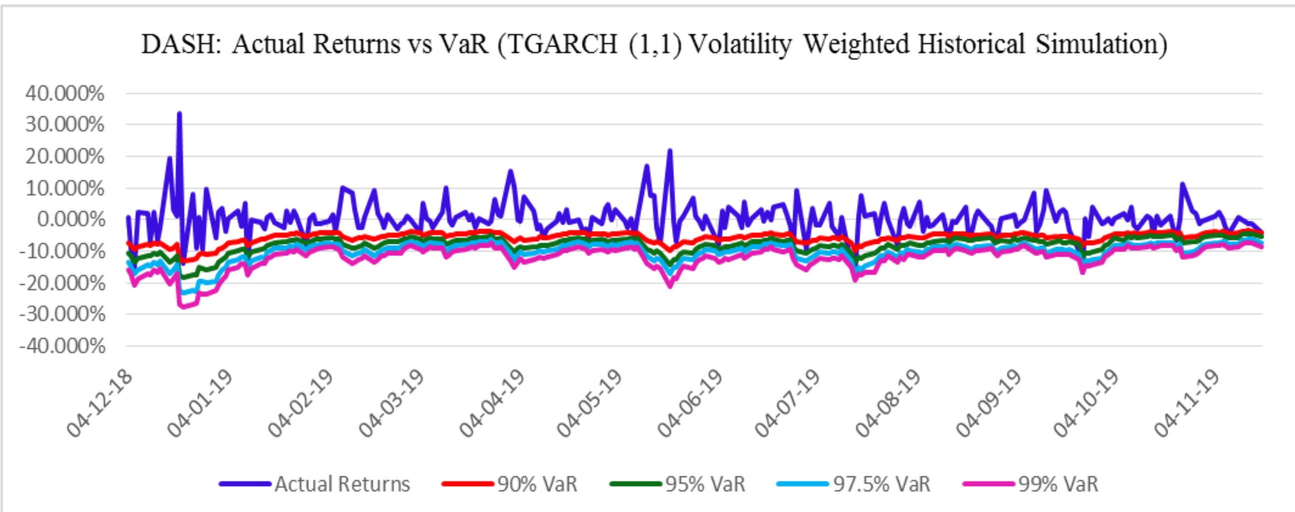
The VaR estimates obtained under each confidence level are then compared to the actual returns for every cryptocurrency and fiat currency. Days were the actual return exceeds VaR are recorded as an exception. Figure 5 shows these results by graphically comparing the VaR estimates of each confidence level with the corresponding actual returns over the entire 250 day sampled period for each of the selected cryptocurrencies and fiat currencies.

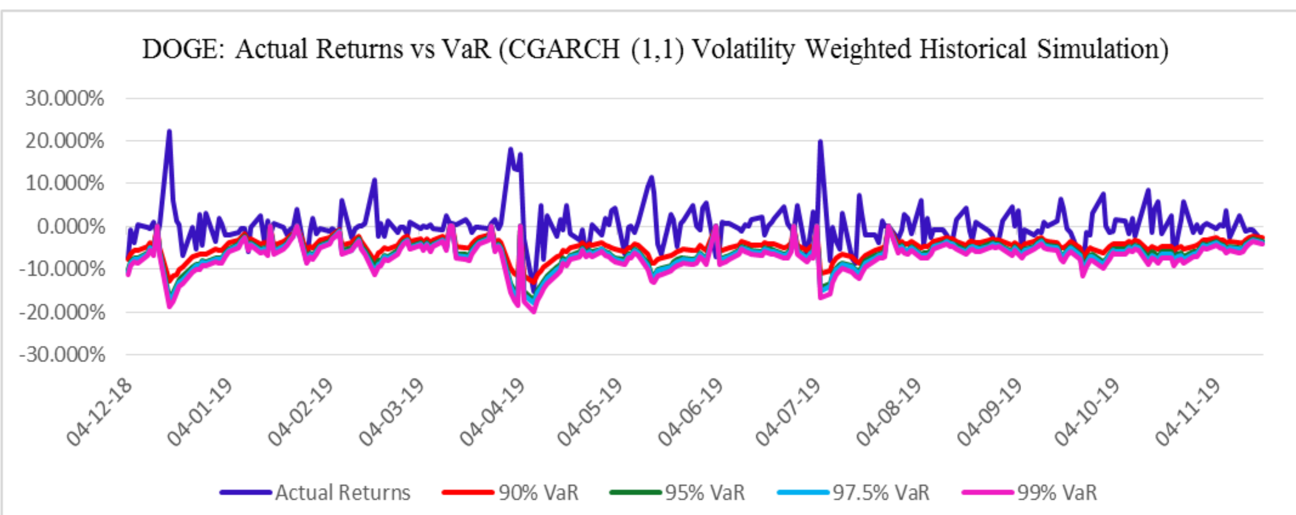

BTC: Actual Returns vs VaR (TGARCH(1,1) Volatility Weighted Historical Simulation)
30.000%
20.000%
10.000%
0.000%
-10.000%
-20.000%
-30.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR

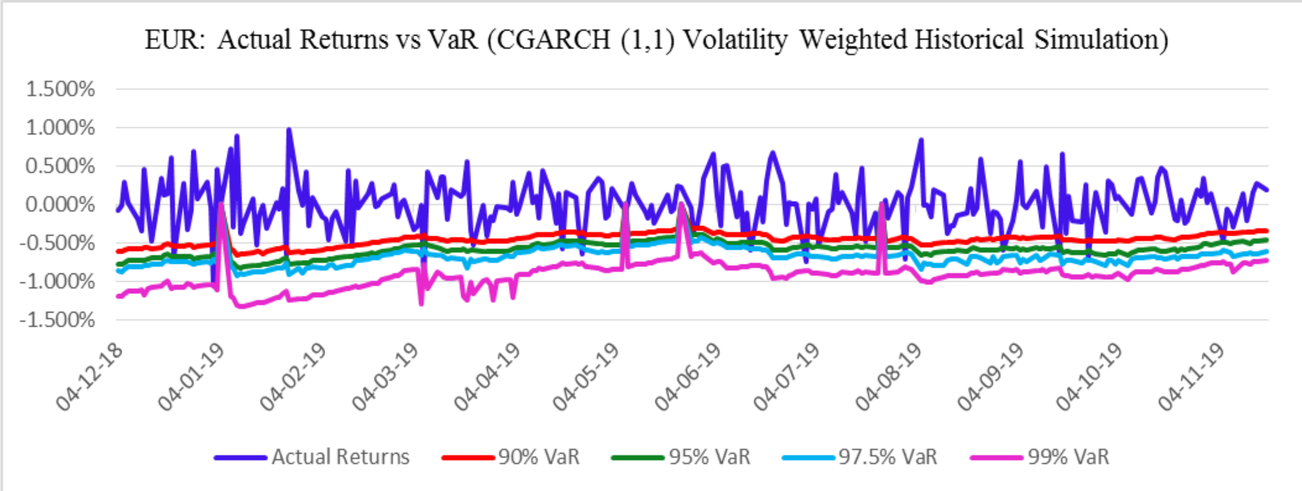

XRP: Actual Returns vs VaR (CGARCH(1,1) Volatility Weighted Historical Simulation)
30.000%
20.000%
10.000%
0.000%
-10.000%
-20.000%
-30.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR

