The Weak Revision as a New Change Operator for Belief Bases

By

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ABSTRACT

This thesis starts by exploring background areas of study such as incomplete and uncertain information in relational databases, belief change in artificial intelligence. It also reminds of how some types of incompleteness and uncertainty can be modeled through disjunctive databases. The paper describes the language being used in addition to some update operators. This thesis then brings to light a new theory in belief change called the weak revision. It first shows the need for such a new operator through a real life example. Second it introduces this new operator by defining its general behavior. Third it proves the correctness of the operator. The thesis will also include a comparative study of the weak revision with the AGM (Alchourron, Gardenfors and Makinson) postulates.

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CHAPTER I

Introduction

1.1 Incomplete and Uncertain Information

One of the distinguishing features of human intelligence is the ability to reason with incomplete and uncertain knowledge. In real life, people are daily confronted with making decisions and actions in situations in which only imperfect knowledge of the relevant facts is available. Very often it is impossible or impractical to obtain complete and certain knowledge since it may even not exist.

In such situations, the information may be deficient in two different ways. If we only have a partial knowledge of the world it is called *incomplete information*. On the other hand, if we have information whose certainty is not entirely guaranteed it is called *uncertain information*. Incompleteness and uncertainty may concern the knowledge relating to the occurrence of events in the world and also the relationships, causal or other, among various events.

While human nature possess tremendous capacity for efficient decision making and acting in situations with large amounts of incomplete and uncertain knowledge, manipulating such imperfect knowledge in artificial intelligence and information systems applications have raised theoretical problems of enormous complexity. In the last decades, researchers have attempted to develop models and methods for representing and manipulating incomplete and uncertain knowledge in order to face up to many real-life situations. The past three decades have also witnessed an extraordinary development of database technology for information systems. From the early database management attempts in the 1960s, many efforts have been made into extending this technology for obtaining richer data models with powerful and efficient data manipulation capabilities necessary to meet the requirements of computer applications of increasing complexity.

The development of the *relational model* after the pioneering work of Codd [Cod70] constituted a milestone in this direction. The relational model allowed to separate two aspects that were closely related in the other database models, namely, the modeling of an application, that is, representing the information, and its physical implementation, that is, the way in which the information is stored in the computer. In this way, the main concern of the user was the semantical aspects of the information he wanted to model.

Further this data independence, the introduction of declarative queries in relational databases allowed the user or application program to get rid of the burden of the navigational retrieval of records from the database.

For these reasons, the relational model knew an increasing success and thus, several relational database systems such as ORACLE, DB2, and INGRES were commercially available in the late 1970s and the early 1980s. At the same time, the research efforts allowed to formalize the concepts of the relational model and the relational database theory was developed.

However, the second half of the 1970s and the beginning of the 1980s brought to light the need to extend relational databases in order to treat other kinds of information that were not considered in the original model proposed by Codd. The introduction of *incomplete* and *uncertain information* was one of the extensions proposed by researchers. One of the basic assumptions of the relational model is that the information contained into a database is *complete* and *certain*, i.e., it represents an accurate picture of the corresponding application domain in the real world. However, since in real-life situations the information is far from being complete and certain, it is necessary to be able to manipulate imperfect information in the context of relational databases. Many possible kinds of incomplete and uncertain information may be introduced into a database. A first concept which may be used for modeling incomplete information is that of null values. A *null value* is a place holder for an attribute of a relation whose value can't be represented by an ordinary constant. Although many possible meanings of null values may be considered, the most frequent ones fall into two categories. The *unknown* null value represents that the attribute's value is missing or unknown; an example is "the salary of director John is unknown". The *inapplicable* null value represents that in a tuple an attribute is inapplicable or its value does not exists; an example is "the wife's name of John is inapplicable since John is not married".

Another type of incompleteness that can be introduced into a database is *disjunctive information*. Information is said to be disjunctive if it is of the form "Jean teaches the course of Physics or the course of Algebra" where it is unknown whether the first fact, the second fact, or both are true. *Maybe information* belongs to the category of uncertainty since it represents facts that are possibly true, such as "Jean is possibly registered in the course of Databases". The introduction of disjunctive information is related with the introduction of maybe information.

Another category of uncertainty is *fuzzy information*. This kind of information generalizes maybe information, since it associates to a maybe fact a value belonging to [0,1], representing how much the fact is likely to be true. An example is "the ship The Mirage has a possibility equal to 1 to be in the Mediterranean Sea and has a possibility equal to 0.75 to be in the Atlantic Sea". Finally, uncertainty may be introduced into a database through *probabilistic information* such as "the next year there will be a 1.25% sales increase with probability 0.75 and 2% with probability 0.25". This type of stochastic information arrives very often in real applications of databases.

When facing imperfect knowledge, the first concern is to establish its "meaning". If the knowledge about the world is incomplete or uncertain, several "scenarios" or states with complete and certain information are possible, but it is not known which one represents the real state of the world.

Thus, a database containing incomplete or uncertain information implicitly represents a set of possible states or a set of possible worlds. A *possible world* is a hypothetical state of the real world that may be completely represented by an ordinary database with complete and certain information. Consider for example an incomplete database containing a disjunctive information "John teaches the course of Physics or the course of Calculus".

If the disjunction above is interpreted inclusively, the database represents three possible states: one in which John teaches Physics, one in which he teaches Calculus, and a third one in which he teaches both courses. The disjunctive fact states that one of these three possible worlds represents the actual situation of the world, but it is unknown which one.

For the rest of this thesis, we shall call a database storing disjunctive information a belief base or a knowledge base.

1.2 Belief Change

In the real life people are constantly changing their beliefs. These changes may be major change, in this case people abandon one group of beliefs in favor of another. Or these could be is minor belief change, for example, a person believed that there was fruit in the basket; but upon investigation found there was none. Not only it's no longer believed that there was fruit in the basket, but it is also actively believed that there is no fruit in the basket. These two sorts of belief change are extremely common in people's everyday life. With humans, these changes occur intuitively; when confronted with new information, we are rarely conscious about the procedure fulfilled in our head to change our beliefs.