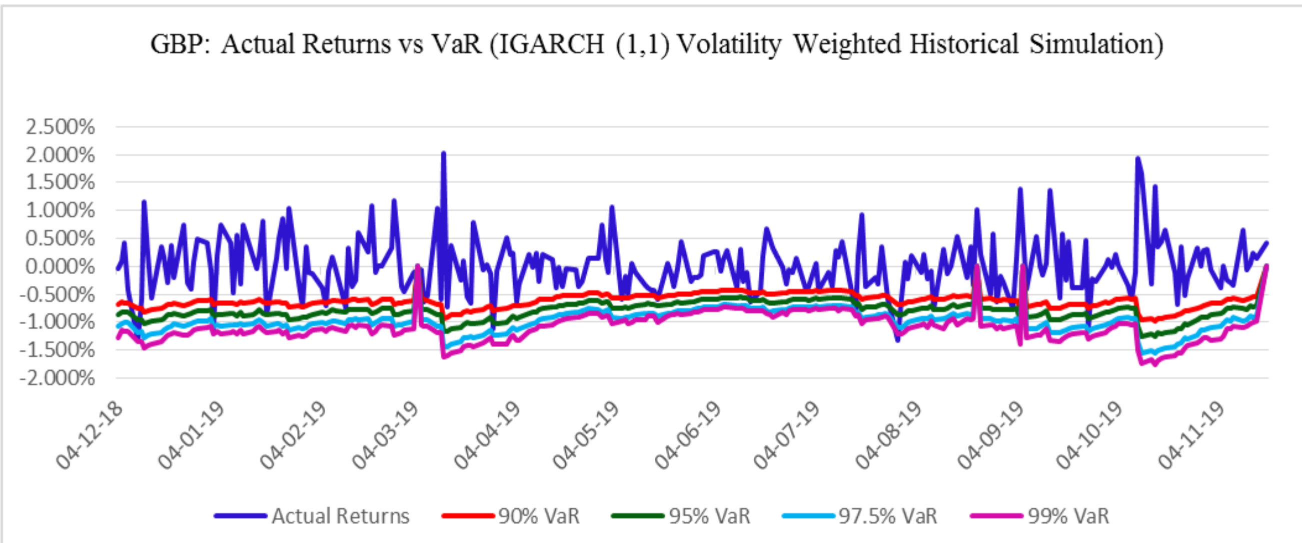

LTC: Actual Returns vs VaR (APARCH(1,1) Volatility Weighted Historical Simulation)
40.000%
30.000%
20.000%
10.000%
0.000%
-10.000%
-20.000%
-30.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR

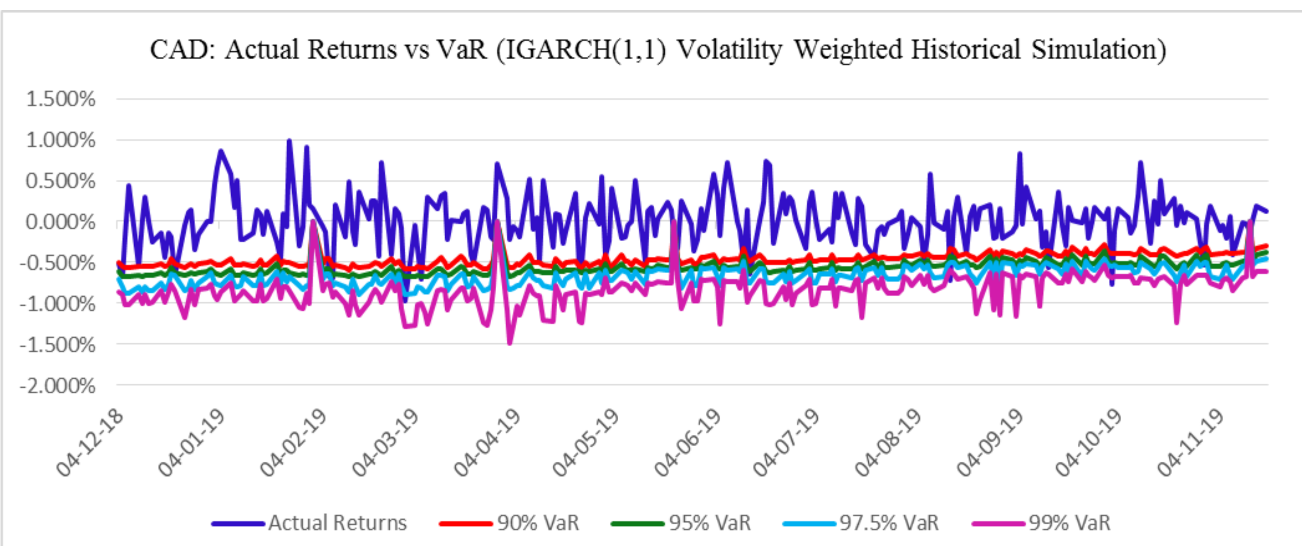

XMR: Actual Returns vs VaR (IGARCH (1,1) Volatility Weighted Historical Simulation)
30.000%
20.000%
10.000%
0.000%
-10.000%
-20.000%
-30.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR

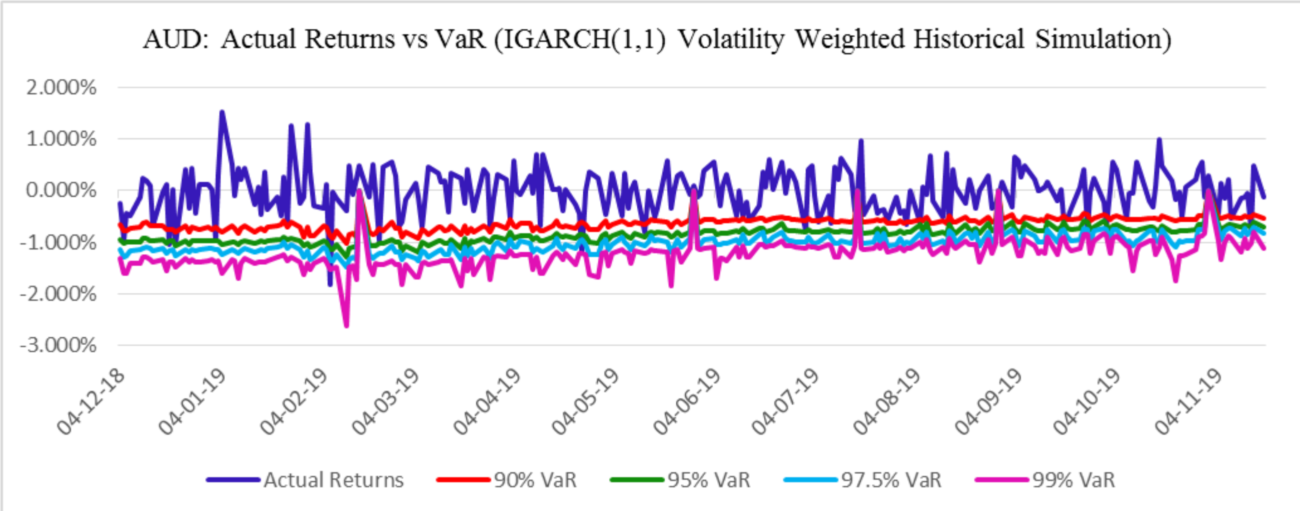

DASH: Actual Returns vs VaR (TGARCH (1,1) Volatility Weighted Historical Simulation)
40.000%
30.000%
20.000%
10.000%
0.000%
-10.000%
-20.000%
-30.000%
-40.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR

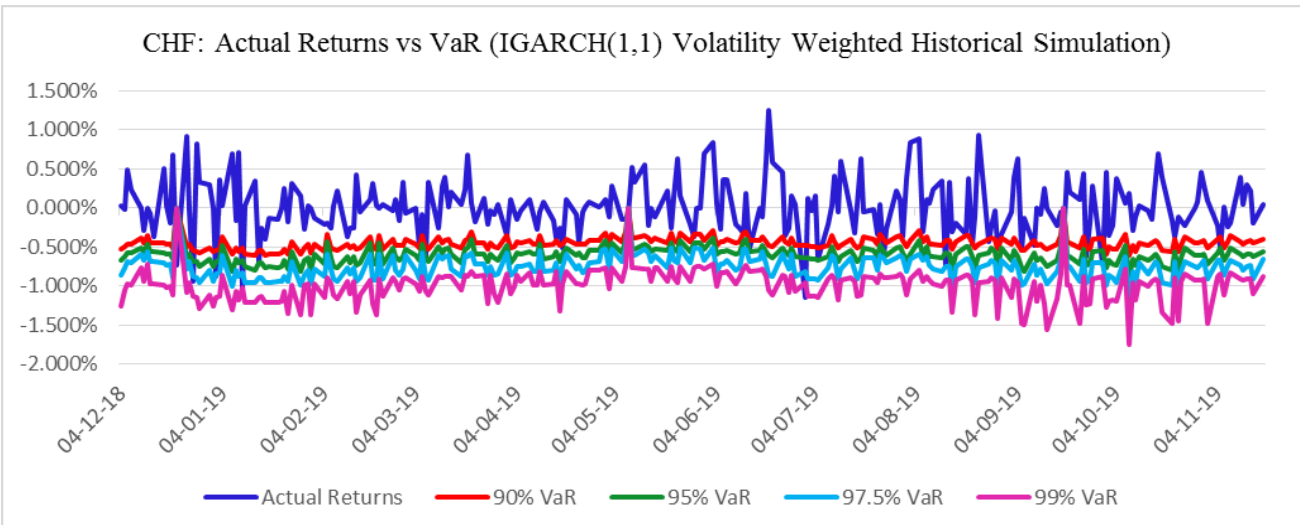

DOGE: Actual Returns vs VaR (CGARCH (1,1) Volatility Weighted Historical Simulation)
30.000%
20.000%
10.000%
0.000%
-10.000%
-20.000%
-30.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR


EUR: Actual Returns vs VaR (CGARCH (1,1) Volatility Weighted Historical Simulation)
1.500%
1.000%
0.500%
0.000%
-0.500%
-1.000%
-1.500%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR


GBP: Actual Returns vs VaR (IGARCH (1,1) Volatility Weighted Historical Simulation)
2.500%
2.000%
1.500%
1.000%
0.500%
0.000%
-0.500%
-1.000%
-1.500%
-2.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR


CAD: Actual Returns vs VaR (IGARCH(1,1) Volatility Weighted Historical Simulation)
1.500%
1.000%
0.500%
0.000%
-0.500%
-1.000%
-1.500%
-2.000%
04-12-18
04-01-19
04-02-19
04-03-19
04-04-19
04-05-19
04-06-19
04-07-19
04-08-19
04-09-19
04-10-19
04-11-19
Actual Returns
90% VaR
95% VaR
97.5% VaR
99% VaR





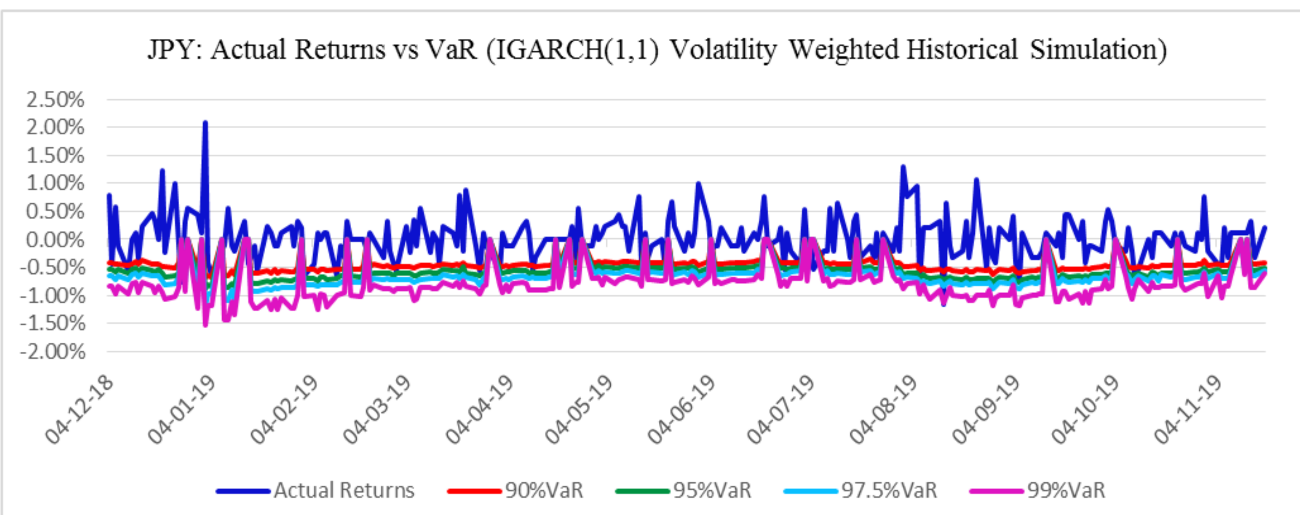


Figure 5: Value at Risk vs Actual Returns for all Cryptocurrencies and Fiat Currencies over the entire 250 Days Sampled Period between December 4th 2018 & November 18th 2019.

4.4.4. Kupiec Test Results

As defined in section 3.5, the above comparison is performed in order to assess the accuracy of the underlying VaR model in forecasting returns. This is addressed by exploiting certain back testing procedures, such as the Kupiec Test.

The Kupiec Test summarizes this level of accuracy into a single number by computing the Likelihood Ratio (LR_K) as depicted in equation (28). The parameters required to compute the LR_K are the number of exceptions, probability of failure and number of VaR trials.

The total number of exceptions "X" is computed 4 times for each cryptocurrency and fiat currency (1 for each confidence level). Noting that the failure rate "*p*" for a 90%, 95%, 97.5% and 99% VaR are 10%, 5%, 2.5% and 1% respectively. The total number of trials "*T*", is 250.

Given that the total number of exceptions for Bitcoin at the 95% VaR confidence level is 6, then the respective LR_K can be computed as:

BTC LR_K at 95% CL $= -2\ln\left[(1-0.05)^{(250-6)} \times 0.05^{6}\right] + 2\ln\left\{\left[1-(6/250)\right]^{(250-6)} \times (6/250)^{6}\right\} = 4.369$

Similar calculations were performed for the remaining cryptocurrencies and fiat currencies at each confidence level following the computation of the number of exceptions. The Kupiec Test results for each cryptocurrency and fiat currency are illustrated in Table 17.