Thus if we want to create an intelligent machine capable of reasoning like human beings, we must investigate more in belief change. The above-mentioned intelligent machines (or agents) hold a set of beliefs about the world. When these agents learn new information, they must be able to incorporate it in an intelligent manner, i.e. they should be able to figure out what information to add or eliminate from the current belief base in order to keep it consistent and to construct and store all consequences of that new information. Although it may intuitively seem a simple concept to implement, belief base change is neither well understood nor agreed upon. Various AI theories have been proposed and operations have been developed to achieve the above-mentioned goals; yet these theories and operations are still being evaluated and discussed.

The study of this field in AI is important in the evolution of traditional databases, and is the future of any computer system that calls for decision-making.

1.3 Background

Although the problem of belief change is an old and crucial problem in Artificial Intelligence, it has been also largely studied in the field of database systems and philosophy. We will briefly present what was achieved in the first two fields since they are relevant to our thesis.

1.3.1 Database Systems

When the relational model was developed, it was assumed that the information contained in the database is *complete* and *certain*, i.e. it was an accurate picture of the world it's representing. However, information in real-life is far from being complete and certain, this is why in the second half of the 1970s and the beginning of the 1980s researchers brought to light the need to extend the original model of relational databases so that incomplete and uncertain information may be introduced into the database. (Grahne [4]; Reiter [2]).

One form of incomplete information is that of null values; a null value is a placeholder for an attribute whose value is either missing (for e.g. the "department in which Martin works is unknown") or inapplicable ("The cellular number of Martin is inapplicable since Martin does not have a cellular"). Incompleteness can be also the result of the introduction of disjunctive information like "Martin is a student in computer science or in electrical engineering ": Major (Martin, CS) $\vee$ Major (Martin, EE). In this type, it is not known whether the first fact, the second fact or both are true.

Maybe information is referred as uncertain information and represents facts which are possibly true: "Paul possibly teaches the course of database".

Fuzzy information and probabilistic information belong to the category of uncertainty, which is still being researched and is beyond the scope of this thesis.

How this research field in relational databases relates to the problem of belief change in Artificial Intelligence? Well, querying a database containing incomplete and uncertain information has been solved in a way or another, but updating such type of databases is still being discussed due to the complexity encountered.

Reiter [2] formalized relational databases by means of particular first order logic theories. His work makes the problem of update in relational databases the general problem of belief change in Artificial Intelligence.

1.3.2 Artificial Intelligence

In Artificial Intelligence, belief revision concerns the modification of a base of knowledge upon the introduction of new information. This new information may or may not conflict with information already contained in or implied by the knowledge base. If a conflict should arise, it is the job of a belief revision agent to resolve that conflict, leaving a consistent knowledge base [22].

Work in belief revision began in the 1980s with Levesque [3] who defined a change operation for knowledge base called *TELL*. This operation allowed the insertion of new information into the knowledge base only if it does not contradict the latter's content. If a conflict should arise, the new information is to be discarded. Although this is an important function, intelligent agents should be able to discard some included beliefs in order to incorporate the new information. In the early 1980s new change operators have been developed under the general title of Revision.

In 1985, Alchourron, Gardenfors and Makinson proposed *rationality postulates*, these are rules that every adequate change operator should be expected to satisfy. Borgida [5], Weber [8], Hegner [11], Winslett [13], Dalal [14], Forbus [17] and others, defined their own change operators each in his research field. At that time, there was no difference between belief revision and belief update.

Katsuno and Mendelzon [21] argued that the postulates proposed by Alchourron, Gardenfors and Makinson are not adequate for every application. They make a fundamental distinction between two kinds of modification to a knowledge base: Revision and update. They showed that AGM postulates described only revision, and defined new rationality postulates that each update operator must satisfy. Since then, researchers tried to develop update operators that meet those new defined postulates.

However Herzig and Rifi [24] showed that most of the KM postulates are problematic and undesirable; this brought to light previously rejected update operators.

1.3 Outline of the thesis

Based on the update and revision operators discussed in the literature of artificial intelligence, in this thesis we will define a new revision operator for disjunctive databases and prove its correctness. We presume that it is intuitive when it comes to real-life situations.

The remaining sections of the thesis are as follows.

Chapter 2 gives an overview of the basic concept of belief change in Artificial Intelligence. A section is dedicated to introduce revision and update and the rest of the chapter describes the contents of the AGM and KM postulates.

Chapter 3 presents some update operators in one and unique language as defined in the work of Herzig and Rifi.

Chapter 4 introduces a new operator for disjunctive databases called the *weak revision.* After defining the operator we shall prove its correctness. Then we shall proceed by studying this new operator in relation to the AGM (Alcourron, Gardenfors, and Makinson) postulates.

CHAPTER II

Belief Change in Artificial Intelligence

2.1 Questions in Artificial Intelligence

As described, the problem of belief change is dependent upon the answers to three methodological questions:

1. How are Beliefs to be represented?
2. What is the relation between beliefs represented explicitly in the knowledge base and those beliefs which can be derived from them?
3. In the face of a contradiction, which beliefs are to be retained and which are to be discarded?

A belief base is generally represented by a set of formulae in a given language. It is important to check if the result of a change operation depends on the language chosen and formulae adopted.

Researches in this field show that belief representation greatly influences their manipulation. There can be different representations:

- A belief is an explicitly stored formula.
- A belief is either explicitly stored or it's logically equivalent to some explicitly stored sentence.
- A belief is either explicitly stored or is derivable through some logic

We will define a belief base as being the set of explicitly stated beliefs, while a belief set as being the of explicitly beliefs plus implicit beliefs which can be derived from them. It has been shown that there are always an infinite number of beliefs implied by any set of beliefs. Thus, for the sake of practicality, we always end up with a finite base of some sort.

Answering question three constitutes the main point of belief revision: which information is to be discarded in order to maintain consistency. To illustrate the problem, consider the following example, where we have to change a knowledge base upon the introduction of

new information P1. If P1 contradicts with a belief P2, it is necessary then to remove P2. If other beliefs are based on P2, these beliefs may then have no justification and could be candidates for deletion. As beliefs can be highly interconnected, there may be delete propagation and many candidates for removal, each of which may or may not be accepted in the new belief base. In more complex cases, P1 is conflicting with a combination P2…Pn, one belief Pi must be removed to preserve consistency, the question is which one to choose in order to minimize delete propagation explained above.
More complications arise when one believe is more believed that the other one. For example, "the President of the USA is Mr. George Bush" is more believed than "Mr. Bush is currently telling the truth". But knowledge base is usually considered a definite quality: either you know something or you don't.