		Model Integrated into the Volatility Weighted Historical Simulation Method	VaR CL	Number of Exceptions	Non-Rejection Interval	LR_K	Critical Value	Result
Cryptocurrency	**BTC**	TGARCH (1,1)	90%	15	[17 , 35]	5.113	3.84	Reject
			95%	6	[7 , 20]	4.369	3.84	Reject
			97.5%	2	[2 , 11]	4.016	3.84	Reject
			99%	1	[0 , 5]	1.176	3.84	Accept
	XRP	CGARCH (1,1)	90%	12	[17 , 35]	9.122	3.84	Reject
			95%	4	[7 , 20]	8.185	3.84	Reject
			97.5%	2	[2 , 11]	4.016	3.84	Reject
			99%	1	[0 , 5]	1.176	3.84	Accept
	LTC	APARCH (1,1)	90%	16	[17 , 35]	4.074	3.84	Reject
			95%	9	[7 , 20]	1.138	3.84	Accept
			97.5%	4	[2 , 11]	0.950	3.84	Accept
			99%	2	[0 , 5]	0.108	3.84	Accept
	XMR	IGARCH (1,1)	90%	18	[17 , 35]	2.389	3.84	Accept
			95%	5	[7 , 20]	6.071	3.84	Reject
			97.5%	1	[2 , 11]	6.947	3.84	Reject
			99%	0	[0 , 5]	-	3.84	Accept
	DASH	TGARCH (1,1)	90%	20	[17 , 35]	1.185	3.84	Accept
			95%	9	[7 , 20]	1.138	3.84	Accept
			97.5%	4	[2 , 11]	0.950	3.84	Accept
			99%	0	[0 , 5]	-	3.84	Accept
	DOGE	CGARCH (1,1)	90%	16	[17 , 35]	3.245	3.84	Accept
			95%	6	[7 , 20]	3.787	3.84	Accept
			97.5%	3	[2 , 11]	1.854	3.84	Accept
			99%	1	[0 , 5]	1.046	3.84	Accept
Fiat Currency	**EUR**	CGARCH(1,1)	90%	18	[17 , 35]	2.149	3.84	Accept
			95%	11	[7 , 20]	0.150	3.84	Accept
			97.5%	8	[2 , 11]	0.522	3.84	Accept
			99%	2	[0 , 5]	0.093	3.84	Accept
	GBP	IGARCH (1,1)	90%	20	[17 , 35]	1.014	3.84	Accept
			95%	8	[7 , 20]	1.796	3.84	Accept
			97.5%	4	[2 , 11]	0.878	3.84	Accept
			99%	1	[0 , 5]	1.128	3.84	Accept

CAD	IGARCH (1,1)	90%	20	[17 , 35]	1.014	3.84	Accept
		95%	12	[7 , 20]	0.008	3.84	Accept
		97.5%	6	[2 , 11]	0.004	3.84	Accept
		99%	2	[0 , 5]	0.093	3.84	Accept
AUD	IGARCH (1,1)	90%	19	[17 , 35]	1.475	3.84	Accept
		95%	7	[7 , 20]	2.783	3.84	Accept
		97.5%	3	[2 , 11]	2.008	3.84	Accept
		99%	1	[0 , 5]	1.116	3.84	Accept
CHF	IGARCH (1,1)	90%	22	[17 , 35]	0.339	3.84	Accept
		95%	8	[7 , 20]	1.833	3.84	Accept
		97.5%	6	[2 , 11]	0.005	3.84	Accept
		99%	1	[0 , 5]	1.140	3.84	Accept
JPY	IGARCH (1,1)	90%	18	[17 , 35]	0.938	3.84	Accept
		95%	6	[7 , 20]	2.940	3.84	Accept
		97.5%	1	[2 , 11]	5.767	3.84	Reject
		99%	1	[0 , 5]	0.852	3.84	Accept

Table 17: The Kupiec Test Results of all Cryptocurrencies and Fiat Currencies.

Remarkably, the Kupiec Test results show that the VaR provides a very accurate measure for the level of downside risk imperiling fiat currencies, where the LR_K values were below the critical value of "3.84" at all confidence levels for each of the Euro, British Pound, Canadian Dollar, Australian Dollar, Swiss Franc, and the Japanese Yen, given that the VaR model was only rejected at the 97.5% confidence level for the latter. Dash and Dogecoin provided similar results to fiat currencies where the VaR results were accepted at all confidence levels.

As for the remaining cryptocurrencies, the results were disparate. The VaR results displayed increased accuracy with an increase in confidence level in the case of Litecoin, where the model was accepted at the 95%, 97.5% and 99% confidence levels and was rejected at the 90% significance level. Perhaps the most peculiar results were that of Monero, where the VaR model was accepted at 90% and 99% confidence levels and rejected at the 95% and 97.5% confidence levels. Nevertheless, it is evident that the VaR provides a poor measure for Bitcoin and Ripple whereby the model was rejected at all confidence levels, noting that it was accepted only at the 99% confidence level which implies that precision was attained only at the highest degree of certainty. Notice that in all rejected situations, the number of exceptions roughly fall behind expectations, signifying that the model overstates the risk in cryptocurrencies due to their distinctively highly volatile feature.

Therefore, it can be deduced that the Value at Risk provides a viable measure of the risk exposure in fiat currencies and some cryptocurrencies, such as Dash and Dogecoin. However, this metric fails in accurately quantifying the level of downside risk in major cryptocurrencies such as the Bitcoin and Ripple. This is mainly due their precarious behavior, which requires further refined and more sophisticated tools such as the Extreme Value Theory. First pioneered by Leonard Tippett, the Extreme Value Theory (EVT) aims to remedy a deficiency in Value at Risk, as it attempts to estimate the probability of extreme values by assuming a separate distribution for extreme losses, as observed in cryptocurrencies.

Chapter 5: Conclusion

5.1. Summary & Thorough Review

This thesis assessed and compared the predictive ability of the GARCH (1,1), IGARCH (1,1), EGARCH (1,1), GJR-GARCH (1,1), APARCH (1,1), TGARCH (1,1) and CGARCH (1,1) in modeling the volatilities of six cryptocurrencies (Bitcoin, Ripple, Litecoin, Monero, Dash and Dogecoin) and six fiat currencies (Euro, British Pound, Canadian Dollar, Australian Dollar, Swiss Franc and the Japanese Yen) over a sampled period extending from October 13th 2015 till November 18th 2019. The sampled period was divided into two sub-sample periods: the in-sample period extending from October 13th 2015 till December 3rd 2018, and the out-of-sample period covering the period from December 4th 2018 till November 18th 2019. The models' parameters were estimated for the in-sample period by maximizing the log likelihood function subject to the assumptions and constraints of each model. Accordingly, the parameters computed during the in-same period were used to forecast volatility for both the in-sample and out-of-sample periods. Then, predicted volatilities under each model were then compared to the realized volatilities using three error metrics (MAE, RMSE and MAPE) to determine the optimal model for each cryptocurrency and fiat currency for both the in-sample and out-of-sample periods.

The results showed consistency among fiat currencies, whereby the Integrated GARCH has proven to be the best performer for most of the fiat currencies, particularly the British Pound, Australian Dollar, Swiss Franc and the Japanese Yen. The IGARCH model was also found to be the most accurate model for the Canadian Dollar, but only for the out-of-sample period given that the Threshold GARCH performed better during the in-sample period. However and quite surprisingly, the Component GARCH modeled the Euro almost "impeccably" during both periods. Therefore, the IGARCH has proven to be the prevailing model when modeling foreign exchange markets.

Exceptionally and among all cryptocurrencies, the Integrated GARCH was also the best performing model for Monero, for both sampled periods. As for the remaining cryptocurrencies, the GJR-GARCH model proved to be superior during the in-sample period while the CGARCH and TGARCH models proved to be best performers during the out-of-sample period. Specifically, for the in-sample period, the GJR-GARCH model is selected for Bitcoin, Litecoin and Dash, APARCH is selected for Ripple, and SGARCH is selected for Dogecoin. For the out-of-sample period, TGARCH performed best for Bitcoin and Dash while CGARCH is selected for Ripple and Dogecoin and APARCH is selected for Litecoin. This validates the assumption that advanced GARCH models better model asymmetries in cryptocurrencies' volatility.

Using the Rolling Window procedure and by incorporating the out-of-sample optimal models into the Volatility-Weighted Historical Simulation method, the Value at Risk was calculated for 250 days between December 4th 2018 and November 18th 2019 at four confidence levels (90%, 95%, 97.5% and 99% confidence levels) for each cryptocurrency and fiat currency. The Value at Risk results were then compared to Actual Returns to determine the number of days or exceptions in which Actual Returns exceed VaR estimates over the 250 days period. Finally, the Kupiec Test was performed using the number of exceptions generated in order to assess the accuracy of the underlying VaR model in forecasting the returns of each cryptocurrency and fiat currency under each confidence level. This was addressed by measuring and comparing Kupiec's Likelihood Ratio (LR_K) to the critical value of "3.84".

The Kupiec Test results showed that the VaR provides a very accurate measure in determining the level of downside risk exposing fiat currencies, where the LR_K values were below the critical value of "3.84" at all confidence levels for each of the Euro, British Pound, Canadian Dollar, Australian Dollar, Swiss Franc, and the Japanese Yen, given that the VaR model was only rejected at the 97.5% confidence level for the latter.

Dash and Dogecoin provided similar results to fiat currencies where the VaR results were accepted at all confidence levels. As for the remaining cryptocurrencies, the outcomes were different. The VaR results displayed increased accuracy with an increase in confidence level in the case of Litecoin, where the model was accepted at the 95%, 97.5% and 99% confidence levels and was rejected at the 90% significance level. As for Monero, the VaR model was accepted at the 90% and 99% confidence levels and rejected at the 95% and 97.5% confidence levels. Nevertheless and quite evidently, VaR provided unconvincing results for major cryptocurrencies, such as Bitcoin and Ripple, whereby the model was rejected at all confidence levels and was accepted only at the 99% significance level.

5.2. Main Findings & Exclusivity of This Research

The findings of this thesis are novel to those of preceding research, as this research is the first and latest to inspect the volatility and the Value at Risk of six major cryptocurrencies along with that of the top six fiat currencies, all together, particularly with the use of several GARCH Models and the Volatility Updating Historical Simulation Method.

Regarding the predictive capacity of the selected models, this research has evidenced the superiority of the IGARCH model in forecasting the volatility of world currencies, namely the British Pound, Canadian Dollar, Australian Dollar, Swiss Franc and the Japanese Yen. The preeminence of the IGARCH model was present in both in-sample and out-of-sample contexts. Up to our knowledge, Holtappels (2018) is the only author to inspect the behavior of the six world currencies considered in this thesis. However, the findings of this thesis contradicts those of Holtappels, who found that the sum of ARCH and GARCH parameters for fiat currencies is less than 1, suggesting there is mean reversion in their variances. Nevertheless, our findings validate his assumption concerning cryptocurrencies, implying that they exhibit an unstable and explosive variance forecast. Indeed, this research has revealed that the volatilities of cryptocurrencies are better vindicated by advanced models such as the CGARCH, GJR-GARCH, APARCH, and TGARCH.

Unfortunately, however, the majority of recent studies have focused entirely on the Bitcoin's behavior or a few other cryptocurrencies and specifically on the in-sample modelling framework and little work has been devoted to the entire cryptocurrency category and out-of-sample context. Nevertheless, our thesis conforms to the findings of Gyamerah (2019) who concluded that the TGARCH model is the best model to forecast time-varying volatility in Bitcoin for the period extending from January 1st 2014 till August 16th 2019. This contradicts the results of Naimy & Hayek (2018) who found that the EGARCH model outperformed the EWMA and GARCH (1,1) models in modeling Bitcoin's volatility. Katsiampa (2017) found that the best conditional heteroskedasticity model for Bitcoin is the AR-CGARCH, highlighting the importance of including both a transitory and permanent component in the conditional variance equation.

To date, the only two papers that have investigated the behavior of the cryptocurrencies examined in our thesis along with their Value at Risk are that of Chu et al. (2017) and Omari et al. (2019).

However, among the twelve GARCH models fitted in the paper of Chu et al. (2017), the IGARCH model gave the best fit for Bitcoin, Dash, Litecoin and Monero, whereas the GJR-GARCH and SGARCH gave the best fit for Dogecoin and Ripple respectively. Nevertheless, the application of this thesis can be perhaps mostly linked with that of Omari et al. (2019), who employed twelve GARCH specifications and nine distributions to eight of the most popular cryptocurrencies. Our findings were quite analogous in that empirical results proved that asymmetric GARCH models with long memory property and skewed and heavy tailed innovations distributions demonstrated better overall performance and that the optimal in-sample GARCH-type models varied from the out-of-sample VaR forecasts models, for all cryptocurrencies. Similar to Chu et al. (2017), the IGARCH was selected for Bitcoin, Dash and Monero while the FIGARCH and TGARCH were selected for Ripple and Litecoin respectively for the in-sample period. Nonetheless, for the out-of-sample period, advanced models displayed supremacy. Regarding the accuracy tests, both papers derived that the best fitting models can be used to provide acceptable estimates for the Value at Risk in cryptocurrencies.

Remarkably, however, it is important to highlight that the EGARCH model which was considered superior in many papers such as Krogt (2018), Abdalla (2012) and Naimy & Hayek (2018) was one of the worst performing models among all fiat and virtual currencies in our research.

Therefore, it is natural to observe some discrepancies especially in the cryptocurrency markets as they are more susceptible to uncertainties and unexpected changes in market sentiment which may eventually alter their volatility structure, given their regulatory concerns and virtual feature which make it continually exposed to internal and external forces. Hence, such contradictions may arise as a result of eternal evolvements in cryptocurrency markets. Another justification is the number of cryptocurrencies involved and models employed in the research. As stated earlier and from the review of available literature, little or no effort has been devoted for the entire cryptocurrency market as most research focused solely on Bitcoin, and besides which most of those papers integrated no more than three models. An alternative explanation might be associated with the time frame involved, as most research were conducted no less than a year ago and from which different periods were selected. The paper of Naimy & Hayek (2018) examined the behavior of Bitcoin fom April 2013 up to March 2016. Chu et al. (2017) considered the period between June 2014 and May

2017 while the period extending from August 2015 through August 2018 was selected by Omari (2019). Essentially, this thesis however, has investigated the behavior of the twelve cryptocurrencies and world currencies over the period extending from October 2015 till November 2019, which makes it novel.