2.2 Revision Vs Update

Consider a knowledge base represented by a theory γ of some logic, say propositional logic. We want to incorporate into γ a new fact, represented by a sentence μ of the same language. What should the resulting theory be? A growing body of work (Dalal [14], Katsuno and Mendelzon [21]) takes as a departure point the rationality postulates proposed by Alchourron, Gardenfors and Makinson [7]. These are rules that every adequate revision should be expected to satisfy. For example: The new fact μ must be a consequence of the revised knowledge base.

In this section, we argue that no such set of postulates will be adequate for every application. In particular, we make a fundamental distinction between two kinds of modifications to a knowledge base. The first one, *revision*, is used when we are obtaining new information about a static world. For example, we may be trying to diagnose a faulty circuit and want to incorporate into the knowledge base the results of successive tests, where newer results may contradict old ones. The AGM postulates describe only revision. The second type of modification, *update*, consists of bringing the knowledge base up to date when the world described by it changes. For example, most database updates are of this variety, e.g. "increase Joe's salary by 5%". Another example is the incorporation into the knowledge base of changes caused in the world by the actions of a robot (Winslett [13]). The AGM postulates had to be drastically modified to describe update. For this reason Katzuno and A. O. Mendelzon [21] provided a set of postulates (The KM postulates) designed to describe update.

The distinction between revision and update was made in the context of extended databases. It was distinguished between *change-recording updates* (which we call updates) and *knowledge-adding updates* (which we call revision).

The difference between the postulates for revision and for update can be explained intuitively as follows. Suppose knowledge base γ is to be revised with sentence μ. Revision methods that satisfy the AGM postulates are exactly those that select from the models of μ -- the models of μ are the possible worlds that satisfy μ -- those that are "closest" to the models of γ, where the notion of closeness is defined by an ordering relationship among models that satisfies certain conditions. The models selected determine the new theory which we denote by $\gamma^{o}\mu$. On the other hand, update methods select, for each model M of the knowledge base γ, the set of models of μ that are closest to M. The new theory describes the union of all such models. Suppose that γ has exactly two models, I and J. This means that there are two possible worlds described by the knowledge base. Suppose that μ describes exactly two worlds, K and L, and that K is "closer" to I than L and also that K is closer to I than L is to J. In this case K is selected for the new knowledge base, but L is not. Note the knowledge base has effectively forgotten that J used to be a possible world. In other words, the new fact μ has been used as evidence for the retroactive impossibility of J. As a result, not only do we refuse to have J as a model of the new knowledge base, but we also conclude that J should not have been in the old knowledge base to begin with.

If we are doing revisions, this behavior is rational. Since the real world has not changed, and μ has to be true in all the new possible worlds, we can forget about some of the old possible worlds on the grounds that they are too different from what we know to be the case. On the other hand, suppose we are doing updates. The models of γ are possible worlds; we think one of them is the real world, but we do not know which one.

Now the real world has changed. We examine each of the old possible worlds and find the minimal way of changing each one of them so that it becomes a model of μ. The fact that the real world has changed gives us no ground to conclude that some of the old worlds were actually not possible.

To illustrate this distinction between update and revision, let us consider two examples that are formally identical to the one above but have different intuitively desirable results. First, suppose that our knowledge base describes five objects A,B,C,D,E inside a room. There is a table in the room, and objects may be on or off the table. The sentence *a* means "object A is on the table," and similarly for sentences *b,c,d* and *e*. The knowledge base γ is the sentence:

$$(a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e) \vee (\neg a \wedge \neg b \wedge c \wedge d \wedge e).$$

That is, either object A is on the table by itself, or objects C, D and E are. This knowledge base has exactly two models I and J. We send a robot to the room, instructing it to achieve a situation in which all or none of the objects are on the table. This change can be modelled by incorporating the following sentence μ:

$$(a \wedge b \wedge c \wedge d \wedge e) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d \wedge \neg e).$$

Let us take Dalal [14] notion of "closeness" and the revision operator that results from it. According to this measure, the distance between two models is simply the number of propositional letters on which they differ. The models selected for the new knowledge base will be those models of μ which are at minimal distance from the models of γ. Now K, the model where nothing is at the table is at distance 1 from I (the model where A is on the table) and at distance 3 from J (the model where C, D, and E are). On the other hand L, the model where every object is on the table, is at distance 4 from I and 2 from J. Dalal's revision operator will therefore select K as the only model of the new knowledge base. But intuitively, it seems clear that this is incorrect. After the robot in done, all we know is that either all objects are on the table or all are off. There is no reason to conclude that they are all off, which is what revision does.

Consider now an example that is formally identical, but where the desired result is given by revision, not by update. Suppose the knowledge base describes the state of a five bits register which we read through noisy communication lines. Each of the propositional letters *a, b, c, d, e* now represents one bit. The state of the register is unchanging. Two different readings have been obtained: 10000 and 00111. By an independent analysis of the circuits that control the register, we learn that all bits must have the same value. That is, only 11111 and 00000 are possible patterns. Dalal's revision method tells us to keep 00000 as the new knowledge base. In other words, we conclude that 00111 is relatively too far from the possible patterns to be an acceptable result. It might be argued that it is better to forget the two readings in the knowledge base and keep both 00000 and 11111 as possible worlds. However, consider an example in which the register is thousands of bits long, the two readings agree on every bit except the first five, and the new fact only says that the first five must be all 0's or all 1's. It is clearly a waste of information now to discard the old knowledge base and just keep the new fact.

2.3 Change Properties

2.3.1 Preserving Consistency

The postulates defined by Alchourron, Gardenfors and Makinson and those defined by Katsuno and Mendelzon agree on the following property: applying a change on a consistent knowledge base must produce a new knowledge base that is also consistent.

However, the AGM postulates enforce also the following property: applying revision on a knowledge base (not necessarily consistent) must generate a consistent knowledgebase. In other words, It must be benefited from the arrival of new information in order to construct a consistent set of beliefs. The latter property can take a more general form:

Adding the notion of the degree of consistency, it can be enforced that the resulting knowledge base have the maximum degree or at least it must not decrease the current degree of consistency.

2.3.2 Importance of the new information

The postulates defined by Alchourron, Gardenfors and Makinson and those defined by Katsuno and Mendelzon agree that when the new information A does not follow from the knowlegdbase B, we have to modify B to take into consideration A.
Indeed, the first postulate of the AGM postulates (U1) and that of the KM postulates (K1) state that we must, in all circumstances, integrate the new information.