Moreover, this thesis revealed that cryptocurrencies generally exhibit a positive leverage effect with positive returns having higher impacts on cryptocurrencies' volatility than negative returns. Using asymmetric GARCH models, our findings conform to those of Naimy & Hayek (2018), Bouri et al. (2017), Baur et al. (2018) and Stavroyiannis (2018) who investigated the response of the conditional variance to past positive and negative shocks and unveiled that an inverted leverage effect exists. The findings of this research also verify those of Kwek & Koay (2006) who showed weak evidence of asymmetries in most currencies' volatility and underlined that strong time varying symmetric effects are apparent in all the series examined, especially in the Australian dollar. Nonetheless, our findings contradict that of Bouoiyour & Selmi (2016) who suggested that Bitcoin prices were driven more by negative shocks than positive shocks.

In addition, this thesis has proven that the Value at Risk provides a viable measure of the risk exposure in fiat currencies and some cryptocurrencies, such as Dash and Dogecoin. However, this metric fails in accurately quantifying the level of downside risk in major cryptocurrencies such as the Bitcoin and Ripple. The high volatile aspect in cryptocurrencies has turned them into a genuinely risky investment and consequently, appropriate risk measures have been deemed increasingly necessary. For this reason, measuring the Value at Risk of cryptocurrencies rather than fiat currencies has gained more enthusiasm, as noticed in the available literature. Our results conform to those of Stavroyiannis (2018), who implemented a large variety of Value-at-Risk measures and back testing criteria. He emphasized that Bitcoin violates VaR and other risk measures. In addition, results from our thesis and those of Omari et al. (2019) and Caporale & Zekokh (2019) demonstrate that the asymmetric GARCH models particularly have better VaR forecasting performance than standard GARCH models for all cryptocurrencies with an increased accuracy at the 99% confidence level. In addition, the presence of outliers in cryptocurrency returns has been observed by many researchers such as Troster et al. (2019); Chaim and Laurini (2019); Charles and Darne (2019), among others. This is mainly due to their precarious behavior. Hence, it has become more evident that cryptocurrencies require further refined and more sophisticated tools such as Extreme Value Theory in order to unravel deficiencies in VaR.

In light of the above, and given the highly volatile distinctive feature of cryptocurrencies, coupled with their significance in the financial field and on the financial system in particular, the need to forecast their volatility has recently become more and more imperative. Therefore, the importance of a comprehensive study encircling the behavior of cryptocurrencies with respect to world currencies has become crucial which may unveil unknown characteristics, amend on or improve existing findings. From here, this thesis contributes to the existing literature by filling the gap in the current research around modeling the behavior of cryptocurrencies with respect to world currencies. It also aims to eliminate any controversies and uncertainties that remain regarding the classification of cryptocurrencies, and whether they are viable alternatives to fiat currencies. Baek and Elbeck (2015) emphasized that the Bitcoin is extremely volatile and speculative. In their research, Uyar and Kahraman (2019) found Bitcoin to be significantly risky with respect to the major currencies; and it is six times riskier than the singular most risky currency. Similarly, Naimy & Hayek (2018) implied that Bitcoin is more susceptible to speculative bubbles and displays higher volatility than traditional currencies. In fact, it had lately lost more than fifty percent of its value, plunging by more than five thousand dollars over the course of one month, between February and March 2020. Therefore, given its virtual nature, unregulatory concerns and uncertainties, it cannot be considered as a currency. Krylov et al. (2018) also revealed that the volatility of Bitcoin is significantly higher than fiat money, whereby the recognition of Bitcoin and cryptocurrencies, in general, as real money is premature mainly due to its violation of the essential requirements for the properties of a currency, which is low level of volatility. This thesis validates the conclusions drawn from earlier studies. Given the relative stability of world currencies, coupled with their low volatility, symmetric behavior to shocks, and their typical response to standard risk measures, all cryptocurrencies and particularly Bitcoin, cannot be considered as viable alternatives to fiat and world currencies as they violate the most crucial element of a standard currency: confidence.

5.3. Limitations of This Research

Although, the original GARCH model assumed the Normal (Gaussian) distribution, this distribution cannot accommodate to the fat-tail disturbance occurring in financial time series. Despite the proven significance of the Student's t and General Error Distributions that have been introduced in this thesis, there are several other distributions that could have been considered: Skew-Normal, Skew-t, Skew GED, Normal Inverse Gaussian Distribution, Generalized Hyperbolic Distribution, and Johnson's SU distribution. For instance, the Johnson's SU distribution has proven to be the best fitted model for four of the eight cryptocurrencies considered in the paper of Omari et al. (2019), namely Litecoin, Dash, Stellar and NEM.

Furthermore, even though the selected models: SGARCH, IGARCH, EGARCH, GJR-GARCH, TGARCH, APARCH and CGARCH models have proven their superiority in predicting the volatility of not only fiat currencies and cryptocurrencies but most securities (stocks, commodities, etc.), this thesis could have integrated further models such as MGARCH as in Holtappels (2018) and NGARCH, NAGARCH, FIGARCH, and ALLGARCH in Omari (2019) and Chu et al. (2017), etc.

Moreover, while the expression "Value at Risk" is widely used, the expression does not refer to one particular methodology or approach for quantifying risk. Although this thesis employed the best possible method, the Volatility Updating Historical Simulation Method, there are other few methods that could have been utilized to measure VaR such as the Basic Historical Simulation Method, Parametric Variance-Covariance Approach and MonteCarlo Simulaton. For example, Al Janabi (2006) provided an excellent primer on the Variance-Covariance Method to measuring Value at Risk while the article of Glasserman, Heidleberger, and Shahabuddin (2002) discusses the use of MonteCarlo Simulation to estimate Value at Risk. Even though those methods were not directly employed in this thesis, the MonteCarlo Simulation was integrated in our Volatility Updating Historical Simulation Method. Nevertheless, the Variance-Covariance Approach was not utilized as it supposes the existence of a normal distribution, which is impractical.

In addition, another limitation in this thesis is that the VaR failed in accurately quantifying the level of downside risk in highly volatile markets such as in cryptocurrencies, particularly Bitcoin and Ripple which are the leading cryptocurrencies today. For this reason, The Extreme Value Theory (EVT) could have been integrated into our research to remedy deficiencies in VaR, as it attempts to estimate the probability of extreme values by assuming a separate distribution for extreme losses, as observed in cryptocurrencies. For instance, the paper of Osterrieder & Lorenz (2017) employed extreme value analysis on Bitcoin and revealed that Bitcoin is more volatile and much riskier than traditional currencies. Then, Osterrieder et al. (2017) further extended on his research to include five additional cryptocurrencies. He deduced that cryptocurrencies are extremely volatile, noting that Bitcoin is the least volatile among the cryptocurrencies considered. Gkillas and Katsiampa (2018) then discovered that Bitcoin Cash is the riskiest cryptocurrency, with results consistent with that of Osterrieder et al. (2017) regarding Bitcoin. Zhang et al. (2019) later on went to employ extreme value analysis on hourly log returns of four cryptocurrencies. The analysis was conducted on high-frequency data, altering the threshold and estimating the Value at Risk and Expected Shortfall for each cryptocurrency. Their findings revealed that Ripple is the riskiest at every percentile and threshold and Bitcoin to be the least volatile.