2.3.3 Minimal Change

Informally, revision or update must generate a new knowledgebase as little different as possible from the original one. To achieve this we have to introduce the notion of closeness between possible worlds. We will define later on the notion of distance between interpretations and this notion is a central device in update and revision operations. This notion has to work on minimizing distances between possible worlds.

2.4 AGM postulates

Alchourron, Gardenfors and Makinson introduced the following definitions and concepts. Three types of belief change can be characterize:

- **Expansion**: The new formula ϕ and all its consequences are inserted in the belief base K. No restrictions for this insertion. Expansion of K by ϕ is denoted by $K+\phi$.
- **Revision**: The new formula ϕ must be added to the set of beliefs K to produce a new consistent belief base. Thus, when ϕ is inconstant with K, we have to discard some formulae to incorporate the new one. The revision of K by ϕ is denoted by $K*\phi$.

- **Contraction**: The formula ϕ must be removed from the set of beliefs K. In some cases, we have to discard also other formulae. The contraction of K by ϕ is denoted by K-ϕ.

The following postulates for revision emphasize three main properties. First, the principle of consistency: a revision operation must produce a consistent belief base. Second, the principle of minimal change: alter the minimum number of beliefs. Finally, they stress the importance of the new information. The AGM postulates for revision are the following:

(G1) $K^*\phi$ is a set of beliefs.

(G2) $\phi \in K^*\phi$.

(G3) $K^*\phi \subseteq K+\phi$

(G4) If $\neg\phi \notin K$, then $K+\phi \subseteq K^*\phi$

(G5) $K^*\phi$ is inconsistent iff ϕ is contradictory.

(G6) If ϕ is equivalent to υ then $K^*\phi = K^*\upsilon$

(G7) $K^*(\phi\wedge\upsilon) \subseteq (K^*\phi)+\upsilon$

(G8) If $\neg\upsilon \notin K^*\phi$, then $(K^*\phi)+\upsilon \subseteq K * (\phi\wedge\upsilon)$

Postulate G1 states that revision preserves belief bases. (G2) emphasizes the introduction of the new information in the belief base. (G3) and (G4) indicate expansion: if ϕ is consistent with the belief base K, ϕ is simply incorporated. (G5) deals with preserving consistency. (G6) requires that revision be independent of the syntax of ϕ. (G7) and (G8) point out that if ϕ is inconsistent with K; K must be revised so as to minimize the extent of change.

2.5 KM Postulates

The KM postulates do not prescribe any particular update operator; they characterize a class of acceptable operators.

The update of a belief base represented by a formula B with a new formula A is denoted by: $B \lozenge A$. The KM postulates for update are the following:

(U1) $B \lozenge A \rightarrow A$

(U2) If $B \rightarrow A$ then $B \lozenge A \leftrightarrow B$

(U3) If B and A are consistent, then $B \lozenge A$ is consistent

(U4) If $B_1 \leftrightarrow B_2$ and $A_1 \leftrightarrow A_2$ then $B_1 \lozenge A_1 \leftrightarrow B_2 \lozenge A_2$

(U5) $(B \lozenge A) \wedge C \rightarrow B \lozenge (A \wedge C)$

(U6) If $(B \lozenge A_1) \rightarrow A2$ and $(B \lozenge A_2) \rightarrow A1$ then $B \lozenge A_1 \rightarrow B \lozenge A_2$

(U7) If B if complete[1] then $(B \lozenge A_1) \wedge (B \lozenge A_2) \rightarrow B \lozenge (A_1 \vee A_2)$

(U8) $(B_1 \vee B_2) \lozenge A \leftrightarrow (B_1 \lozenge A) \vee (B_2 \lozenge A)$

Postulate (U1) states that the new information must be incorporated, (U2) point out that if the new formula A is derivable from B, then the update will not modify B, however if A is consistent with B then $B \lozenge A$ can be different from $B \wedge A$. (U3) guarantees that the result of an update is consistent if the new formula is consistent, however if B is inconsistent, the result of the update is also inconsistent. (U7) is applicable only if the state of the world is completely known. (U8) infers that we perform the update by dealing with each possible world.

Important Remark: Revision is performed on the whole belief base, in other words, when a set of possible worlds representing a belief base is revised, it evolves globally towards the closest set of possible worlds which satisfy the new information. On the other hand, update is performed on each and every possible world trying to find the closest possible worlds verifying the new information.

CHAPTER III

Overview of belief revision

3.1 Introduction to belief revision

Belief revision provides mechanisms for changing repositories of information in the light of new information. These mechanisms can be used to incorporate new information into a knowledge base without compromising its integrity.

If the new information to be incorporated is consistent with the knowledge base then this process is straightforward, we simply add the new information. On the other hand, if the new information contradicts the knowledge base then great care must be exercised, otherwise the introduction of inconsistency will compromise the integrity of the knowledge base.

In order to incorporate new information which is inconsistent with the knowledge base, the system or revision agent must decide what information it is prepared to give up. Belief revision attempts to model decisions concerning modifications to a knowledge base. The guiding principles are that the changes should be both rational and minimal in some sense.

3.2 Example: The System Analyst

Consider the situation where a system analyst is assessing a software module that performs electronic commerce transactions called e-comm. His brief from management is to redesign inefficient and critical modules, and he subsequently learns that the module e-comm is inefficient and critical. He then concludes that e-comm must be redesigned. Shortly after coming to this conclusion, the analyst is informed by management that the module e-comm does not need to be redesigned. How should he go about modifying his knowledge? Not only must the inferred information "e-comm must be redesigned" be retracted, but other beliefs which conflict with the new information must also be removed. There appears to be several possible choices, for example:

(i) Retract the module "e-comm is inefficient"

(ii) Retract the module "e-comm is critical"

(iii) Retract "inefficient and critical modules must be redesigned"

(iv) Any combination of all three

The alternative adopted will depend upon the relative importance attributed to his background information. If the analyst has less confidence in the fact that "e-comm is inefficient" than both "e-comm is critical" and "inefficient and critical modules must be redesigned" he would probably prefer the solution that (i) offers, conversely, if he believes "inefficient and critical modules must be redesigned" with the least confidence then he might prefer (iii). If he is unable to decide which to prefer then he might give up all three to make way for the acceptance of the new information. From this intuitive discussion it is obvious that a preference ordering of our knowledge base can be used to resolve the nontrivial problem of choosing what information to surrender in order to avoid inconsistency. Another approach would be to use a plausibility relation over possible states of the world to help make a decision. For example, if the analyst believes that the most plausible world state is which the e-comm module does not require redesign it is not inefficient, then he could justify adopting (i) above. This is the general approach taken by constructions of belief revision operators.

3.3 More On Belief Change

The framework for belief revision adopted for belief change and more particularly belief revision is known as the AGM postulates. It is a formal framework for modeling ideal and rational changes to repositories of information under the principle of "minimal change". It provides mechanisms for modeling the coherent retraction and incorporation of information.

Technically, the framework models information as logical theories. It also models changes in information content as functions that take a theory (the current knowledge base) and a logical sentence (the new information) to another theory (the new knowledge base). There are several types of change functions: Contraction, withdrawal, expansion, and revision. Contraction and withdrawal functions model the retraction of information, while expansion and revision model various ways of incorporating information. All four functions are interrelated.

Change functions can be described either using rationality postulates, or using certain preference relations or selection functions. The rationality postulates are properties that we would expect rational change functions to satisfy and they characterize various classes of change functions. Moreover they may be satisfied by more than one function. An individual function can be singled out using logical information which help to make necessary choices concerning what information should be given up.

The belief revision framework restricts itself to modeling changes to logical theories that involve the addition and removal of facts. Therefore, we do not consider the possibility of explicitly modifying individual facts as a primitive operation. For example we do not consider transforming "inefficient and critical modules must be redesigned" to "inefficient and critical modules except e-comm must be redesigned". Modifying individual facts is often seen in machine learning, and can be modeled in the belief revision framework by observing that "inefficient and critical modules must be redesigned" entails "inefficient and critical modules except e-comm must be redesigned"; therefore, removing "inefficient and critical modules must be redesigned" and retaining "inefficient and critical modules except e-comm must be redesigned" achieves the same result.

Belief revision models rational modifications to knowledge bases guided by the principle of minimal change. Unfortunately, the notion of rationality and minimality, in the sense we would like to capture, defy explicit definition. Intuitively, by rational we mean things like: The thinker realizes that inconsistency is problematical and thus actively seeks to avoid it, and that given our example, it is not sufficient to retract only the fact that "e-comm does not need to be redesigned" because it is derivable from the remaining information. The principle of minimal change says that, as much information should be conserved as is possible in accordance with an underlying preference relation. The underlying preference relation is used to capture the information content of the knowledge base, the reasoning agent's commitment to this information, and how the information should behave under change.

It has often been incorrectly argued that the choices (i), (ii) and (iii) are more minimal than (iv), because only one basic fact is discarded as opposed to two or more. The problem with that argument is that the interdependencies among our beliefs might force us to discard more than the minimal number of beliefs. The web of causal interdependencies is enmeshed in the preference relation, and cardinality measures are not the only, nor necessarily the most appropriate, when it comes to measuring the magnitude of change. For instance, to take our example a little further, it might have been that the only reason for believing that "inefficient and critical modules must be redesigned" is that "e-comm is inefficient and critical, so if we contract "the modules must be redesigned" then it should be permissible to retract "e-comm is inefficient and critical at the same time. Clearly, cardinality is not the only allowable measure of change. Sometimes the most rational response is to forfeit more than the minimal number of beliefs. For instance, it may be better to remove several weakly held beliefs rather than a single strongly held belief.

CHAPTER IV

Update Operators

4.1 Introduction To The Language

The language is built from a possibly infinite set of atoms ATM = {p, q, r, …} with the classical connectives $\wedge, \vee, \neg$,TRUE,FALSE. L, L_1, … denote literals, LIT is the set of all literals. c, c_1, …denotes clauses, in other words disjunctions of literals.
A, B, C, … denote formulas. We confuse belief bases (that are finite sets) with the conjunction of their elements. As far as possible we shall use B, B_1, … for belief bases, and A, A_1, … for inputs (formulas to be added).

We stipulate that $\neg$ binds stronger than $\wedge$ and $\vee$, which bind stronger than the other connectives. We denote by atm(A) the set of atoms appearing in the formula A. For example atm($p \wedge (p \vee q)$) = {p, q}, atm(TRUE) = atm(FALSE) = NULL.

An atom p occurring in A is redundant if there exist an equivalent formula A' such as p does not occur. Hence p's truth value does not affect the truth value of A. For example p is redundant in $q \wedge (q \vee p)$. In order to establish that an atom is redundant, we can use the following facts:

FACT 1: An atom is redundant in a formula A if and only if
A[p:=TRUE] $\leftrightarrow$A[p:=FALSE]

FACT 2: To check redundancy of an atom is a coNP-complete problem.

Fact 1 shows us that it is in coNP. The other way round, we can polynomially transform the problem of theoremhood of a formula A in the propositional calculus into that of redundancy of p in $A[p_1\backslash p] \wedge \ldots \wedge A[p_n\backslash p]$, where atm(A) = $\{p_1, \ldots, p_n\}$ and

$P \notin atm(A)$.

Fact 1 can be turned into an algorithm to get rid of redundant atoms. We note $A\downarrow$ the formula obtained by eliminating all redundant atoms of A. For example: $(q \wedge (q \vee p))\downarrow = q$, $(p \vee \neg p)\downarrow = TRUE$, $((p \wedge \neg p) \wedge q)\downarrow = FALSE$.

Interpretations are sets of atoms. We shall often represent an interpretation by a maximally consistent set of literals. 2^{ATM} is the set of all interpretations (which might be viewed as possible worlds). Given a formula A, we note $||A||$ the set of interpretations where A is true. A is valid ("valid" means that it is always true) if $||A|| = 2^{ATM}$. In other words $||A||$ is the set of models of A.

The notion of distance between interpretations is a central device in update operations. The distance between w and v is the set of atoms whose truth value differs.

$$DIST(w,v) = (w/v) \cup (v/w)$$
$$=\{p : w \in ||p|| \text{ and } v \notin ||p||\} \cup \{p : w \notin ||p|| \text{ and } v \in ||p||\}$$

For example suppose $ATM = \{p, q, r\}$, $w = \{p, q, \neg r\}$ and $v = \{p, \neg q, r\}$.
Then $DIST(w,v) = \{q, r\}$.