As stated previously, the cryptocurrency market is relatively new compared to that of fiat currencies. For this reason, many major cryptocurrencies were excluded, particularly Ethereum, either due to the scarcity of data or because of their non-volatile feature by the time this thesis was prepared. Nevertheless, due to their high volatility, the market prices of the selected cryptocurrencies have since changed, thereby as has their share portion from the entire cryptocurrency market. As a result, there are few uncertainties whether the findings of this thesis and the behavior of the selected cryptocurrencies could be theorized on the entire cryptocurrency market.

5.4. Implications & Recommendations

As such, volatility is a key element around which financial markets revolve. Its preeminence and essence in different areas of risk management, trading, security pricing, asset allocation, portfolio optimization and monetary policy has enticed continuous interest from scholars, investors, governments, and regulators. From this context, modeling and predicting the volatility of financial markets has been, for years, the core of extensive empirical and the theoretical investigation of academics, authorities and practitioners.

Based on the results of this thesis and the assumptions drawn from our findings, multiple recommendations can be inferred for various parties:

For governmental institutions and regulators, it is recommended from authorities to examine the risk enfolding cryptocurrencies and all relevant research on this subject. Academic papers can substantiate political and monetary decisions by providing relevant information surrounding the behavior of cryptocurrencies. This thesis unfolded the risks conveyed from the cryptocurrency market. For instance, it provided further wisdom concerning the reaction of returns in cryptocurrencies compared to world currencies. Based on those results, governments and regulatory authorities could strengthen regulations and arouse further awareness by enforcing policies and restraining investors from devoting too much investment in cryptocurrencies.

Accordingly, financial managers and investors need to be aware before considering an investment in cryptocurrencies, given their extremely volatile behavior. The results from this thesis have shown that the most stable cryptocurrency is ten times more volatile than the most unstable currency. Consequently, stakeholders are recommended to be attentive for outbursts in volatile periods as this thesis has evidenced that these periods can be quite persistent. Thus, investors and senior managers are advised to limit their positions in cryptocurrencies, specifically during strained conditions.

For academicians, this thesis provides a thorough overview and further clarification surrounding the behavior of cryptocurrencies with respect to world currencies, the relative performance of diverse GARCH models, and reliability concerns of the Value at Risk measure. This thesis can be considered the groundwork and motive for further examining and modeling the volatility of cryptocurrencies or employing alternative models to the Value at Risk.

Nevertheless, few attempts have been made so far to examine the extreme value behavior of cryptocurrencies, as the majority of the present research focuses on the extreme value behavior of daily data on Bitcoin. For this reason, the most relevant extension of this research would be to further exploit the Extreme Value Theory on many cryptocurrencies in order to investigate their tail behavior and whether it could accommodate for extreme outliers, which might give further insight surrounding the risk exposure in cryptocurrencies.

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APPENDIX

Ripple									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-6.060%	-5.583%	-7.413%	-6.052%	**…**	-5.288%	-4.223%	-4.050%	-3.728%
Exceptions	-	-	1	-	11	-	-	-	-
VaR at 95% CL	-7.082%	-6.474%	-8.596%	-7.018%	**…**	-6.424%	-5.071%	-4.907%	-4.516%
Exceptions	-	-	-	-	4	-	-	-	-
VaR at 97.5% CL	-7.418%	-6.781%	-9.003%	-7.351%	**…**	-6.952%	-5.530%	-5.325%	-4.902%
Exceptions	-	-	-	-	2	-	-	-	-
VaR at 99% CL	-7.814%	-7.144%	-9.484%	-7.744%	**…**	-7.453%	-5.929%	-5.686%	-5.234%
Exceptions	-	-	-	-	1	-	-	-	-
Actual Returns	0.997%	-3.972%	-8.073%	-2.875%	**…**	0.227%	-0.969%	-3.231%	-2.527%

Litecoin									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-6.728%	-6.630%	-6.983%	-6.975%	**…**	-4.693%	-4.879%	-4.759%	-4.700%
Exceptions	-	-	1	-	15	-	-	-	-
VaR at 95% CL	-8.838%	-8.690%	-8.990%	-8.979%	**…**	-6.747%	-6.713%	-6.681%	-6.591%
Exceptions	-	-	-	-	9	-	-	-	-
VaR at 97.5% CL	-11.397%	-11.419%	-11.814%	-11.799%	**…**	-8.887%	-8.943%	-8.838%	-8.762%
Exceptions	-	-	-	-	4	-	-	-	-
VaR at 99% CL	-12.778%	-12.801%	-13.244%	-13.228%	**…**	-9.604%	-9.555%	-9.510%	-9.492%
Exceptions	-	-	-	-	2	-	-	-	-
Actual Returns	0.365%	-6.063%	-8.027%	-6.202%	**…**	-0.373%	-2.751%	-2.723%	-1.489%

Monero									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-9.062%	-9.323%	-9.874%	-9.268%	**…**	-5.395%	-5.067%	-5.337%	-5.417%
Exceptions	-	-	1	-	16	-	-	1	-
VaR at 95% CL	-11.697%	-11.991%	-12.359%	-11.601%	**…**	-6.541%	-6.107%	-6.432%	-6.534%
Exceptions	-	-	-	-	5	-	-	-	-
VaR at 97.5% CL	-14.772%	-15.218%	-15.684%	-14.722%	**…**	-8.560%	-8.350%	-8.795%	-9.000%
Exceptions	-	-	-	-	1	-	-	-	-
VaR at 99% CL	-16.471%	-16.886%	-17.403%	-16.336%	**…**	-9.598%	-9.357%	-9.856%	-10.596%
Exceptions	-	-	-	-	0	-	-	-	-
Actual Returns	3.906%	-9.323%	-9.933%	-0.156%	**…**	4.935%	0.318%	-5.462%	-4.227%

Dash									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-7.339%	-8.046%	-9.582%	-8.761%	**…**	-3.627%	-3.528%	-3.571%	-3.987%
Exceptions	-	1	1	-	18	-	-	-	-
VaR at 95% CL	-10.545%	-11.349%	-13.631%	-12.463%	**…**	-4.602%	-4.487%	-4.771%	-5.389%
Exceptions	-	-	1	-	8	-	-	-	-
VaR at 97.5% CL	-13.503%	-14.531%	-17.267%	-15.787%	**…**	-6.473%	-6.392%	-6.374%	-7.378%
Exceptions	-	-	-	-	4	-	-	-	-
VaR at 99% CL	-16.098%	-17.325%	-20.586%	-18.822%	**…**	-7.436%	-7.226%	-7.420%	-8.617%
Exceptions	-	-	-	-	0	-	-	-	-
Actual Returns	0.784%	-8.870%	-15.254%	2.435%	**…**	-0.540%	-1.201%	-1.369%	-3.973%