4.2 Update Definition

The update operator is a binary function denoted by $\Diamond$ who takes as argument a belief base B and an input A and outputs a new belief base $B' = B \Diamond A$.
As explained in the previous chapter, we update belief bases by updating each and every possible world.

Let $w \in ||B||$, and $w \cdot A$ the set of representations resulting from the update of w by A. Therefore the set of models of $B \lozenge A$ is obtained by performing the union of all updated models of B. Formally:

$$||B \lozenge A|| = \cup_{w \in ||B||} w \cdot A$$

Each operation defined takes as arguments an interpretation and an input, and outputs a set of interpretations. There are two families of approaches: The first one aims at minimizing distances between worlds so to achieve minimal change, the second constrains distances to be in some set of exceptions computed from the input.

4.3 Possible Models Approach (PMA)

PMA belongs to the first family of operators, which aims at minimizing distances between interpretations. It was introduce by Winslett [13] in the context of reasoning about action and change.

Informally the PMA first finds the models of the input A, then compute the distance *DIST* between those models and w (recall that w is an interpretation of B). The result is the union of the models of A whose distance with w is minimal.

Formally, let A be the formula representing the input. Then the update of w by $||A||$ is defined as:

$$w \cdot_{pma} ||A|| = \{u \in ||A||: \forall u' \in ||A||, DIST(w,u') \not\subset DIST(w,u)\}.$$

In other terms, the set $w \cdot_{pma} ||A||$ contains all those elements of $||A||$ that are minimal with respect to the closeness ordering $\leq_w$, where $\leq_w$ is defined by

$$u \leq_w v \text{ iff } DIST(w,u) \subseteq DIST(w,v)$$

For example, $w=\{\neg p, \neg q\}$ and $A= p \vee q$. The models of A are $||A||= \{\{p,q\}, \{p,\neg q\}, \{\neg p,q\}\}$. We calculate the distances as follows: $DIST(w,\{p,q\})=\{p,q\}$, $DIST(w,\{p,\neg q\}) =\{p\}$, $DIST(w,\{\neg p,q\})=\{q\}$. We notice that $\{p,\neg q\}$, $\{\neg p,q\}$ have minimal distance with w, then

$$w \cdot_{pma} ||p \vee q||=\{\{p,\neg q\}, \{\neg p,q\}\}.$$

We notice that PMA interprets the inclusive disjunction $p \vee q$ as if it was an exclusive disjunction $p \oplus q$.

4.4 FORBUS: numeric minimal change

The operator proposed by Forbus is stronger than the PMA. It is the update counterpart of Dalal's semantics for belief revision [14]. There, the semantical update operation is defined not from the distance between interpretation DIST (w,u), but from its cardinality card(DIST (w,u)):

$$w \cdot_{Forbus} \|A\| = \{u \in \|A\|, \forall v \in \|A\|: card(DIST(w,u)) \leq card(DIST(w,v))\}.$$

The resulting set of interpretations contains those models of A that are minimal with respect to the closeness ordering $\leq_w$, where $\leq_w$ is defined by

$$u \leq_w v \text{ iff } card(DIST(w,u)) \leq card(DIST(w,v))$$

For example, $w=\{\neg p, \neg q, \neg r\}$ and $A = (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg r)$.
The models of A are $\|A\| = \{\{p, \neg q, \neg r\}, \{\neg p, q, r\}, \{p, q, r\}, \{p, q, \neg r\}\}$.
The cardinalities of the distances between w and the models of A are respectively (1, 2, 3, 2). Then we get:

$$w \cdot_{Forbus} \|A\| = \{\{p, \neg q, \neg r\}\}$$

4.5 MCD: going beyond the PMA

Both PMA and FORBUS have been criticized for their handling of disjunctive input. It has been argued that input such as $p \vee q$ is interpreted as if it was an exclusive disjunction $p \oplus q$. Motivated by that, MCD (Minimal Change with Maximal Disjunctive inclusion) was introduced by Zhang and Foo [25]. MCD is built on top of PMA.

Let U be the set of models of the input A, and let w be some interpretation of the belief base. Let $V = w \cdot_{pma} U$ be the set of models resulting from PMA-updating w with U, and let $S = 2^V$. For $s \in S$, the 'cone' C(s) is the set of those interpretations in U that are beyond all elements of s with respect to the PMA closeness ordering $\leq_w$:

$$C(s) = \{u \in U: \forall v \in s, v \leq_w u\}$$

The set $\{C(s)\}: s \in S\}$ is a covering of $U:U = \cup\{C(s): s \in S\}$. The key idea is that PMA-minimization in that set allows to obtain more interpretations than $w \cdot_{pma} U$ would give us:

$$w \cdot_{mcd} U = \cup_{s \in S} (w \cdot_{pma} C(s)).$$

For example, let $w = \{\neg p, \neg q, \neg r\}$ and $A = (p \vee q)$, we have

$V = w \cdot_{pma} \|p \vee q\| = \{w_1, w_2\}$, where $w_1 = \{p, \neg q\}$ and $w_2 = \{\neg p, q\}$.

Second, $S = \{\varnothing, \{w_1\}, \{w_2\}, \{w_1, w_2\}\}$. We construct $C(\varnothing) = \{w_1, w_2, \{p,q\}\}$,

$C(\{w_1\}) = \{w_1, \{p,q\}\}$, $C(\{w_2\}) = \{w_2, \{p,q\}\}$, and $C(\{w_1, w_2\}) = \{\{p,q\}\}$.

Finally $w \cdot_{mcd} \|p \vee q\| = w \cdot_{pma} \{w_1, w_2, \{p \vee q\}\} \cup w \cdot_{pma} \{w_1, \{p,q\}\} \cup$

$w \cdot_{pma} \{w_2, \{p,q\}\} \cup w \cdot_{pma} \{\{p,q\}\} = \{w_1, w_2\} \cup \{w_1\} \cup \{w_2\} \cup \{p,q\}$

$= \{w_1, w_2\} \cup \{p,q\}$.

4.6 Winslett Standard Semantic (WSS)

WSS belongs to the second family of operators, which constrains distances to be in some set of interpretations. Because the change made by this operator is not minimal and sometimes produce results close to the revision, WSS was considered to be the weakest operation deserving the name of update. We are interested in defining this operator because it handles disjunctions correctly, and its definition is simple.

Informally, WSS is the set of those models of A which preserve the truth value of atoms not occurring in A. Formally:

$$w \cdot_{WSS} A = \{u \in \|A\| : DIST(w, u) \subseteq atm(A)\}$$

For example let $w=\{\neg p, \neg q, \neg r\}$ and $A=p\vee q$. then $w \cdot_{WSS} A=\|(p\vee q)\wedge\neg r\|$.

We notice that WSS handles disjunction in an intuitive way.

CHAPTER V

The Weak Revision Operator

5.1 The Need for a New Revision Operator

Revision operators always give priority to new information over the old. The AGM postulate state that the new information must always be integrated into the belief base. However, in the real world, this is not the case. Consider the following example:

5.2 Example: A Detective Story:

A crime had been committed, a detective is assigned the task of finding out what happened. There happen to be a witness. So the witness tells his story which is the following:

1) Mr. "A" went to the crime building at 5 P.M.
2) Mr. "B" went to the crime building at 6 P.M.
3) Mr. "C" never went to the crime building

So the detective takes this story as a hypothesis of what could have happened. Later on in the investigation, another witness shows up. So the second witness –We shall call him witness 2 – also tells his version of the story which is the following:

1) Mr. "A" and Mr. "B" went together to the crime building at 5 P.M.
2) Mr. "C" went to the crime building at 6 P.M.

When comparing the two stories, the detective finds out that there are some parts where the witnesses agree and parts where the witnesses disagree. So the detective will believe the parts where the witnesses agree and doubt the part where the witnesses disagree. He will have no sure information on the area of disagreement.

5.3 Reflection on the example:

If we were to apply the revision technique to solve this problem and store it in a disjunctive database we will believe only witness 2 because revisions will always believe the newest change. However, in this example, the detective doesn't give more weight for the last witness as we do in traditional revisions. Instead he gives the same weight to both witnesses. For this reason, we need to create a new revision operator. This revision operator will help the detective solve his investigation. We shall call this new revision operator a "weak revision".

Let us summarize what we are looking for: A revision operation is a function (noted "$\leftarrow$") mapping a belief base B and an input A to a new belief base $B \leftarrow A$.

First, we want that when we face two contradicting facts we doubt both. For example, let p and q be propositions, let the belief base $B = \{\neg p, q\}$ and let the revision $A = \{p\}$, then $B \leftarrow A = \{q\}$. This is to say that we doubt both the old belief base and the input A.

Second, we want to believe non-contradicting facts, for example, let p and q be propositions, let the belief base $B = \{q\}$ and let the revision $A = \{p\}$, then $B \leftarrow A = \{p, q\}$. Also, let p and q be propositions, let the belief base $B = \{p, q\}$ and let the revision $A = \{p\}$, then $B \leftarrow A = \{p, q\}$.

We shall use the "$\vdash$" symbol for "infer", for example: "$B \vdash \neg A$" means that the knowledge base B infers NOT A. We shall also use the "$\nvdash$" symbol for "does not infer", for example: "$B \nvdash \neg A$" means that B does not infer NOT A.

5.4 General behavior of the weak revision

If we want to fulfill the above requirements, the weak revision would have the following behavior. Let B be a belief base, and A be a revision to B, there are two cases: Either "$B \vdash \neg A$", or "$B \nvdash \neg A$".

Case 1:

if

$B \vdash \neg A$

Then

$B \leftarrow A \nvdash \neg A$

$B \leftarrow A \nvdash A$

"$B \vdash \neg A$" means that the knowledge base infers what is opposite to A. In other words, if there is a contradiction, the knowledge base must doubt both facts, the fact which was previously inserted into the belief base which is $\neg A$ and the input A.

Case 2:

if

$B \nvdash \neg A$

Then

$B \leftarrow A \vdash A$

"$B \nvdash \neg A$" means that the knowledge base does not infer what is opposite to A. In other words, if there is no contradiction, we believe the input A.

5.5 Definition of the weak revision:

We shall denote the weak revision by "*".

Let D be the minimal distance between any model of ||B|| with any model of ||A||

$$D = \bigcup_{\forall w \in \|B\| \text{ and } \forall u \in \|A\|} \text{Min DIST}(u,w)$$

$$B*A = \{w,u : \text{DIST}(u,w) \in D\}$$

For example, let B = {p,¬q} and let A = {p}. The possible worlds for B and A are: {{p,¬q}} and {{p,q},{p,¬q}} respectively

We notice that one model of B {p,¬q} is the same as one model of A. Therefore, the distance between these two models DIST = {} or the empty set. And the resulting model for D is {p,¬q}. So the set of models ||B*A|| = {{p,¬q}}

⇒ The belief base B*A = {p,¬q}

Example 2:

Let B = {p,¬q} and A = {q}

The possible worlds for B and A are

{{p,¬q}} and {{p, q},{¬p, q}} respectively.

We notice that the two closest models are {p,¬q} from ||B|| and {p, q} from ||A||. Therefore, in this case the minimal distance DIST = {q}. So the resulting models for ||B*A|| are {p,¬q} and {p, q}

⇒ The belief base B*A = {p}.

Example 3:

Let B = {p∨q,¬r} and A = {¬p, r}

The possible worlds for B and A are

{{p, q, ¬r},{¬p, q, ¬r},{p, ¬q, ¬r}}, and {{¬p, q, r},{¬p, ¬q, r}} respectively.

We notice that the two closest models are $\{\neg p, q, \neg r\}$ from $||B||$ and $\{\neg p, q, r\}$ from $||A||$. Therefore, in this case the minimal distance DIST = $\{r\}$. So the set of models $||B*A||$ is $\{\{\neg p, q, \neg r\},\{\neg p, q, r\}\}$

$\Rightarrow$ The belief base $B*A = \{\neg p, q\}$.

5.6 Proof of correctness

In this section we shall prove that the general behavior of the weak revision corresponds to its definition.