Dogecoin									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-7.733%	-6.404%	-5.561%	-5.615%	…	-3.166%	-2.611%	-2.185%	-2.636%
Exceptions	-	-	-	-	15	-	-	-	1
VaR at 95% CL	-9.987%	-8.235%	-7.357%	-7.505%	…	-4.174%	-3.395%	-2.851%	-3.503%
Exceptions	-	-	-	-	6	-	-	-	-
VaR at 97.5% CL	-10.545%	-8.718%	-7.693%	-7.831%	…	-4.538%	-3.779%	-3.189%	-3.776%
Exceptions	-	-	-	-	3	-	-	-	-
VaR at 99% CL	-11.319%	-9.338%	-8.320%	-8.499%	…	-5.092%	-4.245%	-3.571%	-4.216%
Exceptions	-	-	-	-	1	-	-	-	-
Actual Returns	-7.381%	-0.729%	-3.807%	0.334%	…	-0.991%	-0.667%	-0.858%	-3.425%

Euro									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-0.619%	-0.617%	-0.601%	-0.580%	…	-0.370%	-0.360%	-0.355%	-0.348%
Exceptions	-	-	-	-	18	-	-	-	-
VaR at 95% CL	-0.779%	-0.781%	-0.760%	-0.735%	…	-0.512%	-0.479%	-0.475%	-0.467%
Exceptions	-	-	-	-	11	-	-	-	-
VaR at 97.5% CL	-0.869%	-0.872%	-0.849%	-0.820%	…	-0.629%	-0.640%	-0.643%	-0.616%
Exceptions	-	-	-	-	8	-	-	-	-
VaR at 99% CL	-1.196%	-1.199%	-1.167%	-1.128%	…	-0.773%	-0.752%	-0.746%	-0.735%
Exceptions	-	-	-	-	2	-	-	-	-
Actual Returns	-0.079%	-0.009%	0.282%	0.018%	…	-0.018%	0.145%	0.263%	0.181%

British Pound									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-0.691%	-0.638%	-0.658%	-0.650%	…	-0.563%	-0.554%	-0.549%	-0.578%
Exceptions	-	-	-	-	20	-	-	-	-
VaR at 95% CL	-0.872%	-0.829%	-0.829%	-0.821%	…	-0.701%	-0.720%	-0.705%	-0.732%
Exceptions	-	-	-	-	8	-	-	-	-
VaR at 97.5% CL	-1.074%	-1.024%	-1.009%	-0.998%	…	-0.890%	-0.905%	-0.880%	-0.894%
Exceptions	-	-	-	-	4	-	-	-	-
VaR at 99% CL	-1.277%	-1.148%	-1.175%	-1.162%	…	-1.022%	-0.998%	-0.971%	-0.986%
Exceptions	-	-	-	-	1	-	-	-	-
Actual Returns	-0.031%	0.102%	0.408%	-0.438%	…	0.047%	0.233%	0.155%	0.411%

Canadian Dollar									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-0.502%	-0.548%	-0.560%	-0.561%	…	-0.345%	-0.369%	-0.326%	-0.578%
Exceptions	1	1	-	-	18	-	-	-	-
VaR at 95% CL	-0.614%	-0.668%	-0.672%	-0.673%	…	-0.467%	-0.480%	-0.410%	-0.732%
Exceptions	-	1	-	-	11	-	-	-	-
VaR at 97.5% CL	-0.704%	-0.804%	-0.869%	-0.870%	…	-0.512%	-0.533%	-0.482%	-0.894%
Exceptions	-	-	-	-	6	-	-	-	-
VaR at 99% CL	-0.861%	-0.899%	-1.021%	-1.022%	…	-0.633%	-0.671%	-0.616%	-0.986%
Exceptions	-	-	-	-	2	-	-	-	-
Actual Returns	-0.515%	-0.676%	-0.187%	0.442%	…	-0.132%	0.013%	0.185%	0.132%

Australian Dollar									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-0.651%	-0.784%	-0.782%	-0.731%	…	-0.523%	-0.518%	-0.473%	-0.578%
Exceptions	-	1	-	-	17	-	1	-	-
VaR at 95% CL	-0.952%	-1.012%	-1.009%	-1.003%	…	-0.694%	-0.700%	-0.608%	-0.732%
Exceptions	-	-	-	-	6	-	1	-	-
VaR at 97.5% CL	-1.137%	-1.276%	-1.272%	-1.174%	…	-0.843%	-0.772%	-0.718%	-0.894%
Exceptions	-	-	-	-	3	-	-	-	-
VaR at 99% CL	-1.305%	-1.596%	-1.591%	-1.418%	…	-1.130%	-1.018%	-0.813%	-0.986%
Exceptions	-	-	-	-	1	-	-	-	-
Actual Returns	-0.258%	-0.954%	-0.454%	-0.498%	…	-0.058%	-0.746%	0.486%	-0.132%

Swiss Franc									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-0.530%	-0.500%	-0.458%	-0.458%	…	-0.431%	-0.425%	-0.456%	-0.578%
Exceptions	-	-	-	-	22	-	-	-	-
VaR at 95% CL	-0.672%	-0.627%	-0.572%	-0.572%	…	-0.594%	-0.591%	-0.624%	-0.732%
Exceptions	-	-	-	-	8	-	-	-	-
VaR at 97.5% CL	-0.866%	-0.771%	-0.691%	-0.698%	…	-0.756%	-0.737%	-0.905%	-0.894%
Exceptions	-	-	-	-	6	-	-	-	-
VaR at 99% CL	-1.257%	-1.044%	-0.969%	-0.982%	…	-0.915%	-0.902%	-1.094%	-0.986%
Exceptions	-	-	-	-	1	-	-	-	-
Actual Returns	0.030%	-0.020%	0.489%	0.238%	…	0.298%	0.218%	-0.198%	0.040%

Japanese Yen									
Day	**1**	**2**	**3**	**4**	**5 → 246**	**247**	**248**	**249**	**250**
Date	**4-12-18**	**5-12-18**	**6-12-18**	**7-12-18**	**10-12-18 → 12-11-19**	**13-11-19**	**14-11-19**	**15-11-19**	**18-11-19**
VaR at 90% CL	-0.425%	-0.427%	-0.448%	-0.435%	…	-0.431%	-0.444%	-0.443%	-0.578%
Exceptions	-	1	-	-	17	-	-	-	-
VaR at 95% CL	-0.532%	-0.529%	-0.586%	-0.542%	…	-0.544%	-0.557%	-0.556%	-0.732%
Exceptions	-	-	-	-	6	-	-	-	-
VaR at 97.5% CL	-0.638%	-0.644%	-0.714%	-0.651%	…	-0.629%	-0.646%	-0.646%	-0.894%
Exceptions	-	-	-	-	1	-	-	-	-
VaR at 99% CL	-0.829%	-0.837%	-0.972%	-0.845%	…	-0.850%	-0.866%	-0.865%	-0.986%
Exceptions	-	-	-	-	1	-	-	-	-
Actual Returns	0.795%	-0.451%	0.566%	-0.113%	…	0.109%	0.326%	-0.325%	0.218%

Table 18: Partial Illustration of the Actual Returns, Value at Risk, and Number of Exceptions estimates for the remaining Cryptocurrencies and Fiat Currencies at the Different Levels of Significance between December 4th 2018 and November 18th 2019.