Case 1:

If $B \vdash \neg A$

We need to prove that

1) $B \leftarrow A \nvdash A$

2) $B \leftarrow A \nvdash \neg A$

Proof of 1:

If $B \vdash \neg A$

Then $||B|| \subseteq ||\neg A||$

Therefore $\forall v \in ||B|| \quad v \vdash \neg A$

But $||B*A|| = \{\forall v \in ||B||, \forall u \in ||A||: DIST(v,u) \in D\}$

Then $\exists w \in ||B \leftarrow A||$ such that $w \vdash \neg A$

Therefore

$B \leftarrow A \nvdash A$

Proof of 2:

$$D = \bigcup_{w \in \|B\| \text{ and } \forall u \in \|A\|} \text{Min DIST}(u,w)$$

Since

$B \vdash \neg A$

Then $\exists u', w'$ such that $\text{DIST}(w', u') \in D$

It follows that D is not empty, and

$u' \vdash A \in \|B \leftarrow A\|$

Therefore

$B \leftarrow A \nvdash \neg A$

Case 2:

If $B \nvdash \neg A$

We need to prove that

$B \leftarrow A \vdash A$

Proof:

Since

$B \nvdash \neg A$

Then $\exists u \subseteq \|B\|$ such that

$u \vdash A$

This implies that

$$D = \bigcup_{w \in \|B\| \text{ and } \forall u \in \|A\|} \text{Min DIST}(u,w) = \{\{\}\}$$

Since D contains only the empty set, then

$\|B \leftarrow A\| \subseteq \|A\|$

and

$B \leftarrow A \vdash A$

5.7 The Weak Revision Versus The AGM Postulates

In this section we shall provide a comparative study of the weak revision in regard to the AGM postulates:

- Postulate (G1): $K * \phi$ is a set of beliefs:

  This postulate is respected by the weak revision. It states that revision must preserve the belief base

- Postulate (G2): $\phi \in K * \phi$

  This postulate infers that the new information must be inserted into the belief base. We have shown that this postulate does not reflect the real world.

  The postulate (G2) is not satisfied by the weak revision, as a counter example:

  Let $k = \{p, q\}$ and $\phi = \{\neg p\}$, we will have $K * \phi = \{q\}$

  $\Rightarrow \phi \notin K * \phi$

- Postulate (G3): $K * \phi \subseteq K + \phi$

  This postulate assumes that the models of the revision must be included in the models of the expansion. Postulate (G3) is respected by the weak revision.

  Proof:

  <u>Case 1:</u>

  If $K \vdash \neg\phi$

  Then $K + \neg\phi = F$

  $\Rightarrow K * \phi \subseteq F$

  $\Rightarrow K * \phi \subseteq K + \phi$

Case 2:

If K $\nvdash \neg\phi$

$\Rightarrow$ K* ϕ infers ϕ

Since K does not infer $\neg\phi$ then $||K|| \cap ||\phi|| \neq \varnothing$

$\Rightarrow \exists w \in ||K||, \exists u \in ||\phi||$ such that w = u

$\Rightarrow$ w and u are chosen to be in $||K^*\phi||$

$\Rightarrow$ DIST(w,u) = {}

$\Rightarrow$ D = {{}}

$\Rightarrow$ All models of K kept in K*ϕ are also models of ϕ

$\Rightarrow u \in ||K||$

$\Rightarrow K * \phi \subseteq K + \phi$

- Postulate (G4): If $\neg\phi \notin K$, then $K + \phi \subseteq K * \phi$

  This postulate indicates that if ϕ is consistent with the belief base K, we simply incorporate ϕ. It is respected by the weak revision.

  Proof:

  Since $\neg\phi \notin K$ then

  K $\nvdash \neg\phi$

  $\Rightarrow$ K + ϕ infers K * ϕ

  $\Rightarrow$ D = {{}}

  $\Rightarrow \forall u \in ||K+\phi||, u \in ||K|| \cap ||\phi||$

  $\Rightarrow u \,.\, ||\phi|| = \{u\}$

  $\Rightarrow u \in ||K^*\phi||$

  $\Rightarrow K + \phi \subseteq K * \phi$

- Postulate (G5): K * ϕ is inconsistent iff ϕ is contradictory

  This postulate deals with preserving consistency. It is respected by the weak revision.

  Proof:

If ϕ is not contradictory, then $K*\phi$ is consistent (Postulate (G1)).

If ϕ is contradictory, then $K*\phi$ will include models from ϕ, in follows that $K*\phi$ is contradictory.

- Postulate (G6): if ϕ is equivalent to v then $K * \phi = K * v$

  Postulate (G6) indicates that revision must be independent from the syntax of ϕ, and it is respected by the weak revision.

  Proof:

  If ϕ is equivalent to v then $\|\phi\| = \|v\|$

  $\Rightarrow K * \phi = K * v$

5.8 The postulates of the weak revision

If we put together all the postulates respected by the weak revision we will get the following:

- Postulate 1): $K * \phi$ is a set of beliefs:
- Postulate 2) if $K \vdash \neg\phi$

  Then $K * \phi \nvdash \neg\phi$ and $K * \phi \nvdash \phi$
- Postulate 3) If $K \nvdash \neg\phi$

  Then $K * \phi \vdash \phi$
- Postulate 4): $K * \phi \subseteq K + \phi$
- Postulate 5): If $\neg\phi \notin K$, then $K + \phi \subseteq K * \phi$
- Postulate 6): $K * \phi$ is inconsistent iff ϕ is contradictory
- Postulate 7): if ϕ is equivalent to v then $K * \phi = K * v$

5.9 The benefits of the weak revision

The weak revision operator will allow belief bases to doubt contradicting facts. This approach is the closest to the real world. In the real world, when we face two contradicting facts we do not select one of them to belief and discard the other. What will happen in reality is that we will look for further proofs and belief the fact the proofs supports. The weak revision applies this intuitive technique. It allows us to doubt the previous fact and the new fact. The belief base will then believe the new proof it is revised with. This is an improvement on traditional revision techniques where the new information is selected to be true without the need for a proof even when it contradicts what is in the belief base.

Conclusion

In this thesis we have shown how new information in the real world is not always believed, especially when it contradicts previous information. We have introduced a new type of revision where the input information is not always accepted to be inserted into the knowledge base. We have defined postulates for such behavior. We have proved that the new revision operator satisfies the behavior postulates. We have studied our operator with respect to the AGM postulates. All are satisfied except postulate (G2) which infers that new information must be inserted into the knowledge base and contradicts our behavior postulates. And finally, we have set the postulates that the weak revision respects.

To conclude, belief change is still an area of study. Various artificial intelligence theories have been proposed by researchers and they are still being evaluated today. The weak revision represents a simple and efficient method to handle belief change among other revision and update operators.

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